Scaling of the Superfluid Fraction and T_c of ^{3}He in Aerogel

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We have investigated the superfluid transition of ^{3}He in different samples of silica aerogel. By comparing new measurements on a 99.5% sample with previous observations on the behavior of ^{3}He in 98% porous aerogel, we have found evidence for a scaling of the transition temperature and superfluid density of ^{3}He to the correlation length of the aerogel.

The properties of bulk ^{3}He are well understood. The extreme purity of ^{3}He at low temperatures makes it an ideal system to study the agreement between theoretical and experimental results on nonconventional Cooper pairing in the absence of disorder. Disorder plays a crucial role in suppressing the pairing interaction in high T_c superconductors, the other well established non-s-wave paired system. The superfluid transition of ^{3}He confined to a sample of very porous silica aerogel was first reported four years ago [1,2]. The aerogel provides a structural disorder background to the liquid. ^{3}He is compressible, and the density can be continuously tuned by $\approx 30\%$ while maintaining a fixed disorder. The ^{3}He zero temperature coherence length ξ_0, defined as $\xi_0 = \hbar v_f / k_B T_c$, varies from 180 Å to over 700 Å as a function of density. Because the Cooper pairs in ^{3}He form in a p-wave state, quasiparticle scattering from the aerogel strands is pair breaking [3]. Thus the ^{3}He-in-aerogel system is well suited to the exploration of the effect of impurity scattering and disorder on the superfluid transition and phase diagram.

The superfluidity of ^{3}He in silica aerogel has been studied using torsional oscillators [1,4–6], NMR [2,6–9], and sound propagation [10,11] techniques. These measurements show that both the superfluid transition temperature (T_c) and superfluid density (ρ_s) of the ^{3}He are suppressed by the disorder, but that the transition remains sharp [1]. This suppression is sensitive to both the density and the microstructure of the aerogel sample.

The simplest model for the effect of impurity scattering on the ^{3}He superfluid transition is the homogeneous scattering model (HSM) which is based on the Abrikosov-Gorkov model for a superconductor with magnetic impurities that induce pair breaking via spin flip scattering [12]. This mechanism is similar to that of diffuse scattering of Cooper paired ^{3}He from a surface [13] and is unable to explain the observed behavior. Specifically, the observed suppression of the superfluid density is much greater than predicted by this model. More sophisticated models, such as the isotropic inhomogeneous scattering model (IISM) [14–16] proposed by Thuneberg and co-workers, are able to quantitatively predict the superfluid transition temperature of ^{3}He in aerogel (for small suppressions) and have

had success at qualitatively explaining the observed superfluid densities.

In this Letter we present data from several different experiments on ^{3}He in aerogel, including new results on ^{3}He confined to a 99.5% porosity sample. This sample is a factor of 4 more dilute than any previously investigated [17] and is crucial for understanding the evolution from bulk ^{3}He to a regime where impurity scattering dominates. In comparing these different samples we find evidence that the relation between superfluid density and the superfluid transition temperature of ^{3}He in aerogel can be scaled, independent of the aerogel sample. This is significant because both of these quantities are individually sensitive to the microstructure of the aerogel and vary greatly from sample to sample. We also present evidence that the suppression of T_c can be related to the correlation length (ξ_a) of the aerogel sample.

The aerogels used in the experiments discussed in this Letter were grown under basic conditions [18–21]. Under these conditions gelation is the result of diffusion-limited aggregation of small (≈ 30 Å diameter) primary silica particles. The aerogels are characterized by a fractal dimension (D_f) related to the real space correlations and a long length scale cutoff to these correlations (ξ_a) above which the sample appears homogeneous. The fractal exponent depends only on the gelation process, while the cutoff length is also dependent on the average density. Simulations based on the diffusion-limited cluster-cluster aggregation (DLCA) algorithm predict that the fractal exponent should lie between 1.7 and 1.9, which is in good agreement with small-angle x-ray scattering (SAXS) measurements (Table I). We note that ξ_a in the most dilute sample, D, could not be inferred from the data as the SAXS did not extend to sufficiently small q.

