Anomalous Scaling Dimensions and Stable Charged Fixed Point of Type-II Superconductors

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The critical properties of a type-II superconductor model are investigated using a dual vortex representation. Computing the propagators of gauge field **A** and dual gauge field **h** in terms of a vortex correlation function, we obtain the values $\eta_{\mathbf{A}} = 1$ and $\eta_{\mathbf{h}} = 1$ for their anomalous dimensions. This provides support for a dual description of the Ginzburg-Landau theory of type-II superconductors in the continuum limit, as well as for the existence of a stable charged fixed point of the theory, not in the 3D *XY* universality class.

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Determining the universality class of the phase transition in a system of a charged scalar field coupled to a massless gauge field, such as a type-II superconductor, has been a long-standing problem [1]. Analytical and numerical efforts have recently focused on the use of a *dual* description of the Ginzburg-Landau theory (GLT) of type-II superconductors, pioneered by Kleinert [2], in investigating the character of a proposed novel *stable* fixed point of the theory for a charged superconducting condensate, in which case the 3D *XY* fixed point of the neutral superfluid is rendered unstable [3–6]. The dual formulation has also been employed to investigate the possibility of novel broken symmetries in the vortex liquid phase of such systems in magnetic fields [4,5].

The GLT is defined by a complex matter field ψ coupled to a massless fluctuating gauge field **A** with a Hamiltonian

$$H_{\psi,\mathbf{A}} = m_\psi^2|\psi|^2 + \frac{u_\psi}{2}|\psi|^4 + |(\nabla - i2e\mathbf{A})\psi|^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2. \qquad (1)$$

Here, e is the electron charge, and $H_{\psi,\mathbf{A}}$ is invariant under the local gauge transformation $\psi \to \psi\exp(i\theta)$, $\mathbf{A} \to \mathbf{A} + \nabla\theta/2ie$. The GLT sustains stable topological objects in the form of vortex lines and vortex loops; the latter are the critical fluctuations of the theory [4,5]. These are nonlocal in terms of ψ, but local in a dual formulation. The continuum dual representation of the topological excitations ($D = 3$ only) consists of a complex matter field ϕ coupled to a *massive* gauge field **h** [2], with coupling constant given by the dual charge e_d, and with dual Hamiltonian

$$H_{\phi,\mathbf{h}} = m_\phi^2|\phi|^2 + \frac{u_\phi}{2}|\phi|^4 + |(\nabla - ie_d\mathbf{h})\phi|^2 + \frac{1}{2}(\nabla\times\mathbf{h})^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + ie(\nabla\times\mathbf{h})\cdot\mathbf{A}. \qquad (2)$$

The massiveness of **h** reduces the symmetry to a global $U(1)$ invariance. For details on how to obtain this dual Hamiltonian, we refer the reader to the thorough exposition of this presented in the textbook of Kleinert [7]. For $e \neq 0$ the original GLT in Eq. (1) has a local gauge symmetry, the dual theory in Eq. (2) has a global $U(1)$ symmetry. In the limit $e \to 0$, **A** decouples from ψ in Eq. (1), H_ψ describes a *neutral superfluid*, and the symmetry is reduced to global $U(1)$. The dual Hamiltonian $H_{\phi,\mathbf{h}}$ describes a charged superfluid coupled to a massless gauge field **h** with coupling constant e_d, and the global symmetry is extended to a local gauge symmetry. Hence, when $e \to 0$, *the dual of a neutral superfluid is isomorphic to a superconductor*. Integrating out the **A** field in Eq. (2) produces a mass term $e^2\mathbf{h}^2/2$, where an exact renormalization-group equation for the mass of **h** is given by $\partial e^2/\partial \ln l = e^2$ [8]. Therefore, when $e \neq 0$, then $e^2 \to \infty$ as $l \to \infty$. This suppresses the dual gauge field, and the resulting dual theory is a pure $|\phi|^4$ theory. Hence, in the long-wavelength limit, *the dual of a superconductor is isomorphic to a neutral superfluid* [2].

