Electron-Positron Outflow from Black Holes

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Cosmological gamma-ray bursts (GRBs) appear as the brightest transient phenomena in the Universe. The nature of their central engine is a missing link in the theory of fireballs to stellar mass progenitors, and may be associated with low mass black holes. In contact with an external magnetic field B, black hole spin produces a gravitational potential on the wave function of charged particles. We show that a rapidly rotating black hole of mass M produces outflow from initially electrostatic equilibrium with normalized isotropic emission $\sim 10^{48}(B/B_c)^2(M/7M_\odot)^2 \sin^2\theta$ erg/s, where $B_c = 4.4 \times 10^{13}$ G. The half-opening angle satisfies $\theta \geq \sqrt{B_c/3B}$. The outflow proposed as input to GRB fireball models.

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Long gamma-ray bursts (GRBs) are now believed to be at cosmological distances, in view of their isotropic distribution in the sky [1], redshifts of order unity when detected [2], and $\langle V/V_{\max}\rangle = 0.334 \pm 0.008$ distinctly less than the Euclidean value $1/2$ [3]. The GRB990123 event [4] reminded us that they can be luminous with an apparent energy ΔE exceeding $1M_\odot$, assuming isotropic emission. The emissions are well described by the internal-external shock fireball model [5–8]. Their engines should be compact and ultrarelativistic as indicated by $\alpha = G\Delta E/c^5\delta t$, where δt is the time scale of variability, G is Newton's constant, and c is the velocity of light. Typically, $\alpha = 10^{-4}$–10^{-2} is extremely large compared to other transients such as supernovae, including the GRB/SN 1998bw event [9], and outbursts in microquasars such as GRS 1915+105 [10]. This suggests a GRB association with low mass black holes.

Black holes are a natural outcome of stellar evolution, particularly in binaries. Young massive stars evolve towards catastrophic events [11,12] with the possibility of subsequent binary coalescence of their descendants (see, e.g., Ref. [13]). These events probably produce low mass black holes, and most likely do so through an intermediate black hole–torus or disk state [11,14]. Emissions in this state may arise from accretion or tapping the spin energy of the black hole, the latter up to 30% of its total mass.

This Letter focuses on the central engine of GRBs. A theory is described for electron-positron outflow powered by black hole spin when brought into contact with a strong magnetic field. The magnetic field is supported by surrounding matter, as in the aforementioned black hole–torus or disk systems. Black hole spin is known to exert Maxwell stresses through its magnetosphere, for instance, which is expected to delay accretion [15]. A torus magnetosphere around a rotating black hole is, furthermore, expected to be intermittent on a time scale of 0.15–1.5 s [15]. If the torus is formed from tidal breakup of a neutron star, it may remain at nuclear density and reach magnetic field strengths up to 10^{17} G by linear amplification [16]. Here, we focus on the coupling of black hole spin to matter fields of charged particles in these strong magnetic fields. The calculations are perturbative about a Wald field in electrostatic equilibrium [17]. A pair-creation process towards this equilibrium has been considered in a previous analysis [18]. We find that black holes continue pair creation by a spin-powered process. The proposed energy extraction mechanism may also apply to active galactic nuclei, where the magnetic field is believed to be of the order of 10^4 G, provided it combines with traditional pair-cascade processes.

Pair creation can be calculated from the evolution of wave fronts in curved space time, which is well defined between asymptotically flat in- and out-vacua. By this device, any inequivalence between them becomes apparent and generally gives rise to particle production [19,20]. This is perhaps best known from the Schwinger process [21,22] and in dynamical cosmologies [20]. Such a particle production process is driven primarily by the jump in the zero-energy levels of the asymptotic vacua, and to a lesser degree it depends on the nature of the transition between them. The energy spectrum of the particles is ordinarily nonthermal, with the notable exception of the thermal spectrum of Hawking radiation from a horizon surface formed in gravitational collapse to a black hole [23]. There are natural choices of the asymptotic vacua in asymptotically flat Minkowski space times, where a timelike Killing vector can be used to select a preferred set of observers. This leaves the in- and out-vacua determined up to Lorentz transformations on the observers and gauge transformations on the wave functions of interest. These ambiguities can be circumvented by making reference to Hilbert spaces on null trajectories—the past and future null infinities $\mathcal{J}^\pm$ in Hawking's proposal—and by working with gauge-invariant frequencies. The latter received some mention in Hawking's original treatise [23], and is briefly as follows.

