Raman Gap Solitons

Herbert G. Winful and Victor Perlin
Department of Electrical Engineering and Computer Science, University of Michigan, 1301 Beal Avenue, Ann Arbor, Michigan 48109-2122
(Received 22 November 1999)

We show that an intense pump pulse, detuned far from the Bragg resonance of a nonlinear periodic structure, can excite a gap soliton at a wavelength within the band gap that corresponds to the Raman shift of the medium. This Raman gap soliton is a stable, long-lived, quasistationary excitation that exists within the grating even after the pump pulse has passed. We find both stationary solitons as well as slow Raman gap solitons with velocities as low as 1% of the speed of light. The predicted phenomena should be observable in fiber Bragg gratings and other nonlinear photonic band gap structures.

PACS numbers: 42.65.Tg, 05.45.Yv, 42.65.Dr, 42.70.Qs

Light wave propagation in periodic dielectric structures is characterized by the presence of stop bands in frequency within which the wave vector is imaginary and the wave amplitude decays exponentially with distance into the structure [1]. This exponential decay is due to the coherent Bragg reflection that occurs when the period of the structure is about one-half the wavelength of light in the medium. Just outside the stop band the transmission of the structure is high and attains values of unity at frequencies that correspond to linear resonances of the finite length structure. At these resonances, the field envelope distribution has peaks whose intensity exceeds that of the input wave due to constructive interference between forward and backward waves [2]. These properties of linear periodic structures are well established and form the basis for the operation of such devices as distributed feedback lasers [2].

In 1979, Winful *et al.* introduced the concept of a nonlinear periodic structure, one whose refractive index depends on the local intensity of the electromagnetic wave [3]. For such a structure, a light wave that satisfies the Bragg condition at low intensity and, hence, suffers substantial reflection, can nevertheless tune itself out of the stop band as the intensity is increased. This intensity tuning of the stop band leads to optical self-switching, bistability, and multistability in the transmitted and reflected intensities [3,4]. For certain critical input intensities, the intensity-dependent detuning is sufficient to pull one of the resonances of the structure into coincidence with the frequency of the incident wave. At such a nonlinear resonance the transmission becomes unity and the spatial distribution of the field envelope takes on a characteristic shape describable in terms of elliptic functions [5]. These stationary field profiles are known as gap solitons [6] and are the subject of much recent theoretical and experimental study. The dynamic behavior of nonlinear periodic structures is also characterized by instabilities such as self-pulsations and chaos [7,8]. Near the stop band the periodic structure is highly dispersive, exhibiting very large values of both positive and negative group velocity dispersion (GVD) [9]. This dispersion, along with the self-phase modulation that arises from the nonlinear index, can result in pulse compression and the propagation of temporal solitons termed Bragg solitons [9,10]. Most of these predicted phenomena have been observed recently with the use of fiber Bragg gratings and other nonlinear photonic band gap structures [11–15].

One of the major challenges in the experimental studies of gap solitons is how to launch them efficiently. The problem is that a periodic structure is highly reflective at frequencies within the band gap. Thus it is extremely difficult to couple enough light into the structure to shift the band gap and create gap solitons. In fact, the grating transmissivity is proportional to the velocity of the gap soliton which implies that an attempt to launch a zero-velocity gap soliton in the middle of the stop band would result in the reflection of the entire input pulse. To date the stationary gap soliton has not been observed. Such a stationary excitation would be promising for applications such as optical buffers and delay lines. As for slowly propagating gap solitons, the slowest observed to date have velocities that exceed 50% of the speed of light.

In this paper we show that it is possible to launch gap solitons by exciting a grating with an intense pulse so far detuned from the Bragg frequency that it does not interact with the grating. Because of this detuning the pump pulse is not reflected, making it possible to couple most of its energy into the medium. Further, we assume that the medium is Raman active, with a Raman shift such that the first Stokes wavelength is in the vicinity of the Bragg resonance. Through stimulated Raman scattering the pump transfers its energy to the Raman pulse whose amplitude grows from noise or can be seeded with a weak signal at the Stokes frequency. When the intensity of the Stokes pulse grows sufficiently large, it can begin to change the refractive index of the grating and create a gap soliton at the Raman shifted frequency. This Raman gap soliton remains trapped inside the grating even after the pump pulse has passed. We find stable, stationary solitons as well as solitons with velocities less than 1% the speed of light in the medium. The lifetime of these resonance excitations

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can be quite long, depending on the length of the grating and the coupling constant. This work thus introduces a new excitation, the Raman gap soliton, and also solves the problem of effective coupling into a gap soliton.