In this paper, we obtain the anomalous scaling dimensions $\eta_{\mathbf{A}}$ of the gauge field [3,9], as well as $\eta_{\mathbf{h}}$ of the dual gauge field. At a 3D *XY* critical point, $\eta_{\mathbf{A}} = \eta_{\mathbf{h}} = 0$. We find that ($\eta_{\mathbf{A}} = 1, \eta_{\mathbf{h}} = 0$) when $e \neq 0$, and that ($\eta_{\mathbf{A}} = 0, \eta_{\mathbf{h}} = 1$), when $e = 0$. We also contrast the anomalous dimension of the dual mass field ϕ at the dual charged (original neutral) and dual neutral (original charged) fixed points, obtaining $\eta_\phi = -0.24$ in the former case, and $\eta_\phi = 0.04$ in the latter.

A duality transformation, to a set of interacting vortex loops, is performed on the London/Villain approximation to the GLT. In this approximation the partition function is

$$Z(\beta, e) = \int D\mathbf{A}D\theta \times \sum_{\{n\}} \exp\left[-\sum_{\mathbf{x}}\left\{\frac{1}{2}(\Delta\times\mathbf{A})^2 + \frac{\beta}{2}(\Delta\theta - e\mathbf{A} - 2\pi\mathbf{n})^2\right\}\right]. \qquad (3)$$

Here, θ is the local phase of the superconducting order parameter ψ, while **n** is an integer-valued *velocity field* (not vortex field) introduced to make the Villain potential 2π-periodic. The symbol Δ denotes a lattice derivative.

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Amplitude fluctuations are neglected in this approach. The validity of this approximation for $3D$ systems has recently been investigated in detail [10].

An auxiliary velocity field $\mathbf{v}$ linearizes the kinetic energy. Performing the θ integration constrains $\mathbf{v}$ to satisfy the condition $\Delta \cdot \mathbf{v} = 0$, explicitly solved by writing $\mathbf{v} = \Delta \times \mathbf{h}$, where $\mathbf{h}$ is forced to integer values by the summation over $\mathbf{n}$. Introducing an integer-valued *vortex field* $\mathbf{m} = \Delta \times \mathbf{n}$, and using Poisson's summation formula, we find

$$S(\mathbf{A}, \mathbf{h}, \mathbf{m}) = \sum_{\mathbf{x}} \Big\{ 2\pi i \mathbf{m} \cdot \mathbf{h} + \frac{1}{2\beta} (\Delta \times \mathbf{h})^2 + ie(\Delta \times \mathbf{h})\mathbf{A} + \frac{1}{2} (\Delta \times \mathbf{A})^2 \Big\}. \quad (4)$$

Integrating the gauge field in Eq. (4) produces a mass term $e^2\mathbf{h}^2/2$, giving an effective theory containing the vortex field $\mathbf{m}$ coupled to a *massive* gauge field $\mathbf{h}$

$$Z(\beta, e) = \int D\mathbf{h} \sum_{\{\mathbf{m}\}} \prod_{\mathbf{x}} \delta_{\Delta \cdot \mathbf{m}, 0} \times \exp\Big[-\sum_{\mathbf{x}} \Big\{ 2\pi i \mathbf{m} \cdot \mathbf{h} + \frac{e^2}{2} \mathbf{h}^2 + \frac{1}{2\beta} (\Delta \times \mathbf{h})^2 \Big\} \Big]. \quad (5)$$

The variables in Eq. (5) are defined on a lattice which is dual to the lattice from Eq. (3), and the behavior with respect to temperature is inverted in the new variables. The θ field in Eq. (3) describes *order,* while the $\mathbf{m}$ field represents the topological excitations of the θ field. These excitations destroy superconducting coherence, and hence quantify *disorder* [7].