Hawking radiation derives from tracing wave fronts from $\mathcal{J}^+$ to $\mathcal{J}^-$, past any potential barrier and through the collapsing matter, to obtain Bogoliubov projections on the Hilbert space of radiative states on $\mathcal{J}^-$. This procedure assumes gauge covariance, by tracing wave fronts associated with gauge-covariant frequencies in the presence of a background vector potential A_a. The

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generalization to a rotating black hole is obtained by taking these frequencies relative to real, zero-angular momentum observers (cf. [24,25]), whose world lines are orthogonal to the azimuthal Killing vector as given by $\xi^a\partial_a = \partial_t - (g_{t\phi}/g_{\phi\phi})\partial_\phi$, where g_{ab} denotes the Kerr metric. Then $\xi^a \sim \partial_t$ at infinity and $\xi^a\partial_a$ assumes corotation upon approaching the horizon. This obtains consistent particle-antiparticle conjugation among all observers, except for the interpretation of a particle or an antiparticle. Thus, Hawking emission from the horizon of a rotating black hole of mass M gives rise to a flux to infinity,

$$\frac{d^2n}{d\omega dt} = \frac{1}{2\pi}\frac{\Gamma}{e^{2\pi(\omega - V_F)/\kappa} + 1}, \tag{1}$$

for a particle of energy ω at infinity. Here, $\kappa = 1/4M$ is the surface gravity, Ω_H is its angular velocity, and Γ is the relevant absorption factor in the presence of a potential barrier. The Fermi level V_F derives from the normalized gauge-covariant frequency as observed by a zero-angular-momentum observer (ZAMO) close to the horizon, namely, $\omega - V_F = \omega_{\mathrm{ZAMO}} + eV = \omega - \nu\Omega_H + eV$ for a particle of charge $-e$ and azimuthal quantum number ν, where V is the electrical potential on the horizon relative to infinity. The result for antiparticles as seen at infinity follows with a change of sign in the charge, equivalent to the usual conjugation rule $\omega \rightarrow -\omega$ and $\nu \rightarrow -\nu$.

Hawking radiation is symmetric under particle-antiparticle conjugation when $V = 0$. Blackbody radiation (1) from a Schwarzschild black hole has a Hawking temperature $T \sim 10^{-7}(M/M_\odot)$ K, which is negligible for astrophysical black holes [26]. The charged case forms an interesting exception, where the Fermi level $-eV$ gives rise to spontaneous emission which brings the black hole into equilibrium on a dynamical time scale [27,28]. In contrast, the Fermi level $\nu\Omega_H$ of a rotating black hole acting on neutrinos is extremely inefficient in producing spontaneous emission [29]. This is due to an exponential cutoff by an angular momentum barrier which is independent of the sign of the orbital angular momentum, and hence acts universally on neutrinos and antineutrinos. This illustrates that (1) should be viewed with two different processes in mind: nonthermal emission in response to V_F, and thermal emission beyond [19].

Emissions from rotating black holes brought into contact with an external magnetic field will be different. Here, the radiative states of charged particles are characterized by conservation of magnetic flux rather than conservation of particle angular momentum. This has some interesting consequences.

A rotating black hole in an external magnetic field assumes a net horizon charge $q = 2BJ$ in electrostatic equilibrium, where M is the mass and $J = aM$ is the angular momentum of the black hole [17,30,31]. The source-free Wald solution of the vector potential of the electromagnetic field, $A_a = Bk_a/2 - (q/2M - aB)\eta_a$, hereby is

$$A_a = \frac{1}{2}Bk_a, \tag{2}$$

where k^a is the azimuthal Killing vector and η^a is the asymptotically timelike Killing vector. In electrostatic equilibrium, then, $\xi^a A_a = 0$. The magnetic flux through the horizon is primarily generated by q when the black hole spins rapidly, allowing it to support open field lines to infinity [32].

