Controlling Chaos with Simple Limiters

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New experimental results demonstrate that chaos control can be accomplished using controllers that are very simple relative to the system being controlled. Chaotic dynamics in a driven pendulum and a double scroll circuit are controlled using an adjustable, passive limiter—a weight for the pendulum and a diode for the circuit. For both experiments, multiple unstable periodic orbits are selectively controlled using minimal perturbations. These physical examples suggest that chaos control can be practically applied to a much wider array of important problems than initially thought possible.

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Since the seminal work of Ott, Grebogi, and Yorke (OGY) [1], the idea of using closed-loop control techniques to tame physical systems exhibiting chaotic behavior suggests an intriguing solution to a number of problems. Variants of the OGY control scheme have been used to control mechanical systems [2], electronic systems [3], solid-state lasers [4], chemical systems [5], and even heart tissue [6]. Chaos control using delay feedback has also been demonstrated [7,8]. The objective of these closed-loop chaos control schemes is to use very small perturbations to select and stabilize specific unstable steady states or unstable periodic orbits (UPO) that exist in chaotic attractors.

In this Letter, we report experimental results that provide a new perspective on controlling chaotic systems. In particular, we focus on the issue of controller complexity, as measured relative to the system being controlled, and demonstrate that chaos control can be accomplished using very simple controllers. The notion of controller complexity has not been adequately addressed in previous reports of experimental chaos control, yet many of the most exciting prospects of chaos control will hinge critically on this issue.

Potential applications of chaos control reach beyond merely the suppression of undesired chaotic behavior. When operated in a chaotic regime, extraordinarily simple devices can generate very complex signals. In deliberately engineered chaotic systems the controller is used to switch among a myriad of possible behaviors. The goal in these systems is to exploit the natural complexity of chaos to achieve an overall reduction in device complexity. One emerging application of this paradigm is the use of symbolic dynamics for communications [9].

Naturally, it is of universal benefit that the controller be as simple as possible. For example, controlling the chaotic pulsing of diode laser systems is very difficult due to the extremely fast (sub-nanosecond) dynamics of these systems [10]. Small latency and large bandwidth requirements severely limit the complexity of a candidate controller. Other examples include the control of spatially extended dynamics, such as turbulent flow, where the unstable dynamics are high dimensional and a large number of distributed sensors and actuators may be required. In this case, a complex controller is impractical due to cost and density requirements. But, perhaps more fundamentally, it is imperative that the complexity of the controller be comparable to, or less than, the device being controlled if we wish to engineer chaos to some useful end. The whole concept of chaos engineering becomes untenable if the simple chaotic oscillator has to be straddled with a massively complex controller.

Generally, the complexity of the controller in published reports of chaos control is far greater than that of the system being controlled. A possible exception is the application of continuous delay feedback [7]. Unfortunately, this type of control can add many unwanted degrees of freedom to the system, and the resulting dynamics can become considerably more complicated. A recent letter reports a new method of chaos control based on transit-time pulse-width-modulation feedback (TPF) [11]. The simplicity of this approach allows the highest frequency example of chaos control to date—a 19 MHz Colpitts oscillator. However, the controller for this system, which is much simpler than previous occasional feedback controllers, still requires several integrated circuits and is vastly more complex than the single-transistor oscillator being controlled.

In this Letter, we show by experimental demonstration that a chaos controller can be extremely simple—even simpler than the physical system being controlled. The first experiment is a driven chaotic mechanical pendulum, for which the chaos controller is an additional mass attached to the pendulum by a string. The second experiment is a double scroll circuit, for which the controller is simply a diode. Notably, these controllers are passive limiters. For both experiments, multiple UPOs are selectively controlled and the average control perturbation is minimal when an actual UPO is stabilized, thereby providing strong evidence that chaos control, by the usual definition, has been attained.