Integrating out the $\mathbf{h}$ field in Eq. (5), we obtain the Hamiltonian employed in the present simulations,

$$H(\mathbf{m}) = -2\pi^2 J_0 \sum_{\mathbf{x_1}, \mathbf{x_2}} \mathbf{m}(\mathbf{x_1}) V(\mathbf{x_1} - \mathbf{x_2}) \mathbf{m}(\mathbf{x_2}), \quad (6)$$

$$V(\mathbf{x}) = \sum_{\mathbf{q}} \frac{e^{-i\mathbf{q}\cdot\mathbf{x}}}{4 \sum_{\mu} \sin^2(\frac{q_\mu}{2}) + \lambda^{-2}}. \quad (7)$$

In Eq. (7), the charge e and lattice-spacing a have both been set to unity, and λ is the bare London penetration depth. At every MC step, we attempt to insert a loop of unit vorticity and random orientation. A new energy is calculated from Eq. (6), and the proposed move is accepted or rejected according to the Metropolis algorithm. This procedure ensures that the vortex lines of the system always form closed loops of random size and shape [5]. A system size of $40 \times 40 \times 40$ was used, and up to 1.5×10^5 sweeps over the lattice per temperature were used.

To investigate the properties of $\mathbf{A}$ and $\mathbf{h}$ at the charged critical point of the original theory, Eq. (1), we have calculated the correlation functions $\langle \mathbf{A_q A_{-q}} \rangle$ and $\langle \mathbf{h_q h_{-q}} \rangle$ in terms of vortex correlations, obtaining

$$\langle \mathbf{A_q A_{-q}} \rangle = \frac{1}{|\mathbf{Q}|^2 + m_0^2} \left(1 + \frac{4\pi^2 \beta m_0^2 G(\mathbf{q})}{|\mathbf{Q}|^2 [|\mathbf{Q}|^2 + m_0^2]} \right), \quad (8)$$

$$\langle \mathbf{h_q h_{-q}} \rangle = \frac{2\beta}{|\mathbf{Q}|^2 + m_0^2} \left(1 - \frac{2\beta \pi^2 G(\mathbf{q})}{|\mathbf{Q}|^2 + m_0^2} \right), \quad (9)$$

where $G(\mathbf{q}) = \langle \mathbf{m_q m_{-q}} \rangle$, $m_0 = \lambda^{-1}$, and $Q_\mu = 1 - e^{-i\mathbf{q}\cdot\hat{\mu}}$. All correlation functions have been calculated in the *transverse gauge* $\nabla \cdot \mathbf{A} = \nabla \cdot \mathbf{h} = 0$.

Invoking the standard form $(q^2 + m_{\text{eff}}^2)^{-1}$ for the correlation functions in the immediate vicinity of the critical point in the limit $q \to 0$, we find the following expressions for the effective masses:

$$(m_{\text{eff}}^{\mathbf{A}})^2 = \lim_{q \to 0} \frac{m_0^2}{1 + 4\pi^2 \beta G(\mathbf{q}) q^{-2}}, \quad (10)$$

$$(m_{\text{eff}}^{\mathbf{h}})^2 = \lim_{q \to 0} \frac{m_0^2}{2\beta [1 - \frac{2\pi^2 \beta G(\mathbf{q})}{m_0^2}]}. \quad (11)$$

When $e \neq 0$ the correlation function for $\mathbf{A}$ assumes the form $\langle \mathbf{A_q A_{-q}} \rangle \propto 1/q^{2-\eta_\mathbf{A}}$ at the critical point. To determine $\eta_\mathbf{A}$, we compute the vortex correlator $G(q)$. For $\lambda \ll L = 40$, we expect the behavior $\lim_{q \to 0} G(q) \propto q^2$, q^η, $C(T)$ for $T < T_c$, $T = T_c$, and $T > T_c$, respectively. When these limiting forms are inserted in Eq. (10), we see that for $T \leq T_c$, $m_{\text{eff}}^{\mathbf{A}}$ will be finite through the Higgs Mechanism (Meissner effect). For $T \geq T_c$ we will have $m_{\text{eff}}^{\mathbf{A}} = 0$ as in the normal case of a massless photon. Assuming $G(q) \propto q^\eta$ precisely at the critical point, it is seen that η corresponds to $\eta_\mathbf{A}$ from the definition of $\langle \mathbf{A_q A_{-q}} \rangle$. *We thus identify the scaling power of $G(\mathbf{q})$ at the critical point with the anomalous dimension of the massless gauge field* $\mathbf{A}$.