To describe the dynamics of the Raman gap soliton and to test the feasibility of its generation we use coupled mode equations for the forward pump amplitude (A_p), the forward Stokes (A_{S^+}) amplitude, and the backward Stokes (A_{S^-}) amplitude. Initially we consider only the first Stokes component. While higher order Stokes signals can be generated at sufficiently high intensities, these components lie outside the grating stop band and hence do not experience any reflective feedback. As a result their amplitude will be small compared to the first Stokes component. We allow for pump depletion but ignore the material group velocity dispersion since it is much smaller than the dispersion due to periodic structure [9]. While the ideas presented here apply to general nonlinear periodic structures, we will use terminology and parameters suitable for fiber Bragg gratings since they are the most likely candidates for experimental verification. The coupled mode equations are as follows:

$$\frac{\partial A_p}{\partial z} + \frac{1}{v_p}\frac{\partial A_p}{\partial t} = -\frac{g_p}{2}(|A_{S^+}|^2 + |A_{S^-}|^2)A_p + i\gamma_p(|A_p|^2 + 2|A_{S^+}|^2 + 2|A_{S^-}|^2)A_p\,, \tag{1}$$

$$\frac{\partial A_{S^+}}{\partial z} + \frac{1}{v_s}\frac{\partial A_{S^+}}{\partial t} = \frac{g_s}{2}|A_p|^2 A_{S^+} + i\gamma_s(|A_{S^+}|^2 + 2|A_p|^2 + 2|A_{S^-}|^2)A_{S^+} + i\kappa A_{S^-} + i\Delta\beta A_{S^+}\,, \tag{2}$$

$$\frac{\partial A_{S^-}}{\partial z} - \frac{1}{v_s}\frac{\partial A_{S^-}}{\partial t} = -\frac{g_s}{2}|A_p|^2 A_{S^-} - i\gamma_s(|A_{S^-}|^2 + 2|A_p|^2 + 2|A_{S^+}|^2)A_{S^-} - i\kappa A_{S^+} - i\Delta\beta A_{S^-}\,. \tag{3}$$

Here κ is the grating coupling constant which is proportional to the amplitude of the periodic refractive index modulation. The intensity-dependent refractive index is described by the parameters $\gamma_p = \pi n_2/\lambda_p$ and $\gamma_s = \pi n_2/\lambda_s$, with n_2 the nonlinear index coefficient, λ_p and λ_s, respectively, the pump and Stokes wavelengths. The gain coefficients g_p and g_s are related to the Raman gain coefficient g_R through $g_s = g_R/A_{\mathrm{eff}}$ and $g_p = (\lambda_s/\lambda_p)g_s$, where A_{eff} is the effective area of the guided mode [16]. The group velocities of the pump and Stokes waves in the unperturbed medium are v_p and v_s, respectively. The detuning of the Stokes frequency (ω_s) from the Bragg frequency (ω_B) is described by the parameter $\Delta\beta = \overline{n}(\omega_s - \omega_B)/c$, where $\overline{n}$ is the average refractive index of the fiber core. This set of equations includes forward and backward Raman amplification, grating coupling between the Stokes waves, as well as self- and cross-phase modulation. These equations are solved numerically with the boundary condition that at $z = 0$, the pump pulse is a given function of time and the initial condition that the forward and backward Stokes amplitudes evolve from a weak cw input seed. The numerical integration uses a stable second-order Runge-Kutta scheme along the forward and backward characteristics.

The parameter values used in the simulations were chosen as appropriate for a typical fiber Bragg grating. The grating length L varies between 2 and 4 cm, the coupling constant κ ranges between 2 and 6 cm^{-1}, the nonlinear index is taken as 1.2×10^{-22} $(\mathrm{m/V})^2$, the Raman gain coefficient is 1.1×10^{-13} m/W, and the fiber effective area is 50 $\mu\mathrm{m}^2$. The Raman shift of silica fiber is 440 cm^{-1} and the width of the Raman gain ($\sim$100 cm^{-1}) is much larger than the bandwidth of the fiber Bragg grating ($\sim$0.1 cm^{-1}). The input pump pulses are taken as Gaussian with pulse width τ (FWHM) between 100 and 700 ps. The system of equations obeys the following scaling rule: the output is invariant under the transformation $(L, \kappa, P_0, \tau) \rightarrow (aL, \kappa/a, P_0/a, a\tau)$, where a is a constant and P_0 is the peak power of the input pump pulse. This scaling rule helps to reduce the parameter range over which one must scan in order to uncover the variety of qualitative behaviors.

