Coherent Λ-Σ Coupling in s-Shell Hypernuclei

Y. Akaishi,[1] T. Harada,[2] S. Shinmura,[3] and Khin Swe Myint[1,4]
[1]*Institute of Particle and Nuclear Studies, KEK, Tanashi, Tokyo 188-8501, Japan*
[2]*Sapporo Gakuin University, Ebetsu, Hokkaido 069-8555, Japan*
[3]*Gifu University, Gifu 501-1112, Japan*
[4]*Mandalay University, Mandalay, Myanmar*
(Received 29 December 1999)

It is found that the suppression due to two-body ΛN-ΣN coupling solves the overbinding problem in ${}^{5}_{\Lambda}$He but it, in turn, causes a severe underbinding in the four-body systems. The shortage of this binding is overcome by introducing explicitly the Λ-Σ coupling which is equivalent to the ΛNN three-body force. This three-body force becomes strong in the 0^+ states of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He according to the coherently added enhancement. The 0^+-1^+ splitting in ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He is found partly due to the ΛN spin-spin interaction and partly due to the Λ-Σ coupling in the recent Nijmegen soft-core potential.

PACS numbers: 21.10.Dr, 13.75.Ev, 21.80.+a, 27.10.+h

There is a long standing problem of fitting the experimental Λ-separation energies of ${}^{3}_{\Lambda}$H, ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He, and ${}^{5}_{\Lambda}$He consistently. Dalitz *et al.* [1] determined a reasonable ΛN central force that produces the correct Λ-separation energies in ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H. Then this potential is found to be so strong for ${}^{5}_{\Lambda}$He that it gives the binding energy value $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ of 5.46 MeV, which is much larger than the experimental value of 3.12 ± 0.02 MeV. Shinmura *et al.* [2] gave a solution to this overbinding problem by including a phenomenological ΛN tensor force. On the other hand, the significant effect of coupling between Σ and Λ has also been proposed in theoretical treatments of the hypernuclear problem [3]. In this Letter, we investigate the binding mechanism of the s-shell hypernuclei and show that some realistic hyperon-nucleon (YN) interactions will reproduce all the s-shell hypernuclear data by solving the overbinding problem with ΛNN three-body force due to the Λ-Σ coupling.

In order to find some keys to solve the overbinding problem we prepare various types of potentials simplified from realistic YN interactions. The potential which is designated as D0 has only a central part of ΛN interaction, D1 has central and tensor parts of ΛN channel interaction, and D2 has central parts of both ΛN and ΣN channels. All central and tensor parts of both channels are included in D3. These D0-3 potentials give identical phase shifts at low energies which are equivalent to the Nijmegen D interaction [4]. The potentials SC89(S) and SC97f(S) are obtained in the same way as D3 from the Nijmegen soft-core SC89 [5] and SC97f [6] interactions.

The Λ-separation energies of four-body and five-body hypernuclei are calculated by the Brueckner-Hartree-Fock method on Gaussian basis: Reaction matrices (g_{YN}) [7] from YN interaction are expanded on 20-range Gaussian functions and are used to obtain a hyperon-nucleus Hartree-Fock potential by folding procedure with nucleon wave functions of Gaussian combination. This hyperon-nucleus potential is again expanded on 10-range Gaussian bases, and is used to solve the relative motion between the hyperon and the core nucleus. Those bases are prepared in geometrical progressions [8]. Matrix elements of potentials with respect to Gaussian functions are calculated without approximation, and the center-of-mass motion of the system is removed by using the hyperon-nucleus reduced mass.

Now let's see some results. The D0 potential of central type ΛN interaction gives a good result $B_\Lambda = 2.44$ MeV for ${}^{4}_{\Lambda}$He compared to the experimental one 2.39 ± 0.03 MeV, but causes the overbinding of 6.66 MeV in ${}^{5}_{\Lambda}$He as it was calculated by Dalitz *et al.* [1]. By employing the D2 potential which includes the ΛN-ΣN coupling of central type the binding energy in the five-body system is reduced to 3.01 MeV, which is well close to the experimental one. This is due to the large suppression of the ΛN-ΣN coupling in the nucleus. While this coupling remedies the overbinding problem in ${}^{5}_{\Lambda}$He, the suppression effect in turn causes a serious underbinding problem in the four-body systems. Namely, we obtain $B_\Lambda = 1.04$ MeV for ${}^{4}_{\Lambda}$He, which is short by 1.35 MeV, by restricting the hyperon wave function in the Λ space where Σ-space effects are taken into account only via intermediate states of reaction matrices.