Figures 1 and 2 show $G(q)$ and the gauge field masses $m_{\text{eff}}^{\mathbf{A}}$ and $m_{\text{eff}}^{\mathbf{h}}$, respectively. At the critical point $G(q) \propto q$, so that $\eta_\mathbf{A} = 1$. Note that, while $m_{\text{eff}}^{\mathbf{A}}$ vanishes at $T = T_c$, $m_{\text{eff}}^{\mathbf{h}}$ is finite but nonanalytic. *As a result of the vortex*

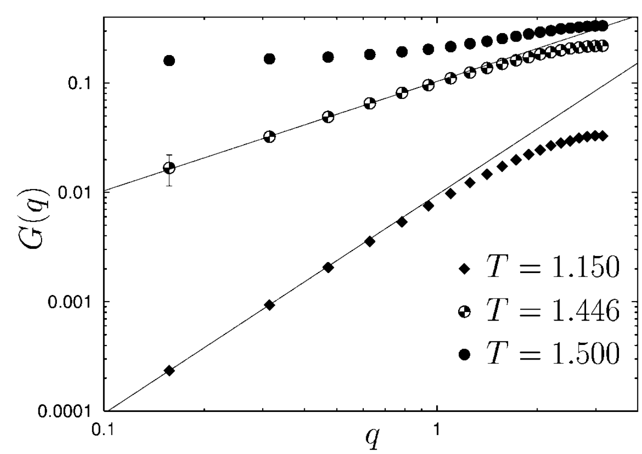

FIG. 1. A log-log plot of $G(q)$ for $T < T_c$, $T = T_c$, and $T > T_c$, with $\lambda = a/2$. For this λ, $T_c = 1.446$. Apart from the point $q = q_{\text{min}}, T = 1.446$ the error bars are smaller than the symbols used.

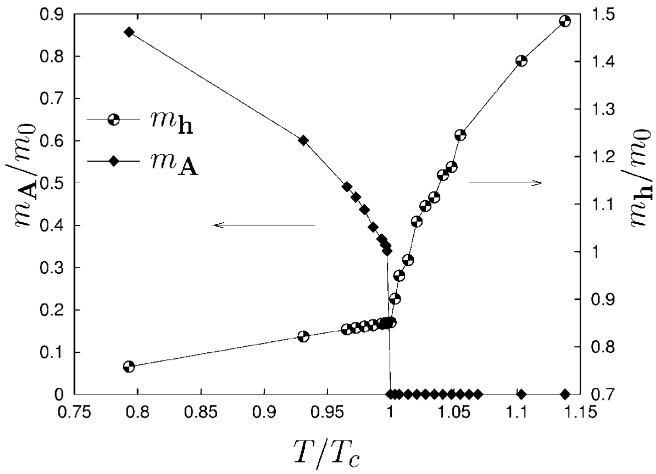

FIG. 2. $m_{\rm eff}^{\mathbf{A}}/m_0$ and $m_{\rm eff}^{\mathbf{h}}/m_0$ as functions of T.

loop blowout, the screening properties of the vortices are dramatically increased, and $m_{\rm eff}^{\mathbf{h}}$ *increases sharply*.

To find $\eta_{\mathbf{h}}$ independently, we consider first the uncharged case $\lambda \to \infty$, $m_0 \to 0$. First, at an intermediate step in the transformation Eqs. (3)–(5), the action reads

$$S(\beta, e) = -\sum_{\mathbf{x}} \left\{ \frac{1}{2\beta} \mathbf{l}^2 + ei\mathbf{A} \cdot \mathbf{l} + \frac{1}{2} (\nabla \times \mathbf{A})^2 \right\}. \quad (12)$$

Here, $\mathbf{l}$ is an integer field of closed current loops. Setting $e = 0$ in Eq. (5), the action of the dual Villain model is obtained,

$$\tilde{S}_V(\beta, \Gamma) = -\sum_{\mathbf{x}} \left\{ 2\pi i \mathbf{m} \cdot \mathbf{h} + \frac{1}{2\beta} (\Delta \times \mathbf{h})^2 + \frac{\Gamma}{2} \mathbf{m}^2 \right\}. \quad (13)$$