The wave function ψ of charged particles in a Wald field can be expanded as $e^{-i\omega t}e^{i\nu\phi}e^{ip_s s}f(\rho)$ in coordinates (ρ, ϕ, s, t), where ρ labels a flux surface, ϕ is the azimuthal angle, and s is along B. Comparison with plane-wave solutions [33] shows a confinement on the νth flux surface with $g_{\phi\phi}^{1/2} = \sqrt{2\nu/eB}$ in Landau levels $E_{n\alpha} = \{m_e^2 + p_s^2 + |eB|(2n + 1 + \alpha)\}^{1/2}$, where m_e is the electron mass and $\alpha = \pm 1$ refers to spin orientation along B. These states enclose a magnetic flux $A_\phi = \frac{1}{2}Bk^2 = \nu/e$. The gauge-covariant frequency of the Landau states near the horizon is given by $-\xi^a(i^{-1}\partial_a + eA_a)\psi = (\omega - \nu\Omega_H)\psi$. The jump

$$V_F = [-\xi^a(i^{-1}\partial_a + eA_a)]_\infty^H\psi = \nu\Omega_H \tag{3}$$

between the horizon and infinity defines the Fermi level of the particles at the horizon. In contrast, the Wald field about an uncharged black hole has $V_F = \nu\Omega_H - eaB_0$, which is out of electrostatic equilibrium. Note that the canonical angular momentum of the Landau states vanishes: $k^a\hat{\pi}_a\psi = (i^{-1}\partial_\phi - eA_\phi)\psi = 0$. The Fermi level (3) makes explicit the black hole–spin coupling to ψ in the presence of A_a. We shall now study the effect of $\nu\Omega_H$, starting from electrostatic equilibrium to infer aspects of the late time evolution.

Black hole spin, therefore, couples to matter fields by $\nu\Omega_H$ and to the electromagnetic vector potential A_a. The latter is commonly expressed in terms of the electromagnetic force EMF_ν over an infinite loop, fixed in Boyer-Lindquist coordinates, which runs over the axis of rotation, the horizon, the νth flux surface with flux Ψ_ν with closure at infinity. Thus, we have $\mathrm{EMF}_\nu = \Omega_H\Psi_\nu/2\pi$ [24,34], and hence a new identity

$$\nu\Omega_H = e\mathrm{EMF}_\nu. \tag{4}$$

In electrostatic equilibrium, $\xi^a A_a = 0$, and, following (3), the extension of (4) to (s, ν) off the horizon is $[-\xi^a(i^{-1}\partial_a + eA_a)]_\infty^{(s,\nu)} = -\nu g_{t\phi}/g_{\phi\phi} = -eBg_{t\phi}/2 = -eA_t$ (for $-e$). Here, $V = A_t$ is the electric potential in Boyer-Linquist coordinates, while zero-angular momentum observers detect a zero electric potential.

The luminosity in (1) is now set by the transmission coefficient through the potential barrier in the so-called level-crossing picture [22]. The WKB approximation (i.e., using zero-angular-momentum observers) gives the inhomogeneous dispersion relation

$$(\omega - V_F)^2 = m_e^2 + |eB|(2n + 1 - \alpha) + p_s^2, \quad (5)$$

where $V_F = V_F(s, \nu)$ is the s-dependent Fermi level on the νth flux surface [35]. Pair creation is now set by the gradient (cf. the case of neutrinos [29]) $\eta = |\partial V_F/\partial s| \sim |\partial_r(eBg_{t\phi}/2)| = |\partial_r(eA_t)|$, i.e.,

$$\eta \sim eBaM \frac{r^2 - a^2\cos^2\theta}{(r^2 + a^2\cos^2\theta)^2}\sin^2\theta, \quad (6)$$

using $\partial_s \sim \partial_r$. Radiation states at infinity are separated from those near the horizon by a barrier, where $p_s^2 < 0$ about $V_F(s_0) = \omega$. The WKB approximation gives the transmission coefficient

$$|T_{n\alpha}|^2 = e^{-\pi[m_e^2 + |eB|(2n+1+\alpha)]/\eta}. \quad (7)$$

Since the Wald field is highly uniform, magnetic mirror effects and curvature radiation, for example, are neglected. Evidently, T is dominant in $n = 0$ and $\alpha = -1$ (for e^-).

