Wave Front Reversal of a Dipolar Spin Wave Pulse in a Nonstationary Three-Wave Parametric Interaction

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The effects of wave front reversal and inversion of the time profile are observed experimentally for a dipolar spin wave pulse (carrier frequency ω_1) interacting in a yttrium-iron garnet film with a weakly localized nonstationary parametric pumping (carrier frequency $\omega_p \cong 2\omega_1$) in a three-wave process with conservation law $\omega_p = \omega_1 + \omega_2$, where ω_2 is the carrier frequency of the reversed pulse. Theoretical analysis based on the solution of the system of equations for envelopes of interacting wave packets by means of a Green's function formalism gives a quantitative description of the observed phenomena.

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Dipolar spin waves at microwave frequencies propagating in yttrium-iron garnet (YIG) films provide a superb opportunity to study nonlinear wave processes in dispersive, diffractive media with weak dissipation. For example, interesting novel effects like formation [1] and collisions [2] of strongly self-focused two-dimensional dipolar spin wave packets (spin wave "bullets"), and self-generation of spin wave envelope solitons [3] have been observed recently in YIG films.

Here we report the observation of another new nonlinear effect—*wave front reversal* (or phase conjugation) *and inversion of time profile* of a dipolar spin wave pulse at microwave frequency happening as a result of a *nonstationary three-wave* parametric interaction with spatially localized electromagnetic pumping.

The carrier frequencies and wave vectors of the incident (signal) pulse (ω_1, $\mathbf{k}_1$), reversed pulse (ω_2, $\mathbf{k}_2$), and pumping pulse (ω_p, $\mathbf{k}_p$) satisfy the conservation laws

$$\omega_p = \omega_1 + \omega_2, \qquad \mathbf{k}_p = \mathbf{k}_1 + \mathbf{k}_2 . \tag{1}$$

In this process the frequency of electromagnetic pumping ω_p must be almost 2 times larger than the carrier frequencies of the incident (signal) and reversed pulses ($\omega_p \cong 2\omega_1 \cong 2\omega_2$). Since the wave number k_p of the electromagnetic pumping is much smaller than the carrier wave numbers of the dipolar spin waves ($k_p \ll k_1, k_2$), it is possible to obtain front reversal ($\mathbf{k}_2 \cong -\mathbf{k}_1$) of the incident pulse in the process (1). At least two properties make the process of wave front reversal (or phase conjugation) different from a usual wave reflection (see, e.g., [4,5]). First of all, while the direction of the reflected wave is determined by the Snell's law, the reversed wave always propagates in the direction opposite to that of the incident wave. Second, under certain conditions the leading front in the incident wave pulse might become the trailing front of the reversed wave pulse, i.e., the time profile of the reversed pulse might be inverted relative to the incident pulse.

In our present work we have found and practically realized the conditions under which the *inversion of the time profile* of the incident pulse can be observed experimentally in the parametric process (1). It turned out that the inversion of the time profile could happen only if the parametric interaction (1) is *nonstationary* (see [6]), i.e., when the pumping is pulsed, and the duration of the pumping pulse τ_p is small compared to all the other characteristic time scales of the problem

$$\tau_p \ll \tau_1 < \tau_L , \tag{2}$$

where τ_1 is the duration of the incident pulsed signal, and $\tau_L = L/v$ is the time necessary for the incident signal to pass the region of pumping localization (L is the size of the pumping localization region, and v is the group velocity of the incident pulse).

The phenomenon of a wave front reversal (or phase conjugation) has been studied previously for light waves in nonlinear optics [4,5]. In optics, however, the wave front reversal ($\mathbf{k}_2 = -\mathbf{k}_1$) was realized in a second-order *four-wave* parametric process with the conservation laws

$$2\omega_p = \omega_1 + \omega_2, \qquad \mathbf{k}_{p1} + \mathbf{k}_{p2} = \mathbf{k}_1 + \mathbf{k}_2 , \tag{3}$$

when pumping was done by a pair of oppositely directed light waves ($\mathbf{k}_{p1} = -\mathbf{k}_{p2}$) having frequency close to that of the incident and reversed signals $\omega_p \cong \omega_1 \cong \omega_2$. The wave front reversal caused by a three-wave interaction (1) was never observed in optics due to the difficulties in fulfilling the conservation laws (1) in the spectrum of light waves in optical fibers.