How can this underbinding problem of the four-body system be solved? Gibson *et al.* [9] first introduced Σ-space components explicitly into the wave function of ${}^{4}_{\Lambda}$He as

$$|{}^{4}_{\Lambda}\mathrm{He}\rangle = \phi_\Lambda(\mathbf{r})|{}^{3}\mathrm{He}\rangle + \sqrt{\frac{2}{3}}\,\phi_{\Sigma^+}(\mathbf{r})\,|{}^{3}\mathrm{H}\rangle - \sqrt{\frac{1}{3}}\,\phi_{\Sigma^0}(\mathbf{r})|{}^{3}\mathrm{He}\rangle, \quad (1)$$

where $\mathbf{r}$ denotes the relative coordinate between the hyperon and the core nucleus. We adopt this idea in order to take into account ΛNN three-body force effects due to the Λ-Σ coupling. Then, the mean potential between the

0031-9007/00/84(16)/3539(3)$15.00

hyperon and the nucleus has a Λ-Σ coupling term, to which the two-body ΛN-ΣN coupling interaction contributes as follows [10]:

$$\frac{3}{2}\,{}^3g_{\Sigma N,\Lambda N} - \frac{1}{2}\,{}^1g_{\Sigma N,\Lambda N} \quad \text{for } 0^+, \qquad \frac{1}{2}\,{}^3g_{\Sigma N,\Lambda N} + \frac{1}{2}\,{}^1g_{\Sigma N,\Lambda N} \quad \text{for } 1^+. \tag{2}$$

Since the ΛN-ΣN coupling interaction is much stronger in the spin-triplet state than in the spin-singlet state, the Λ-Σ coupling effect on B_Λ in the 0^+ state becomes much larger than that in the 1^+ state. This is due to coherent enhancement in the $J^\pi = 0^+$, $J_z = 0$ state where contributions from $[|\Lambda p\rangle \leftrightarrow \sqrt{2/3}\,|\Sigma^+ n\rangle - \sqrt{1/3}\,|\Sigma^0 p\rangle]^{S=1}_{S_z}$ with $S_z = -1, 0, +1$ are added constructively with weight 1/2 each. On the other hand, contributions from $[|\Lambda p\rangle \leftrightarrow \sqrt{2/3}\,|\Sigma^+ n\rangle - \sqrt{1/3}\,|\Sigma^0 p\rangle]^{S=1}_{S_z=+1}$ and $[|\Lambda n\rangle \leftrightarrow \sqrt{1/3}\,|\Sigma^0 n\rangle - \sqrt{2/3}\,|\Sigma^- p\rangle]^{S=1}_{S_z=+1}$ are canceled out and only $[|\Lambda p\rangle \leftrightarrow \sqrt{2/3}\,|\Sigma^+ n\rangle - \sqrt{1/3}\,|\Sigma^0 p\rangle]^{S=1}_{S_z=0}$ remains with weight 1/2 in the $J^\pi = 1^+$, $J_z = +1$ state. Thus, the attractive effect due to the Λ-Σ coupling in the 0^+ state is by about 1 order of magnitude, $9 = [(3/2)/(1/2)]^2$, as large as that in the 1^+ state. In this respect the 0^+ state is an extraordinary state. We define the coupling process of Eq. (1) "coherent Λ-Σ coupling" in which a nucleon remains in its ground state after converting Λ to Σ, giving all the other nucleons an equal footing to interact with the Σ. The other process where a nucleon changes to an excited state after the interaction is called "incoherent Λ-Σ coupling," which is incorporated into reaction matrices in the present treatment.

By using the potential D2 the ΛNN three-body force effect due to the coherent Λ-Σ coupling is estimated to be a 1.23 MeV attraction with 1.9% Σ-mixing for the 0^+ state and only a 0.01 MeV attraction with 0.01% Σ-mixing for the 1^+ state. The D2 potential gives $B_\Lambda = 0.06$ MeV for ${}^3_\Lambda$H and $B_\Lambda = 3.01$ MeV for ${}^5_\Lambda$He. Thus, all the existing experimental energies of s-shell Λ hypernuclei are well reproduced by the simple D2 potential which is given as

$$v(r) = V_a \exp[-(r/a)^2] + V_b \exp[-(r/b)^2], \tag{3}$$

with $a = 0.5$ fm and $b = 1.2$ fm. The strength parameters are given in Table I.