Here, a term $\Gamma \mathbf{m}^2/2$ has been added, and $\tilde{S}_V(\beta, \Gamma)$ corresponds to the Villain action in the limit $\Gamma \to 0$. However, it is physically reasonable to propose that the limit $\Gamma \to 0$ is nonsingular, since the added term is short ranged. It should therefore be an irrelevant perturbation to the long-ranged Biot-Savart interaction governing the fixed point. Rescaling $\mathbf{h} \to \mathbf{h}e/2\pi$ in Eq. (13), we have [11] $Z(\beta, e) = \tilde{Z}_V(e^2/4\pi^2, 1/2\beta)$, leaving Eqs. (12) and (13) interchangeable; $\eta_{\mathbf{h}}$ from Eq. (13) should have the same value as $\eta_{\mathbf{A}}$ from Eq. (12). The above is demonstrated by our simulations based on Eqs. (6)–(9), *which are independent of the proposed form Eq. (13)*.

To determine $\eta_{\mathbf{h}}$ we study the correlation function $\langle \mathbf{h_q h_{-q}} \rangle$ [Eq. (9)] in the limit $m_0 \to 0$. At the uncharged fixed point of the original theory, which is the charged fixed point of the dual theory, we have $\lim_{q \to 0} 2\pi\beta^2 G(\mathbf{q}) = [1 - C_2(T)]q^2 + \dots$, $q^2 - C_3(T)q^{2+\eta_{\mathbf{h}}} + \dots$, and $q^2 - C_4(T)q^4 + \dots$, for $T < T_c$, $T = T_c$, and $T \geq T_c$, respectively. Here, $C_2(T)$ corresponds to the helicity modulus (superfluid density) [12], $C_3(T)$ is a critical amplitude, and $C_4(T)$ is the inverse of the mass of the dual gauge field for $T \geq T_c$. Correspondingly, we have $\lim_{q \to 0} \langle \mathbf{h_q h_{-q}} \rangle = 2\beta C_2/q^2$, $2\beta C_3/q^{2-\eta_{\mathbf{h}}}$, and $2\beta C_4$, for $T < T_c$, $T = T_c$, and $T \geq T_c$, respectively. Note that $\mathbf{h}$ is massless for $T < T_c$, while it is massive for $T > T_c$, the dual system exhibits a "dual Meissner effect" for $T \geq T_c$. At $T = T_c$, we have $q^2 \langle \mathbf{h_q h_{-q}} \rangle \simeq C_3(T)q^{\eta_{\mathbf{h}}}$. A plot of $q^2 \langle \mathbf{h_q h_{-q}} \rangle$ is shown in Fig. 3. A linear behavior at $T = T_c$ is found, implying that $\eta_{\mathbf{h}} = 1$ when $e = 0$. Since $\eta_{\mathbf{h}} = 1$ in the uncharged case, this provides further support for the Hamiltonian Eq. (2).

We now set $e \neq 0$. The gauge field $\mathbf{h}$ becomes massive via the term $e^2\mathbf{h}^2/2$, which appears after integrating out the $\mathbf{A}$ field in Eq. (2). In this case, $\lim_{q \to 0} \langle \mathbf{h_q h_{-q}} \rangle = 2\beta/m_0^2$ from Eq. (9), and $\mathbf{h}(r)$ would naively have the trivial scaling dimension $(2 - d)/2$. However, the mass term offers us a freedom in assigning dimensions to e and $\mathbf{h}$, by introducing renormalization Z-factors, here $e' = Z_{\mathbf{h}}^{1/2} e$ and $\mathbf{h}' = Z_{\mathbf{h}}^{-1/2} \mathbf{h}$.