The approach we take for controlling chaos in these experiments is an extension of the TPF method described in

[11]. In TPF, a system parameter is perturbed by a constant amount while the system state remains within a predefined window of state space. The strength of the control perturbation is determined by the length of time the system remains within the window. For select window positions, a periodic orbit is stabilized with minimal control strength, indicating that a UPO of the uncontrolled system has been stabilized. In [11], the state window and control perturbation are implemented using separate, active devices. For the present experiments, we realize both of these elements using a single, passive device—a weight for the pendulum and a diode for the electronic oscillator. In effect, each of these controllers is simply a limiter, which turns on when the system state exceeds a threshold and directly pulls the system back below threshold. In terms of a control window, the limiter forms a wall in state space at the threshold level. By adjusting the threshold level, different periodic orbits are stabilized. This is consistent with the results obtained by Glass and Zeng for flattened one-dimensional maps [12]. The stabilized orbits also include UPOs of the uncontrolled system. When a UPO is stabilized, the system state just reaches, but does not penetrate, the limiter wall; consequently, the average control strength is very small in this state.

For the first experiment, a driven chaotic pendulum was purchased from Pasco Scientific. The operation of this system and mathematical model have been described previously [13]. We chose a vertical configuration for the pendulum, as shown schematically in Fig. 1. The pendulum is driven with an electric motor and is equipped with a rotary motion sensor for recording the angular position of the pendulum mass. The motor has an angular frequency of 4.9 rad/s. Without control, the angular position and velocity trace out the chaotic attractor shown in Fig. 2(a). The angular velocity is derived from the measurements of the angular position.

To control this system, a simple mechanical limiter is attached to the pendulum. This limiter consists of a 20-g weight attached to one of the pendulum springs via a loose string, as shown in Fig. 1(a). The rotating part of the pendulum itself weighs 145 g. As the angular position of the pendulum exceeds some threshold, the string goes taut and lifts the weight, as shown in Fig. 1(b). Below this threshold, the weight has no effect on the dynamics. When lifted, the weight applies an additional force to the system, and this new force is used as a control signal. The exact nature of this force may be quite complicated, since the weight also induces a lateral deflection of the spring assembly; however, the weight clearly limits the pendulum's motion beyond a critical angle by effectively increasing the pendulum's mass. A standard lab jack, also shown in Fig. 1, is used to change the rest height for the control weight, thereby providing an adjustment for the limiter position.

The motor's angular frequency is not affected significantly by the additional mass and, in general, the motion of the pendulum repeatedly lifts the weight. By adjusting the rest height, we make the critical angle for the limiter coincide with the maximum angular travel for a particular UPO in the system. The subsequent dynamical behavior of the pendulum is periodic and does not lift the weight

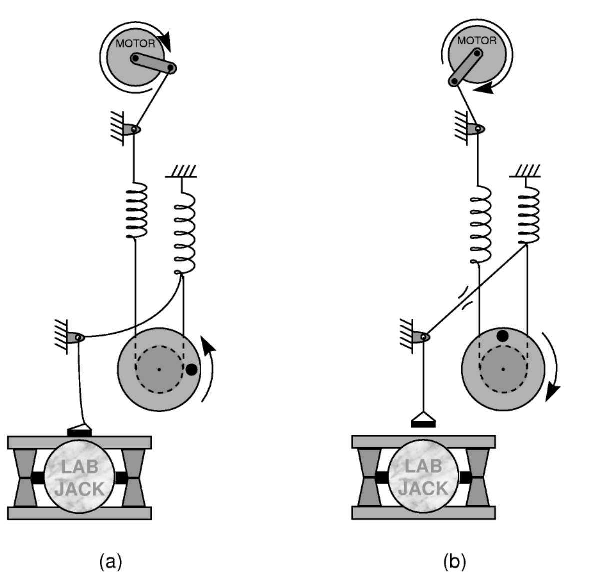

FIG. 1. Configuration of the chaotic driven pendulum, including the mechanical limiter in the (a) slack and (b) taut positions.