We find a number of distinct operating regimes that depend on the input pump power. For low input power there is no observable output at the Stokes wavelength and the pump propagates through the structure without any distortion or loss. As the pump power is increased, energy begins to be transferred to the Stokes wave through stimulated Raman scattering. There is a Raman threshold beyond which the Stokes power becomes comparable to the pump power. This threshold power is given roughly by [17]

$$P_{\mathrm{th}} \simeq 16A_{\mathrm{eff}}/g_R L\,. \tag{4}$$

For a 4-cm-long piece of fiber this threshold power is about 200 kW. As this threshold value is exceeded, an intense Raman pulse is generated. This pulse proceeds to shed energy until it reaches a value of intensity that is just enough to create a gap soliton at the Raman frequency. This Raman gap soliton is trapped inside the grating and executes slow oscillations within the structure as it bounces back and forth between turning points that mark the locations of equivalent lumped mirrors. At each reflection a small amount of energy leaks out. Eventually the magnitude decays to a value such that there is no intensity-dependent contribution to the dynamics. At that point the pulse evolution is linear and the rate of energy loss increases. Figure 1 shows the evolution of a slow Raman gap soliton

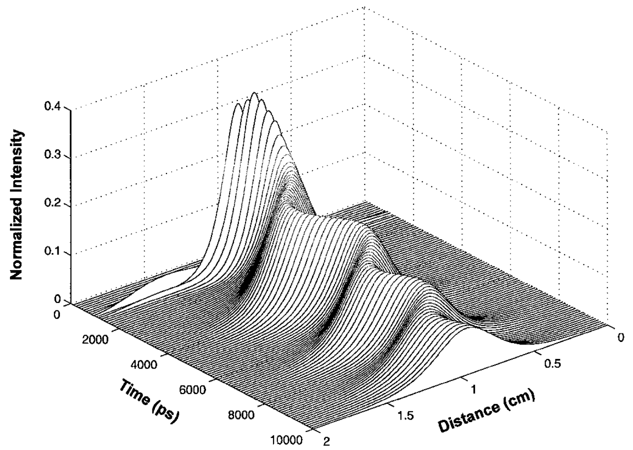

FIG. 1. Space-time evolution of a slow Raman gap soliton. Here $L = 2$ cm, $\kappa = 6\ \mathrm{cm}^{-1}$, $\tau = 350$ ps, and $P_0 = 318$ kW.

with a velocity only 1% the speed of light in the unperturbed medium. This gap soliton consists of forward and backward components of nearly equal amplitude that are coupled together by the grating. The small difference in their amplitudes (here about 1%) is what causes the gap soliton to move with the correspondingly low velocity.

Stable, immobile Raman gap solitons have also been found for certain parameter values. Figure 2 shows such a stable, stationary gap soliton. For this soliton the amplitudes of the forward and backward components are equal. The lifetime of this soliton is at least 20 ns, which is 40 times the duration of the pump pulse. The steady state Raman gap soliton is accurately described by a sech^2 profile. Figure 3 shows the numerically computed intensity profile of the stationary Raman gap soliton, given by the sum of the intensities of the forward and backward Stokes waves. The dotted curve is a sech^2 fit to the data. The excellent agreement confirms our identification of the field profile as a gap soliton. Note that the Raman gap soliton persists after the pump has left the grating. The dynamics of the Raman gap soliton can then be analyzed by setting the pump intensity equal to zero. The remaining equations are exactly the same equations that govern nonlinear pulse propagation and soliton formation in periodic structures [9].

Since the Raman gap soliton is trapped in the Bragg grating, the question remains as to the experimental signature of such an excitation. The gap soliton does couple to the outside world for any finite length grating. The energy leakage through the ends of the structure can be used to characterize the soliton. Figure 4 shows the output intensity at the left end of the grating as a function of time for the slow moving Raman gap soliton of Fig. 2. After the initial transient, the output oscillates as a result of the periodic motion of the soliton. The energy loss at each reflection leads to a broadening of the soliton which in turn results in higher intensities at the grating edges and consequently a higher rate of energy loss. Here the output intensity (after the transient) peaks at about 12 ns after which point it decays exponentially, consistent with the evolution of a linear resonance.

In the linear case, no significant field amplitude can exist inside the structure for frequencies inside the stop band. This is well known from the theory of distributed-feedback lasers. What makes it possible for the Raman gap soliton to live for long periods inside the band gap are the following effects: (1) Through cross-phase modulation, the pump shifts the band gap so that the frequency of the weak seed Stokes input, located at the Bragg peak in the absence of the pump, now lies somewhat to the side of this peak. (2) Through stimulated Raman scattering this seed begins to grow to high intensities. Eventually the Stokes pulse becomes intense enough to maintain the band gap shift through its own intensity-dependent refractive index changes even after the pump has exited the grating. The excitation of the Raman gap soliton thus involves both an "active" component and a "reactive" component. The active component transfers energy from the pump wave to