TABLE I. Parameters that characterize aerogel samples.

	Porosity	D_f	ξ_a
Sample A	0.98	1.9	1300 Å
Sample B	0.98	1.8	≈ 900 Å
Sample C	0.98	1.8	840 Å
Sample D	0.995	1.7	≥ 2000 Å

Samples *A* and *C* have the same density but were made under different conditions and have slightly different fractal dimension and significantly different ξ_a. Both samples *A* and *C* have been studied with SAXS. Sample *B* was made under conditions very similar to sample *C*. We do not have direct information on its microstructure from SAXS but assume here from T_c suppression that samples *B* and *C* are essentially identical. The correlation length for sample *D* is within the range obtained from simulations based on the DLCA algorithm and is consistent with SAXS on a 99.4% sample [22] ($\xi_a \geq 2000$ Å). For a more extensive discussion the reader is referred to Ref. [4].

The 99.5% porosity aerogel used for our experiment was grown inside the (on average) 100 μm large pores of a coarse silver sinter. Previous torsional oscillator experiments have been affected by the presence of spurious resonances resulting from composite modes of ^{3}He and aerogel whose frequency crosses the resonant frequency of the cell [1,6]. The strength of these resonances grow as the porosity of the aerogel sample increases, affecting the quality of data [1,5,6]. In our cell the aerogel is clamped to the silver sinter, and the effect of spurious resonances is strongly reduced.

We operated our torsional oscillator in self-resonant mode near 483 Hz. The temperature in the cell was measured using a lanthanum diluted cerous magnesium nitrate ac susceptibility thermometer, thermally connected to the sample through a shared reservoir of ^{3}He. Data were collected while the temperature increased at a rate of 20 μK per hour. The period shift of the oscillator as the superfluid ^{3}He decoupled from the torsion head provided both the transition temperature of the ^{3}He in aerogel and the superfluid density.

Figure 1 shows the superfluid transition temperature for several different aerogel samples. Three of these samples (*A*, *B*, and *C*) have a nominal porosity of 98%; the fourth one (*D*) is our 99.5% aerogel. All of these measurements were done by monitoring the period shift in torsional oscillators filled with the aerogel and ^{3}He. The difference in transition temperature between samples *A*, *B*, and *C* arises from differences in the microstructure of the aerogels (see Table I and Ref. [4]). The relative suppression of the transition temperature $(1 - T_c/T_{c0})$ (with T_{c0} the transition temperature in bulk ^{3}He) is larger at lower pressures (larger ξ_0) than at higher pressures (smaller ξ_0). In view of the fact that all the aerogels used in these experiments have a similar fractal dimension and primary particle size, one would expect them to be mutually self-similar on length scales shorter than the fractal cutoff length. Thus for temperature dependent ^{3}He coherence lengths $\xi(T)$ shorter than ξ_a, the ratio of density of silica sampled at two different $\xi(T)$ is independent of aerogel density. Experimentally one observes a strong dependence of the suppression of T_c on the aerogel density.

In Fig. 2 we show the dependence of the relative suppression of T_c as a function of ξ_0. At short coherence
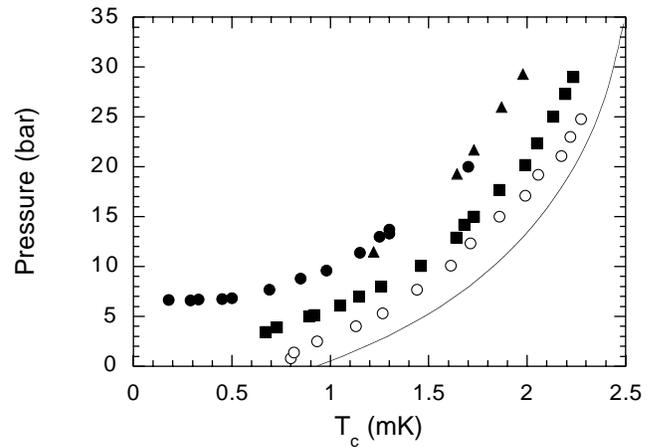
FIG. 1. The pressure is plotted versus T_c for different aerogel samples. The legend is as follows: filled squares are cell *A* (98% aerogel) [1], filled triangles are cell *B* (98% aerogel) [4], filled circles are for cell *C* (98% aerogel) [5], and empty circles are cell *D* (99.5% aerogel). The solid line is the superfluid transition curve for bulk ^{3}He.