In the D2 case the 0^+-1^+ splitting in ${}^4_\Lambda$H and ${}^4_\Lambda$He comes not from the ΛN spin-spin interaction, but from the ΛNN three-body force. This confirms Gibson's statement that the 0^+-1^+ energy difference is not a measure of the ΛN spin-spin interaction [11]. However, the D3 potential which simulates most closely the original Nijmegen D potential brings only a small 0^+-1^+ splitting energy of 0.13 MeV in contrast to the D2 potential. This is due to the difference of ΛN-ΣN potential type, which is of central type for D2 but is mainly of tensor type for D3. The success of D2 in solving the four-body underbinding problem does not apply to the Nijmegen D potential.

TABLE I. Parameters of the D2 potential in the even states (units in MeV).

	States	$T=1/2$ V_a	$T=1/2$ V_b	$T=3/2$ V_a	$T=3/2$ V_b
	ΛN-ΛN	1165.0	−105.12	⋯	⋯
${}^1E$	ΣN-ΛN	90.547	−45.085	⋯	⋯
	ΣN-ΣN	927.16	−9.9459	1047.0	−119.24
	ΛN-ΛN	763.11	−83.938	⋯	⋯
${}^3E$	ΣN-ΛN	43.344	55.049	⋯	⋯
	ΣN-ΣN	539.41	−116.48	1756.1	−35.821

Recently, the Nijmegen group proposed a new version of soft-core model for YN interaction [6]. We try to solve the overbinding/underbinding problem by employing the realistic soft-core interaction models [5,6]. Figure 1 summarizes the results calculated for ${}^4_\Lambda$He together with the experimental data. The left part and the right part for each interaction are the cases without and with the Σ-space component of Eq. (1), respectively. The level splitting at the left part is due to the ΛN spin-spin interaction, and the level shifts shown by the arrow at the right part are the ΛNN three-body force effect due to the Λ-Σ coupling term of the hyperon-nucleus potential. The coherent Σ admixture is shown in Fig. 1 for the 0^+ state and it is negligibly small for the 1^+ state.

The SC89(S) potential gives $B_\Lambda({}^4_\Lambda\text{He}, 0^+)$ of 2.51 MeV. Carlson [12] obtained 1.6 MeV for it by the VMC method which would be increased a little more by the GFMC method. As seen in the figure the 0^+-1^+ splitting result is too large compared to the experimental one due to too strong ΛN-ΣN coupling of SC89(S). This strong coupling causes oversuppression in the five-body system giving $B_\Lambda({}^5_\Lambda\text{He})$ of 0.53 MeV which is not inconsistent with Carlson's unbound result [12].

The SC97f(S) potential fits rather well all the experimental Λ binding energies of s-shell hypernuclei. It gives $B_\Lambda = 0.16$ MeV for ${}^3_\Lambda$H [13]. In order to understand the nature of interaction we divide the ΛN-ΣN coupling interaction into its central and tensor parts. The three-body force effect on the 0^+-1^+ splitting shown in Fig. 1 is due to the ΛN-ΣN central coupling, while the large suppression is mainly due to the ΛN-ΣN tensor coupling. The 0^+-1^+ splitting in ${}^4_\Lambda$He is found partly due to the ΛN spin-spin interaction and partly due to the ΛN-ΣN central coupling in the case of SC97f(S). The binding $B_\Lambda({}^5_\Lambda\text{He})$ is calculated to be 2.38 MeV for SC97f(S) and 3.57 MeV for SC97e(S). Since the experimental value 3.12 ± 0.02 MeV is in between them, the result for SC97f(S) is not bad and its potential parameters would be adjusted so as to fit the data without difficulty. A proper ratio of the ΛN-ΣN central coupling and tensor coupling is significant in hyperon-nucleon interactions. It is found that the Nijmegen SC97f potential almost meets this requirement.