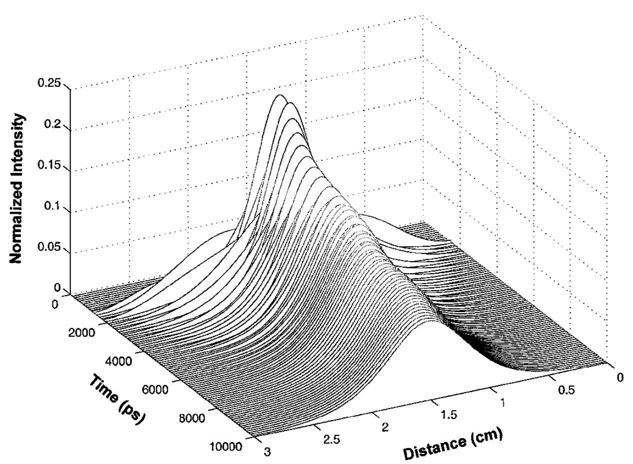

FIG. 2. A stationary Raman gap soliton. Here $L = 3$ cm, $\kappa = 6\ \mathrm{cm}^{-1}$, $\tau = 510$ ps, and $P_0 = 201$ kW.

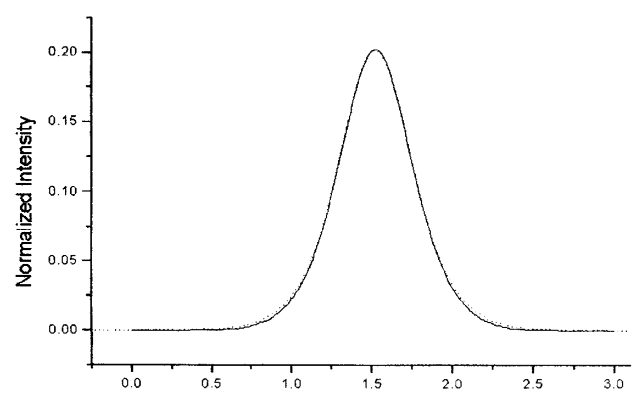

FIG. 3. Solid curve: Intensity distribution of the stationary Raman gap soliton as computed from Eqs. (1)–(3). Dotted curve: Sech^2 fit to the computed intensity profile.

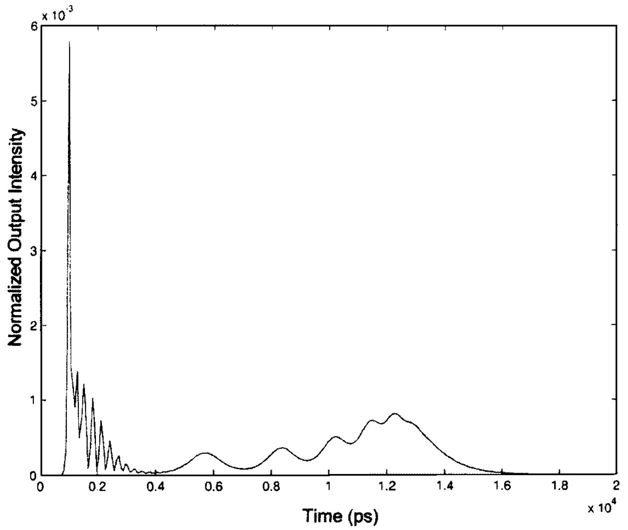

FIG. 4. Output intensity of the backward Stokes wave at $z = 0$ as a function of time. This output corresponds to the slow gap soliton of Fig. 1.

the Stokes wave through the usual Raman gain which is related to the imaginary part of the third-order susceptibility $\chi^{(3)}$. The reactive component, proportional to the real part of $\chi^{(3)}$ tunes the stop band relative to the Stokes frequency through self- and cross-phase modulation. In regular optical fiber solitons, one also speaks of Raman solitons which are created by a pump pulse such that the Stokes wavelength lies in the region of negative GVD [18]. Here too we find both Raman gap solitons, with frequencies in the band gap, and more generally, Raman Bragg solitons whose center frequency lies in the region of negative grating dispersion [19]. Here we have concentrated on the Raman gap soliton which does not require any contribution from negative GVD. The grating feedback also makes it possible to generate intense compressed Stokes pulses through backward stimulated Raman scattering [19] and should enable the operation of a distributed-feedback fiber Raman oscillator.

Recent gap soliton experiments on fiber Bragg gratings have noted certain nonlinear losses that occur at peak intensities of order 20 $\mathrm{GW/cm^2}$ [11,12]. These losses are believed to be due to stimulated Raman scattering [12]. In this paper we turn this loss into our gain with the recognition that what constitutes Raman loss at one wavelength is Raman gain at another. This Raman gain provides the means to efficiently excite a gap soliton at the Raman shifted frequency and should make possible the observation of truly stationary gap solitons.

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