lengths, the relative suppression is small for all samples. However, when the pressure is reduced, increasing ξ_0, this suppression shows a marked dependence on the microstructure of the aerogel. The transition temperature shows an evolution from a strong impurity scattering regime when the ^{3}He is confined to a 98% porosity aerogel sample towards the behavior of bulk ^{3}He in the 99.5% sample.

Motivated by Fig. 2 we plot the relative suppression of T_c against ξ_0 scaled by the aerogel correlation length ξ_a. The aerogel-limited mean free path (l_g) would be another natural choice to compare against ξ_0. However, this length scale has not been independently measured. If we use the HSM to determine l_g from T_c/T_{c0}, the values of l_g
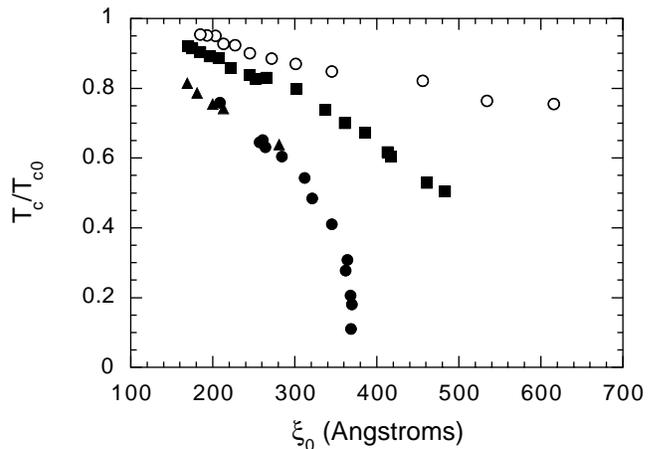
FIG. 2. The superfluid transition temperature for ^{3}He in aerogel, scaled by the bulk transition temperature, is plotted for several different samples as a function of ξ_0. The symbols are the same as shown in Fig. 1.

show a strong pressure dependence [23]. Figure 3 shows that the scaled transition temperature depends solely on the ratio of ξ_0 to ξ_a *independent* of the aerogel density. The error bars for sample D result from the high and low estimates (2000 Å and 3000 Å, respectively) for the correlation length for a 99.5% porosity sample from DLCA simulations.

We determined the temperature-dependent superfluid density of ^{3}He in 99.5% aerogel using the shift in resonant frequency of our torsional oscillator upon warming. A small, temperature-dependent contribution due to bulk ^{3}He in the cell was subtracted. The remaining shift was scaled by the period change due to filling the cell with ^{3}He at 50 mK and by the tortuosity (measured with ^{3}He) to obtain the superfluid density. Figure 4 shows the bare superfluid density (ρ_s^b/ρ) plotted versus the reduced temperature T/T_c for different aerogel samples and pressures. The bare superfluid density is obtained from ρ_s by stripping away the Fermi liquid factor [24] according to

$$\frac{\rho_s^b}{\rho} = \frac{(1 + \frac{1}{3}F_1)\frac{\rho_s}{\rho}}{1 + \frac{1}{3}F_1\frac{\rho_s}{\rho}} \qquad (1)$$

and is equivalent to $1 - Y(T)$, where $Y(T)$ is the temperature-dependent Yosida function for bulk ^{3}He.