Prior to integrating out $\mathbf{A}$ in Eq. (2), the mass appears in the term $ie(\nabla \times \mathbf{h}) \cdot \mathbf{A}$. Integration of the ϕ field, partial or complete, can only produce $(\nabla/i - e_d \mathbf{h})$ terms. In particular, this must hold during integration of fast Fourier modes of the ϕ field. Thus, the term $i(\nabla \times \mathbf{h}) \cdot \mathbf{A}$ *is renormalization group invariant*, i.e., its prefactor must be dimensionless. In terms of scaled fields, at the charged fixed point of the original theory, we have $\mathbf{A}' = Z_{\mathbf{A}}^{-1/2} \mathbf{A}$, with $Z_{\mathbf{A}} \propto l^{\eta_{\mathbf{A}}}$, $\eta_{\mathbf{A}} = 1$ [8]. For $\mathbf{h}$, we use $Z_{\mathbf{h}} \propto l^{\Delta}$, where Δ is *not* an anomalous scaling dimension ($\mathbf{h}$ is massive, cf. Fig. 2), but rather a contribution to the engineering dimension of $\mathbf{h}$. Inserting this into the crossterm $ie(\nabla \times \mathbf{h}) \cdot \mathbf{A}$, we find the scaling dimension $(\eta_{\mathbf{A}} + \Delta)/2 - 1$, which must vanish. This gives the constraint $\Delta = 1$ to avoid conflicting results for $\eta_{\mathbf{A}}$.

Remarkably, therefore, the scaling dimension of $\mathbf{h}$ at $T = T_c$ is the same in both cases $m_0 = 0$ and $m_0 \neq 0$. The results for $\eta_{\mathbf{A}}$ and $\eta_{\mathbf{h}}$ in the previous paragraphs are summed up in Table I.

During the simulations, we sample the distribution of loop sizes $D(p)$, where p is the perimeter of a loop. This

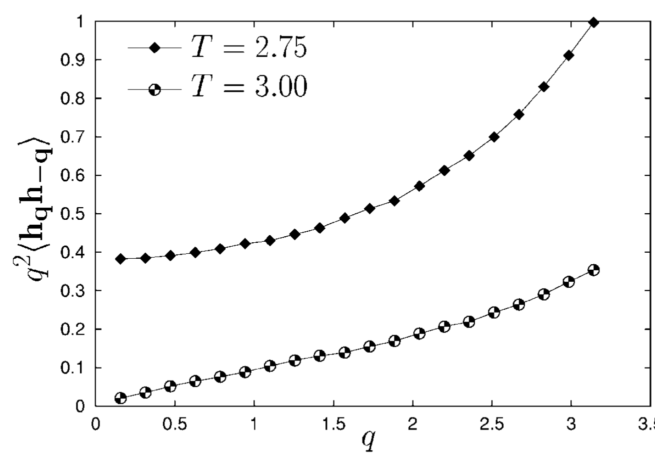

FIG. 3. $q^2\langle \mathbf{h_q h_{-q}} \rangle$ for two different T. For $\lambda = \infty$, $T_c = 3.00$.

TABLE I. Values of $\eta_{\mathbf{A}}$ and $\eta_{\mathbf{h}}$ at the stable neutral and charged critical points of the original and dual theories. FP is an abbreviation for fixed point.

m_0	$\eta_{\mathbf{A}}$	FP original theory	$\eta_{\mathbf{h}}$	FP dual theory
0	0	neutral 3D *XY*	1	charged
Finite	1	charged	0	neutral 3D *XY*

distribution function can be fitted to the form [4,5]

$$D(p) \propto p^{-\alpha} e^{-\beta p \varepsilon(T)}, \tag{14}$$

where $\varepsilon(T)$ is an effective line tension for the loops [5]. The critical point is characterized by a vanishing line tension, and close to the critical point we find that $\varepsilon(T)$ vanishes as $\varepsilon(T) \propto |T - T_c|^{\gamma_\phi}$.