In summary, the Λ-Σ coupling of the hyperon-nucleus potential can be divided into the incoherent and coherent

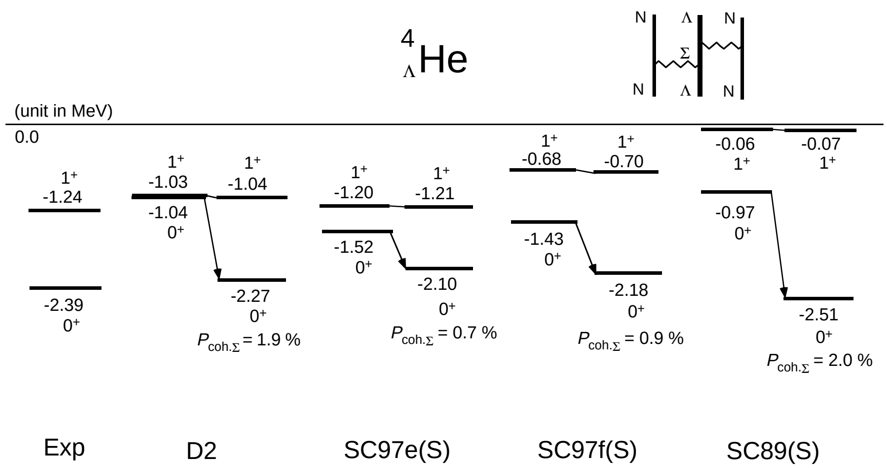

FIG. 1. The Λ energy levels calculated for the 0^+ and 1^+ states of $^{4}_{\Lambda}$He with the D2, SC97e(S), SC97f(S), and SC89(S) potentials. The level shifts shown by the arrow are mainly due to the ΛNN three-body force.

parts where the former gives a suppression effect while the latter provides an attractive effect. The suppression of the incoherent ΛN-ΣN coupling solves the overbinding problem in $^{5}_{\Lambda}$He but it, in turn, causes the underbinding problem in the four-body systems. The shortage of this binding is overcome by the coherent Λ-Σ coupling of which the main part is equivalent to the ΛNN three-body force. This three-body force is strong only in the 0^+ states of $^{4}_{\Lambda}$H and $^{4}_{\Lambda}$He among s-shell hypernuclei according to the coherently added enhancement. It is found that two different types of YN interactions, D2 and SC97f(S), can solve the overbinding/underbinding problem. The SC97f(S) results show that the 0^+-1^+ spin doublet splitting in $^{4}_{\Lambda}$He is half due to the ΛN spin-spin interaction and half due to the Λ-Σ coupling. Thus, the importance of the coherent Λ-Σ coupling in the four-body systems is revealed for the first time on the basis of realistic YN interaction. The coherent Λ-Σ coupling can also explain the ^{4}He(K^-,π^-) spectra including the $^{4}_{\Sigma}$He unstable bound state [14].

The authors would like to thank Professor Th. A. Rijken for valuable discussion and Professor K. Miyagawa for the calculation of $^{3}_{\Lambda}$H with SC97f(S). This work was supported by the Grants-in-Aid for Scientific Research of the Japanese Ministry of Education, No. 08239104.

[1] R. H. Dalitz, R. C. Herndon, and Y. C. Tang, Nucl. Phys. **B47**, 109 (1972).
[2] S. Shinmura, Y. Akaishi, and H. Tanaka, Prog. Theor. Phys. **65**, 1290 (1981).
[3] A. R. Bodmer, Phys. Rev. **141**, 1387 (1966).
[4] M. M. Nagels, Th. A. Rijken, and J. J. de Swart, Phys. Rev. D **12**, 744 (1975).
[5] P. M. M. Maessen, Th. A. Rijken, and J. J. de Swart, Phys. Rev. C **40**, 2226 (1989).
[6] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).
[7] Y. Akaishi and S. Nagata, Prog. Theor. Phys. **48**, 133 (1972).
[8] M. Kamimura, Phys. Rev. A **38**, 621 (1988).
[9] B. F. Gibson, A. Goldberg, and M. S. Weiss, Phys. Rev. C **6**, 741 (1972).
[10] J. Dabrowski, Phys. Rev. C **8**, 835 (1973).
[11] B. F. Gibson, in *Proceedings of the U.S.-Japan Seminar on Properties and Interactions of Hyperons, Maui, 1993*, edited by B. F. Gibson, P. D. Barnes, and K. Nakai (World Scientific, Singapore, 1994), p. 55.
[12] J. A. Carlson, in *LAMPF Workshop on (π, K) Physics*, edited by B. F. Gibson, W. R. Gibbs, and M. B. Johnson, AIP Conf. Proc. No. 224 (AIP, New York, 1991), p. 198.
[13] K. Miyagawa (private communication).
[14] T. Harada, Phys. Rev. Lett. **81**, 5287 (1998).