Thus, our present experiments performed on microwave spin waves in magnetic films provide the first experimental evidence of the *wave front reversal* effect, caused by *the first-order three-wave* parametric process (1). In contrast with optics, it is easy to realize the three-wave process

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(1) for dipolar spin waves at microwave frequencies, and this process has been used for the amplification of linear spin wave pulses and spin wave envelope solitons [6–8]. We also note, that in a media with dissipation the power threshold of the three-wave parametric process (1) is much lower than the threshold of the four-wave process (3) (see pp. 23–28 in [9]).

Preliminary attempts to observe the wave front reversal process for dipolar spin waves in a three-wave interaction in a YIG films have been reported in [10]. A Green's function formalism for the solution of the problem of parametric three-wave interaction was also proposed there. The experiments in [10] were performed, however, in a *stationary* regime of parametric interaction, when pumping pulse duration was relatively long $\tau_p > \tau_L$. Therefore, it was not possible to observe the *inversion of the time profile* of the incident pulse in [10].

In this Letter, we elucidate the conditions under which the *inversion of the time profile* of the incident pulse becomes possible in the process of wave front reversal caused by the three-wave interaction (1). To do this, we analyze theoretically the interaction (1) using the system of reduced parametric equations for the wave packet envelopes $a_{1,2}(z,t)$ dependent on time t and spatial coordinate z along the direction of wave propagation [10–12]

$$\left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial z} + \Gamma_1\right) a_1(z,t) = V a_p(z,t) a_2^*(z,t), \quad \text{(4a)}$$

$$\left(\frac{\partial}{\partial t} - v_2 \frac{\partial}{\partial z} + \Gamma_2\right) a_2^*(z,t) = V^* a_p^*(z,t) a_1(z,t), \quad \text{(4b)}$$

where $v_1 = v_2 = v$, and $\Gamma_1 = \Gamma_2 = \Gamma = g\Delta H$ are group velocities and dissipation parameters of the incident and reversed pulses, correspondingly, g is the gyromagnetic ratio ($g/2\pi = 2.8$ MHz/Oe), ΔH is the half-linewidth of the ferromagnetic resonance in a YIG film, $a_p(z,t)$ is the amplitude of the nonstationary localized pumping, V is the coupling coefficient between spin waves and parallel pumping defined in [8] (in our experiment $V/2\pi = 0.67$ MHz/Oe), and the asterisk denotes complex conjugation. Equations (4) are correct in the case when the interacting wave packets are spectrally narrow $2\pi/\tau_{1,2} \ll \omega_{1,2}$ (where τ_1 and τ_2 are the durations of the input and reversed pulses correspondingly), and when the parametric parallel pumping is localized weakly, so that $\Delta k_p \approx 2\pi/L \ll k_{1,2}$.

In some particular cases the system of Eqs. (4) can be solved analytically using the Green's function formalism described in [10]. In particular, in the case of a nonstationary locally homogeneous pulsed pumping in the form of a rectangular pulse of duration $\tau_p \ll \tau_L$, which has constant amplitude a_p in the localization region $0 \le z \le L$ and is vanishing everywhere else, it is possible to obtain an explicit expression for the time profile $a_2^*(z=0,t)$ of the reversed pulse at the left boundary ($z=0$) of the pumping localization region

$$a_2^*(z=0,t) = \sinh(Va_p\tau_p) \times \left[a_1(2t_p + \tau_p - t) + \left(\frac{Va_p\tau_p}{\tanh(Va_p\tau_p)} - 1\right)\frac{1}{2(Va_p)^2}\frac{\partial^2 a_1}{\partial t^2}\right], \quad (5)$$

where $a_1(z=0,t)$ is the profile of the incident pulse at $z=0$, and t_p is the moment when the pumping is switched on.

The expression (5), obtained in the case when the pumping pulse duration has a finite value τ_p, is a generalization of the classical expression for the time profile of a reversed signal obtained in nonlinear optics (see, e.g., [5]) for the case when pumping has the form of the Dirac's δ function [$a_p(t) \propto \delta(t - t_p)$, i.e., $\tau_p \to 0$]

$$a_2^*(z=0,t) = Ca_1(2t_p - t), \quad (6)$$

where C is the constant coefficient dependent on the pumping power. Equation (6) describes the ideal inversion of the incident signal time profile.