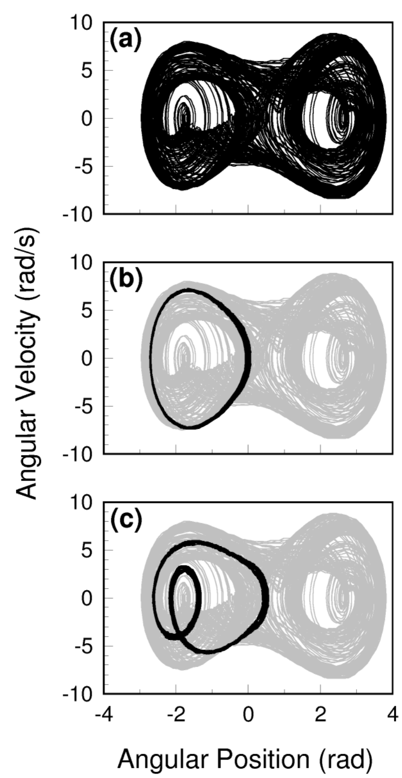

FIG. 2. Measured phase space projections for the pendulum in the (a) uncontrolled state, (b) controlled period-1 UPO, and (c) controlled period-2 UPO.

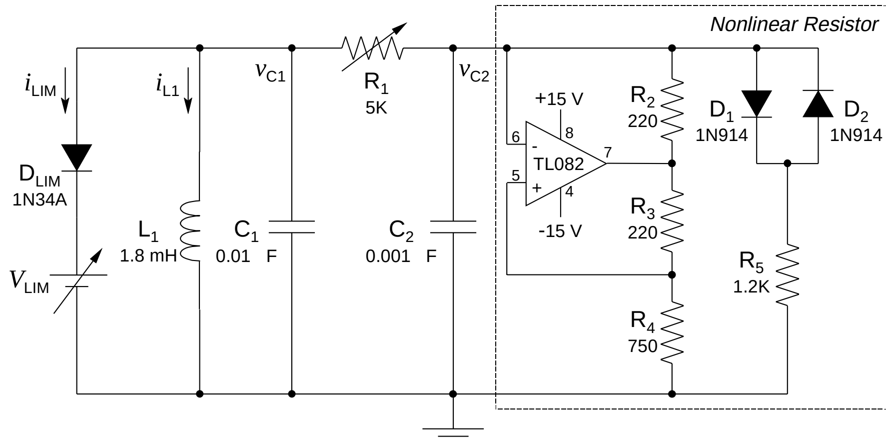

FIG. 3. Double scroll circuit including the diode limiter D_{LIM} and control level adjustment V_{LIM}.

at all. The only controlling force on the system is a small tug as the string is just pulled taut. At these positions of minimal control force, the pendulum is stabilized to an actual UPO of the uncontrolled system. Figures 2(b) and 2(c) show period-1 and -2 UPOs controlled and identified in this manner. The periodicity of the UPOs is measured relative to the drive period.

The second experiment uses a double scroll circuit, which is a simple electronic oscillator known to exhibit a wide variety of chaotic behaviors [14]. The variant shown schematically in Fig. 3 was built using discrete components on a solderless breadboard. Setting $R_1 \cong 1.45$ kΩ, the uncontrolled circuit is tuned to provide the folded-band attractor shown in Fig. 4(a). The circuit oscillates at approximately 30 kHz.

To control this circuit a diode, labeled D_{LIM} in Fig. 3, is included in the circuit. When the voltage in the LC tank portion of the circuit exceeds a threshold, the diode turns on and excess charge is drained from the tank. In effect, the diode acts as a limiter analogous to the weight used to control the pendulum. To adjust the control level, the voltage V_{LIM} is varied.

Figures 4(b) and 4(c) show period-1 and -2 UPOs that were stabilized using the diode limiter. The waveforms were recorded using a digital sampling oscilloscope (Tektronix TDS680B). Both UPOs were identified by local minima in the peak control current as V_{LIM} was changed. In Figures 5(a) and 5(b), the measured control current i_{LIM} is compared to the tank current i_{L1} for the controlled states shown in Fig. 4. The control current was monitored using a current-to-voltage converter at V_{LIM}, and the tank current was derived from measurements of the voltage v_{C1}. In both comparisons, the peak control current is less than 0.5% of the maximum tank current, thereby providing strong evidence that the orbits are actual UPOs of the uncontrolled oscillator. Although this experiment was performed at kHz frequencies, a diode could perform the same control function on a chaotic device operating at much higher, MHz or GHz frequencies.