As with the transition temperature T_c, the ρ_s^b for these two different samples are similar at high pressures and are both close to the bulk value. At the lowest pressure, there is a factor of 5 difference in ρ_s^b/ρ at the same reduced temperature between the 99.5% sample and the 98% sample; ρ_s^b/ρ is more strongly suppressed by the aerogel than T_c. There is also a large difference between the suppression factors of T_c and ρ_s^b in the same aerogel sample. This large suppression in ρ_s with respect to T_c is consistent with measurements made on other samples [1,4] and cannot be explained with homogeneous scattering models for ^{3}He in aerogel [14].
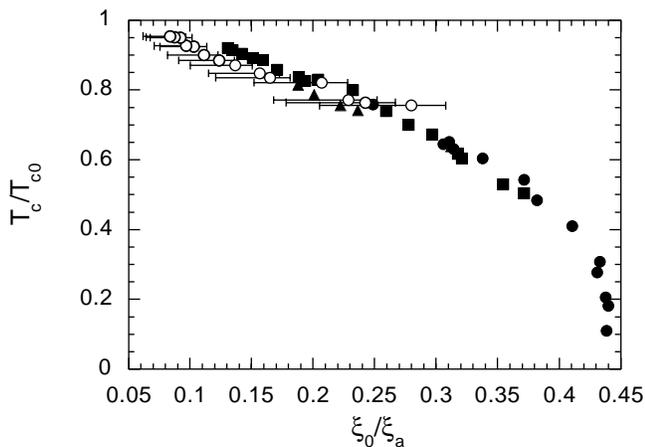
In order to better understand the superfluid transition of ^{3}He in aerogel, in Fig. 5 we plot ρ_s^b/ρ_s at $0.8T_c$ against $(T_c/T_{c0})^2$ for several aerogel samples. The error bars shown for cell B arise from a spurious resonance that introduces uncertainty in the determination of ρ_s. As in Fig. 3, the data collapse onto a common curve. The dashed line is the prediction for a homogeneous scattering model based on the Abrikosov-Gorkov equation [14]. This plot compares aerogels with *different* densities—there is a factor of 4 difference in the average impurity density between the 98% samples and the 99.5% aerogel sample. Furthermore, the coherence length of the Cooper pairs varies from 180 to 600 Å over this data, yet the very strong pressure dependence shown in Figs. 2 and 4 has been factored out in this plot.

The correlations in the aerogel will affect the suppression of T_c and the evolution of ρ_s^b relative to a homogeneous disorder, but this suppression apparently depends only on the correlation length of the sample and possibly the fractal exponent. The steep slope of the data in Fig. 5 is evidence that the fractal nature of the aerogel plays an important role in the development of ρ_s [25]. Since all of these samples were base catalyzed, the behavior of ^{3}He in each aerogel is determined mainly by ξ_a. As long as ξ_0 is much less than ξ_a, the disorder sampled by the ensemble of Cooper pairs should be insensitive to changes in the temperature-dependent coherence length $\xi(T)$, until $\xi(T) \approx \xi_a$. That is, the system has a conformal symmetry normally absent in disordered systems. The evidence of this one-parameter scaling is displayed in Fig. 5; ρ_s^b is a function of T_c/T_{c0} (or equivalently ξ_0/ξ_a) only. This behavior is reminiscent of the compilation of data from disordered high T_c materials by Franz *et al.* [26] with the exception that the relative suppression of T_c/T_{c0} and ρ_s/ρ_{s0} in the high T_c materials do not evidence this scaling behavior, presumably because the impurities are not fractally correlated.