It is clear from Eq. (5), that in the case of finite pumping pulse duration $\tau_p > 0$ the time profile of the reversed pulse is distorted. These distortions are proportional to the second derivative of the incident signal time profile $a_1(t)$, and become larger with the increase of τ_p.

This effect is illustrated by Fig. 1 where the time profiles of the reversed pulse $a_2^*(t)$ are calculated numerically using Eqs. (4) for a triangular-shaped incident pulse (with base duration τ_Δ) and different durations τ_p of a rectangular pumping pulse. It is clear from Fig. 1, that distortions of the reversed pulse shape increase sharply with the increase of the pumping pulse duration. Clear inversion of the incident pulse time profile takes place only when $\tau_p < 0.1\tau_\Delta$. For the incident pulse of an arbitrary shape this condition can be written as $\tau_p \ll \tau_1$. It is also necessary for the incident pulse not to leave the region of pumping localization during the action of the pumping pulse, so that $\tau_1 + \tau_p < \tau_L$. Thus, if we wish to observe the inversion in time of the input pulse profile in a real experiment, the regime of parametric interaction (1) must be *nonstationary,* so that the characteristic time scales of the problem obey the inequality (2) (for details, see [6]).

Our experiments on the front reversal of dipolar spin waves were performed using a standard delay-line structure [8,10] shown in Fig. 2. This structure consists of input (1) and output (2) microstrip transducers of the width 50 μm separated by the distance $l = 5$ mm. To create a spatially extended region of parallel pumping localization ($L = 3.3$ mm in our experiment) the pumping pulse was supplied to an open dielectric resonator situated between the microstrip transducers and made of a thermostable ceramics (dielectric permeability $\epsilon \approx 80$). A YIG film sample (in-plane sizes 1.6 mm $\times$ 18 mm, thickness

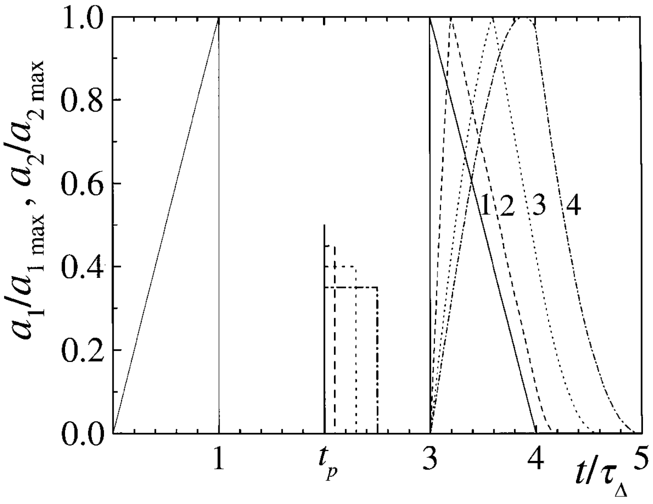

FIG. 1. Time profiles of a reversed pulse (right) created by the triangular input pulse of base duration τ_Δ (left) interacting with rectangular pumping pulses of different duration τ_p (middle). The pumping pulse is switched on at the moment $t = t_p = 2\tau_\Delta$, and its duration is expressed in the units of $\xi = \tau_p/\tau_\Delta$. 1: $\xi \to 0$; 2: $\xi = 0.1$; 3: $\xi = 0.3$; 4: $\xi = 0.5$.

4.9 μm, saturation magnetization $4\pi M_0 = 1750$ Oe, ferromagnetic resonance linewidth measured at 5 GHz $2\Delta H = 0.4$ Oe) was placed on top of the microstrip transducers, while the central part of it was inside the pumping dielectric resonator (see Fig. 2). The YIG film was magnetized tangentially to its surface by the external magnetic field of 1020 Oe. The effective bias magnetic field, which accounts for the combined effects of cubic and uniaxial anisotropies, was $H_0 = 1042$ Oe. This geometry corresponds to the excitation of backward volume magnetostatic waves (BVMSW) propagating along the direction of the bias magnetic field $\mathbf{H}_0$. The calculated value for the upper boundary of the BVMSW spectrum was $\omega_\perp = g\sqrt{H_0(H_0 + 4\pi M_0)} = 2\pi \times 4775$ MHz, which turned out to be in good agreement with experiment.