From these examples it is clear that the usual components of chaos control—measurement, sample and hold, control signal calculation, and amplification—can all be replaced by a simple limiter. As in all chaos control schemes, the perturbation caused by the limiter becomes small as the system approaches the target state. However, in contrast to many other control techniques, the addition of a limiter does not add complexity to the system by increasing the size of the system's state space, since the

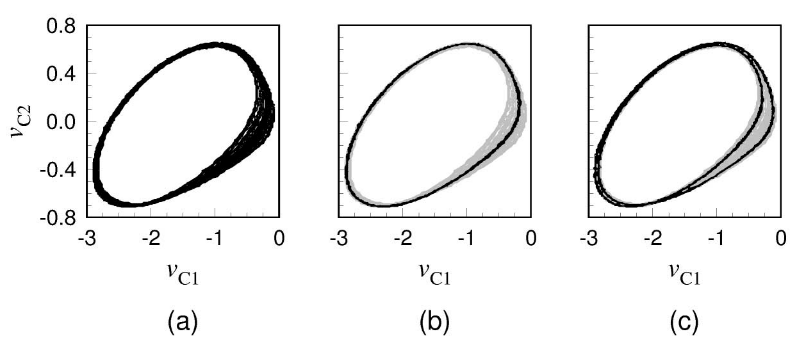

FIG. 4. Measured phase space projections of the double scroll oscillator in the (a) uncontrolled state, (b) controlled period-1 UPO, and (c) controlled period-2 UPO.

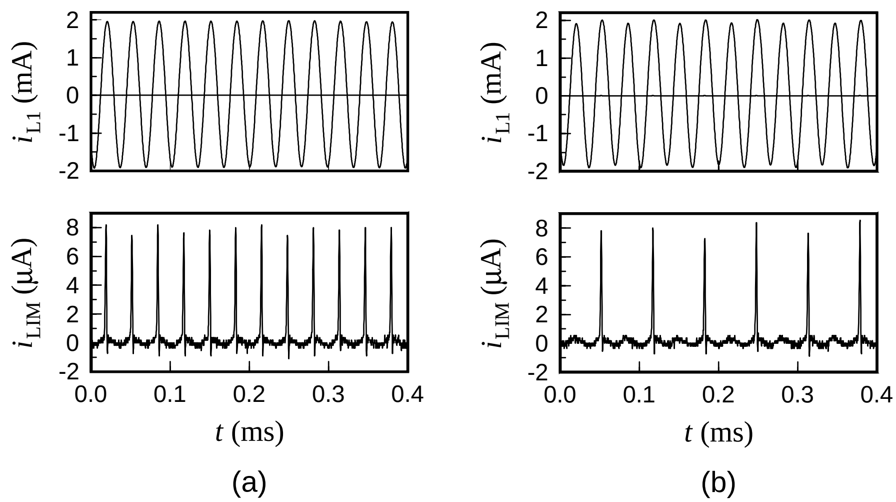

FIG. 5. Control current i_{LIM} and tank current i_{L1} compared for stabilized (a) period-1 and (b) period-2 UPOs of the double scroll circuit.

limiter is an instantaneous function of the current state. Importantly, the limiter can be constructed of components natural to the system at hand. For the pendulum, the limiter is simply a mass whose position can be adjusted; for the circuit, it is a diode with a voltage offset. For other physical systems, one can identify natural limiters that could be used for chaos control. For an optical system, it could be an adjustable photonic band gap filter that presents a barrier in frequency space. For a ship crane, it might be a well-positioned block. For a chemical process, it could be an energy barrier due to a temperature and pressure dependent phase transition. In these problems, the challenge is to see the chaos controller not as an external, active device, but as a simple extension of the system itself.

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