The net pair-production rate by the gradient η in (6) can be derived from the analogous electrostatic process produced by an electric field E along B. This [22,28] shows an outflow of particles

$$\dot{N} = \frac{e}{4\pi^2}\int \frac{\eta B e^{-\pi m_e^2/\eta}}{\tanh(\pi eB/\eta)}\sqrt{-g}\, d^3x. \quad (8)$$

For a rapidly spinning black hole, the small angle approximation gives

$$\dot{N} \sim \frac{N_H^2}{128\sqrt{3}\,\pi^2 M}\left(\frac{a}{M}\right)^4 c^{-7/2} e^{-8\pi c/\theta^2}\theta^7 \quad (9)$$

asymptotically as $8\pi c/\theta^2 \gg 1$. Here $c = m_e^2 a/eBM$, $N_H = m_e^2 M^2$ is characteristic for the number of particles on the horizon, and θ is the half-opening angle of the outflow. The right-hand side of (9) forms a lower limit in the case of $8\pi c/\theta \leq 1$. When $a \sim M$, N_H/c is characteristic for the total number of flux surfaces ν_* which penetrate the horizon and $c \sim B_c/B$, where $B_c = 4.4 \times 10^{13}$ G is the field strength which sets the first Landau level at the rest mass energy. A similar calculation shows a luminosity in particles L_p in a jet of half-opening angle θ, the normalized isotropic emission

$$L_p' \sim \frac{L_p}{\theta^2} \sim \frac{\sqrt{3}}{4}\, eBM\dot{N}. \quad (10)$$

This calculation shows that the black hole spin continues pair production towards a departure from electrostatic equilibrium $q > 2BJ$, due to the outflow of e^- (with the sign convention $B\Omega_H > 0$). As $q > 2BJ$, the outflow evolves towards an inner jet of e^+ produced by the electrostatic contribution to V_F near the polar caps, and an outer jet of e^- produced by the $\nu\Omega_H$ contribution to V_F. A full calculation of the resulting equilibrium outflow falls outside the present scope. Nonetheless, the luminosity (10) is expected to remain characteristic for the evolved outflow.

A saturation of the outflow (9) follows by nondissipative and dissipative backreactions. The magnetic field diminishes by azimuthal currents from the charged particles, and the horizon potential V_F diminishes by dissipation in the horizon, whose electrical surface conductivity is 4π (see [24]). These effects go beyond the present Wald field approximation. In angular dependence the dissipative backreaction is dominant, so that

$$4\pi e\dot{N} < \nu\Omega_H \quad (11)$$

as a limit on the outer spin-driven e^- outflow, up to a logarithmic factor of order $\ln(\pi/2\theta)$, where ν is taken at the half-opening angle θ of the outflow. Consistency with (9) gives rise to a minimum opening angle θ_0 in the e^- outflow

$$\theta_0 \sim \sqrt{\frac{B_c}{3B}}, \quad (12)$$

again up to logarithmic corrections. For $\theta > \theta_0$, the outflow is effectively set by the saturation limit (11), whereby the particle luminosity (10) is bounded by

$$L_p' \sim \left(10^{48}\,\frac{\text{erg}}{\text{sec}}\right)\left(\frac{B}{B_c}\right)^2\left(\frac{M}{7M_\odot}\right)^2\sin^2\theta, \quad (13)$$

where $\theta > \theta_0$ is the half-opening angle of the outflow. This calculation applies formally to the initial jet. A full calculation of the evolved jet, which consists of combined $e^\pm$ outflow saturated against dissipative losses in the horizon (in the sense as described above) falls outside the present scope. Nevertheless, it is expected that the luminosity (13) remains characteristic for particle-antiparticle outflow in the evolved jet, whose opening angle will be bounded below by the initial value (12).

A connection to fireballs [5,8] in the theory of GRBs [6] is at hand when B reaches 10^{16} G, for which (13) represents an outflow in electron-positrons consistent with the GRB 9901023 event. The outflow is along open field lines, and essentially baryon-free in view of the exponential suppression of the transmission coefficient (7) for particles with higher mass.

The theory of fireballs accounts for the nonthermal spectrum by some baryon loading [5,36] to circumvent thermal emission in earlier models [37]. Baryonic contamination may arise, for instance, by entrainment of the interstellar medium, a wind from the surrounding torus, or the hydrogen envelope in hypernovae. On the other hand, intermittency at the source (see, e.g., Refs. [6,15,38]) produces unsteadiness in the flow, as discussed in the compact fireball model of Eichler and Levinson [8] with angular variations (both in θ_0 and in orientation) and internal shocks even when baryon-free. Shocks also result from interactions with collimating baryonic material, perhaps subject to radiative viscosity, which also contributes to nonthermal emission from a broad range of radii [8]. These aspects imply an emission spectrum substantially different from that in the aforementioned steady-state models, also given the fact that (13) is nonthermal in origin.

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