The carrier frequency ω_1 of the incident (signal) pulse was chosen to be $\omega_1 = 2\pi \times 4720$ MHz which corresponds to the carrier wave number of $k_1 = 155\ \mathrm{cm}^{-1}$ and group velocity $v = -2.21\ \mathrm{cm}/\mu\mathrm{s}$, $\tau_L = L/v = 150$ ns. This position of the frequency ω_1 in the BVMSW spectrum allows us to work with rectangular signal pulses of duration $\tau_1 \geq 20$ ns without large distortions in their spectrum. The maximum power of the input pulse was smaller than 5 mW, which guaranteed a linear regime of signal pulse propagation when pumping was switched off. The carrier frequency of the pumping pulse was chosen to be $\omega_p = 2\omega_1 = 2\pi \times 9440$ MHz, and the maximum pumping power was $(P_p)_{\max} = 5$ W.

The experimental curves demonstrating the effect of wave front reversal for a BVMSW pulse in a *nonstationary* regime of parametric interaction ($\tau_p \ll \tau_1 < \tau_L$) are presented in Fig. 3. To observe explicitly the reversal of the time profile of the incident signal, we used the incident signal in the form of two successive rectangular pulses of duration 25 ns separated by the interval of 40 ns (see curves 1 and 2 in the left side of Fig. 3), so the full duration of the input signal was $\tau_1 \cong 90$ ns. The pumping pulse duration was $\tau_p \cong 20$ ns as shown in the middle of Fig. 3.

In the case when two rectangular pulses in the incident signal had equal amplitudes, two maxima in the reversed signal were exactly equal, and it was not possible to determine in which order the pulses arrive at the input transducer (see dotted curves 2 and 2′ in Fig. 3). In contrast,

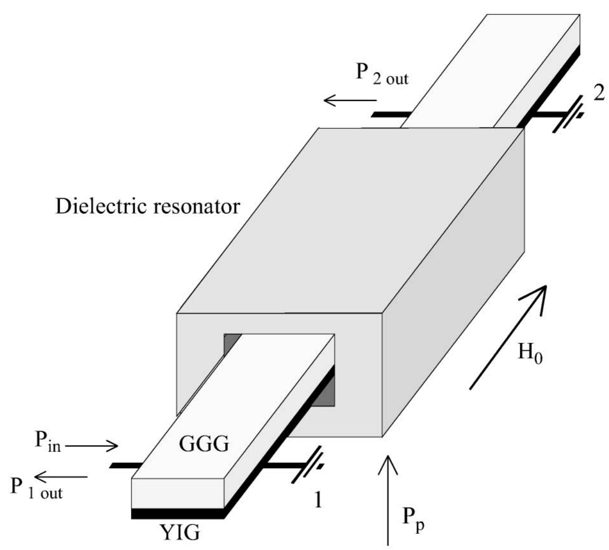

FIG. 2. Diagram of the experimental delay-line structure: 1 and 2 are the input and output microstrip transducers. The dielectric resonator in the middle is used to supply a pumping pulse.

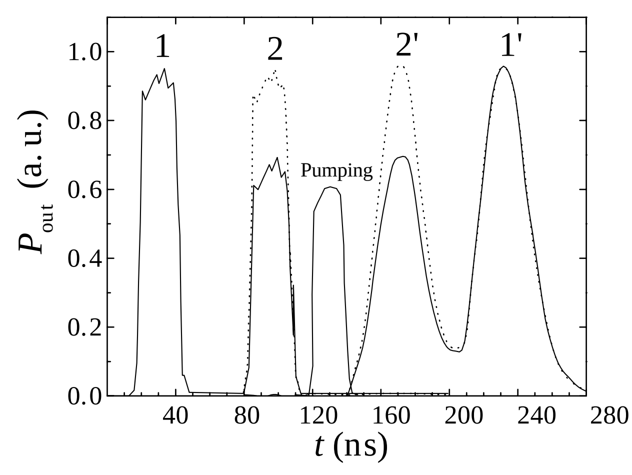

FIG. 3. Reversal of the time profile of the incident signal. 1,2: Signals reflected from the input transducer before interaction with pumping (amplitudes reduced by 20 dB); 1′, 2′: Signals at the input transducer created by the reversed wave formed in a first-order parametric interaction with pumping. Dotted lines show the case of input signal pulses of equal amplitudes, solid lines show the case when input pulses have different amplitudes.