FIG. 3. Relative suppression of T_c versus ξ_0/ξ_a. The correlation length for the 99.5% sample is taken to be 2200 Å with the error bars corresponding to the high and low values of 3000 Å and 2000 Å, respectively.
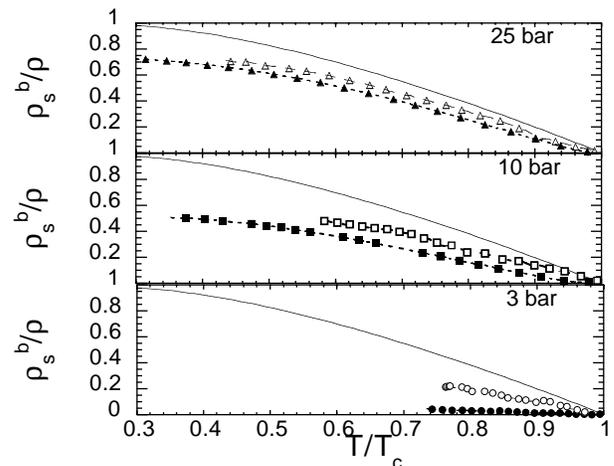
FIG. 4. We plot the bare superfluid density versus reduced temperature for cell A (98%), filled symbols, and cell D (99.5%), empty symbols.
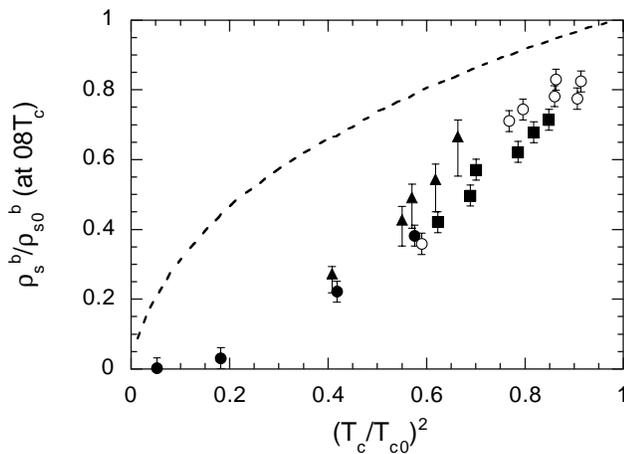
FIG. 5. The bare superfluid density at $0.8T_c$ scaled by the bare bulk superfluid density is plotted versus the square of the scaled T_c. The symbols are the same as in Fig. 1. The dashed line is the theoretical curve from the HSM.

In order to compare the behavior of T_c and ρ_s^b of ^{3}He in aerogel with ^{3}He in bulk it will be necessary to understand precisely *how* the fractal disorder affects the superfluid pairing mechanism. One test for models of nonconventional Cooper pairing in the presence of disorder would be to predict the functional form of curve for ρ_s^b versus $(T_c)^2$ for ^{3}He in (base-catalyzed) aerogel. The IISM of Thuneberg *et al.* predicts a relation between ρ_s^b/ρ_{s0}^b and T_c/T_{c0} similar to the trend illustrated in Fig. 5, showing behavior very different than the Abrikosov-Gorkov model [14,15]. This model does not explicitly consider the fractal nature of the aerogel but shows how inhomogeneities in the disorder can lead to a large suppression of ρ_s^b relative to T_c.

In this Letter we have presented data from our measurements on ^{3}He in a very dilute 99.5% porous silica aerogel. The values of T_c and ρ_s^b fall between those of ^{3}He in bulk and ^{3}He in denser aerogel samples. We also present T_c and ρ_s^b data for ^{3}He in aerogel experiments performed at Cornell that show behavior that can be traced to the fractal structure of the aerogel. In order to more fully understand this exciting physical system, more attention must be devoted to understanding the microstructure of the aerogel. Specifically, the scaling discussed above depends strongly on the fact that the fractal exponent real-space correlation is similar for all the aerogel samples. It would be interesting to study neutrally catalyzed silica aerogels, which have a different fractal exponent than the base-catalyzed samples [19]. ^{3}He would be expected to follow *different* scaling behavior when confined to base-catalyzed and neutrally catalyzed aerogels. As yet, there is no clear theo-retical picture for why the transition temperature T_c should depend on the ratio ξ_0/ξ_a. The explanation for this behavior will provide insight into how correlated disorder affects nonconventional Cooper pairing.

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