The vortex loops are the topological excitations of the GL and 3D *XY* models; at the same time they are the real-space representation of the Feynman diagrams of the dual field theory. By sampling $D(p)$, we obtain information about the dual field ϕ, particularly γ_ϕ can be identified as a *susceptibility* exponent for the ϕ field [5]. Using the scaling relation $\gamma_\phi = \nu_\phi(2 - \eta_\phi)$, and the fact that at the charged dual fixed point $\nu_\phi = \nu_{3DXY}$ [5], this gives a value for the anomalous scaling dimension η_ϕ when the value $\nu_{3DXY} = 0.673$ is used [13].

In Ref. [5] the vortex loops of the 3D *XY* model have been studied meticulously, yielding the value $\eta_\phi(0) = -0.18 \pm 0.07$. Since the dual of this model is isomorphic to a superconductor, $\eta_\phi(0)$ should be similar to $\eta_\psi(e)$ of the original GLT.

We have studied the vortex loop distribution in both the neutral and the charged case. In the former case we find $\eta_\phi \simeq -0.24$, in good agreement with Ref. [5]. In the latter case the dual theory has a $U(1)$ symmetry, and we would expect to find $\eta_\phi = \eta_{3DXY}$. The exponent η_{3DXY} has recently been determined with great accuracy to $\eta_{3DXY} = 0.038$ [13], whereas we find $\eta_\phi \simeq 0.04$ which compares well with this value. Figure 4 shows $\varepsilon(T)$ for both the charged and uncharged models. *It is evident that they belong to two different universality classes.*

In the case $e \neq 0$, *which corresponds to the dual neutral case*, the inverse ϕ propagator is given by $G^{-1} = q^2 + \Sigma(q)$, where Σ is a self-energy, and $\Sigma(q) \sim q^{2-\eta}$ by definition. This gives a leading order behavior $G \sim 1/q^{2-\eta}$ provided $\eta > 0$, and we find $\eta = 0.04$ for this case. On the other hand, for the case $e = 0$, *which corresponds to the dual charged case*, dual gauge field fluctuations alter the physics, softening the long-wavelength ϕ field fluctuations. We obtain $G^{-1} = q^4 + \Sigma(q)$, again with $\Sigma(q) \sim q^{2-\eta}$, which now gives a leading order behavior $G \sim 1/q^{2-\eta}$, provided $\eta > -2$. Our result $\eta = -0.24$ for the case $e = 0$ (dual charged) is consistent with this, and also with the absolute bounds $\eta > 2 - D = -1$, in $D = 3$.

A consequence of the above is that in $D = 3$ dimensions, $\lambda \sim \xi^{(D-2)/(2-\eta_{\mathrm{A}})} = \xi$ at the charged critical point,

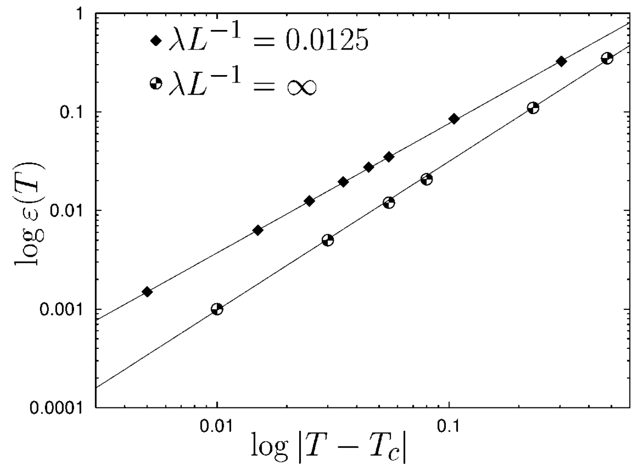

FIG. 4. $\ln \varepsilon(T)$ as a function of $\ln|T - T_c|$. The upper line shows the charged case with finite e, and the lower line shows the neutral case with $e = 0$. The slopes of the two straight lines are $\gamma_\phi = 1.315$ and $\gamma_\phi = 1.51$, corresponding to the anomalous dimensions $\eta_\phi = -0.24$ (neutral, i.e., dual charged) and $\eta_\phi = 0.04$ (charged, i.e., dual neutral), respectively.

in contrast to $\lambda \sim \sqrt{\xi}$ at the 3D *XY* neutral critical point. Our results are valid beyond all orders in perturbation theory.

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