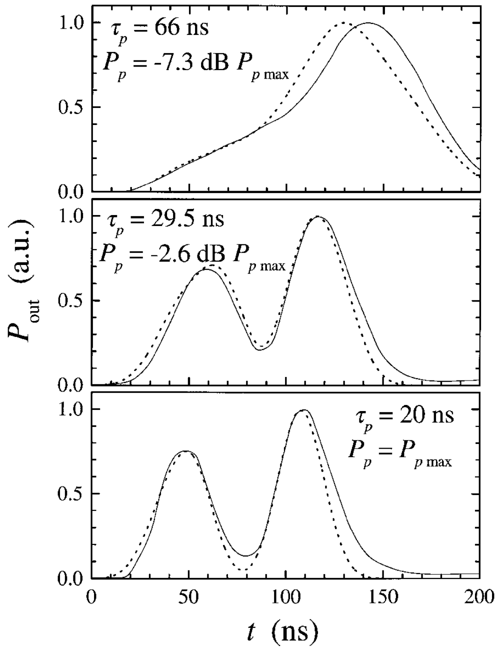

FIG. 4. Comparison of theoretical (dotted curves) and experimental (solid curves) profiles of the reversed signal corresponding to different values of the pumping pulse duration τ_p, and different values of the pumping power P_p. The time profile of the incident signal is the same as in the left side of Fig. 3. $(P_p)_{\max} = 5$ W.

when the amplitudes of the incident pulses were different (solid curves in Fig. 3) it is clear that the order in which the reversed pulses arrive at the input transducer is opposite to the order of these pulses in the incident signal. In other words, the *time profile of the incident signal is inverted*.

It can be seen that the bell-like shapes of maxima $1'$, $2'$ in the reversed signal in Fig. 3 are different from the almost rectangular shapes of the pulses 1, 2 in the incident signal. These distortions are caused by the effect of the finite duration of the pumping pulse, qualitatively described by Eq. (5) and illustrated in Fig. 1.

To make a quantitative comparison of our experimental results with theory, we solved the system of equations (4) numerically for the incident signal profile consisting of two successive rectangular pulses (as shown in the left side of Fig. 3), and for different durations of the pumping pulse τ_p. The results of this solution (dotted curves) are shown in Fig. 4 in comparison with experimentally measured profiles of the reversed signal at the input microstrip transducer.

It is clear from Fig. 4, that for the largest value of the pumping pulse duration $\tau_p = 66$ ns the distortions of the reversed pulse profile are so substantial, that the two maxima in the reversed signal are not resolved. For smaller values of τ_p ($\tau_p = 29.5$ ns and $\tau_p = 20$ ns), when the regime of parametric interaction can be considered *nonstationary* ($\tau_p \ll \tau_L$), the maxima in the reversed signal are clearly resolved, although the distorsions of their fronts are still present. It is worth noting that in all three cases presented in Fig. 4 the numerical solution of the system (4) gives good quantitative description of the observed effect.

In conclusion, we have observed for the first time the *inversion of the time profile* of a dipolar spin wave pulse in the wave front reversal process caused by the *nonstationary three-wave parametric interaction* (1) with spatially localized parallel pumping. We have also formulated conditions under which the experimental observation of the this phenomenon is possible. The most important of these conditions is the *nonstationary* character of parametric interaction (1), when the duration of the pumping pulse τ_p is much smaller than the time τ_L necessary for the incident signal to pass the region of pumping localization ($\tau_p \ll \tau_L$). Our experimental results are in quantitative agreement with the theory based on the solution of the system of the reduced parametric equations (4) for the envelopes of interacting wave packets.

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