Relationship between the Noise-Induced Persistent Current and the Dephasing Rate

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ac noise in disordered conductors causes both dephasing of the electron wave functions and a dc current around a mesoscopic ring. We demonstrate that the dephasing rate τ_φ^{-1} in long wires and the dc current $\langle I\rangle$, induced by the same noise and averaged over an ensemble of small rings, are connected in a remarkably simple way: $\langle I\rangle\tau_\varphi = C_\beta e$. Here e is an electron charge, and the constant $C_\beta \sim 1$ depends on the Dyson symmetry class. The relationship seems to agree reasonably with experiments. This suggests that the two puzzles, anomalously large persistent current and the low-temperature saturation of the dephasing, may have a common solution.

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Since the discovery of universal conductance fluctuations [1,2] physics of mesoscopic systems has made tremendous progress. However, a few important experimental observations still remain unexplained. One of the long-standing challenges is the anomalously large value of persistent current [3]. Levy *et al.* [4] studied the magnetic response of an ensemble of 10^7 mesoscopic copper rings as a function of the applied weak magnetic field. The average current per ring $\langle I_0(\phi)\rangle = I_0 \sin(4\pi e\phi/hc)$ found from such measurements is a $hc/2e$-periodic function of the magnetic flux ϕ threading each ring. The amplitude I_0 has been found to be of the order of $I_0^{\mathrm{exp}} \sim e/\tau_D$, where $\tau_D = L^2/D$ is the time of diffusion around the ring of the circumference L. Other measurements [5–7] of persistent current brought up similar results.

On the other hand, assuming that *the system is in equilibrium* and that electrons do not interact with each other, one gets [8] the amplitude $I_0^{\mathrm{theor}} \sim (e/h)\Delta = g^{-1}I_0^{\mathrm{exp}}$, where Δ is the electron mean level spacing, and $g = \hbar/\tau_D\Delta$ is the dimensionless conductance. The sign of the average persistent current predicted by existing theories of noninteracting electrons, as well as the sign of the contribution of the electron-electron interaction in nonsuperconducting systems, is *paramagnetic*, i.e., $\partial\langle I_0(\phi)\rangle/\partial\phi > 0$ at small ϕ.

In the experiments (Refs. [4–6]) the dimensionless conductance was large $g \sim 10^2$, i.e., the observed persistent current exceeded the theoretical estimation by 2 orders of magnitude. None of numerous attempts (see, e.g., [9] and for discussion [10]) succeeded in explaining the magnitude of the persistent current by electron-electron interaction. Therefore, the amplitude of the persistent current is in striking disagreement with existing theories.

Another major puzzle in mesoscopic physics recently attracted much attention. Mohanty *et al.* [11] experimentally proved that the saturation of the dephasing rate at low temperatures T cannot be explained by conventional arguments such as magnetic impurities or heating. On the other hand, the dephasing due to electron-electron interactions (or electron-phonon interactions) in an equilibrium system is theoretically predicted to disappear at $T = 0$ [12,13]. Several attempts have been made to resolve the puzzle resulting from the noise caused by the two-level systems [14], two-channel Kondo effect [15], or external radiation [16]. In all these explanations electrons interact with an "environment" that displays a *real* time evolution, e.g., real transitions in two-level systems or a time-dependent electric field. From this point of view, the system of free electrons subject to an external ac field captures all the essential features of dephasing. For instance, using the proper correlator of the equilibrium *intrinsic* ac electric noise one can evaluate [12] the dephasing rate due to the e-e interaction, which is in agreement with the experiment at moderately low temperatures [13].

It is known that the ac electric field may also cause a dc current in mesoscopic systems [17,18]. In contrast to classical physics where the rectification exists only in media without an inversion center, the Aharonov-Bohm effect makes the *disorder-averaged* rectified current also possible [18]. This is because of the different behaviors of the magnetic field and the magnetic flux under space inversion: $\mathbf{H} \to \mathbf{H}$ but $\phi \to -\phi$. This rectified dc current leads to the dc magnetic response similar to the one which results from the equilibrium persistent current.

We show below that such a noise-induced dc current averaged over an ensemble of *small* rings of the circumference $L \ll L_\varphi$ and the dephasing rate induced by the same noise in *long* wires of the length $\mathcal{L} \gg L_\varphi = \sqrt{D\tau_\varphi}$ are related in a remarkably simple way:

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$$I_E\tau_\varphi = C_\beta e, \qquad C_\beta = \begin{cases} -4/\pi, & \beta = 1, \\ +2/\pi, & \beta = 4, \end{cases} \tag{1}$$

where e is an absolute value of the charge carriers and C_β is a constant that depends on the Dyson symmetry class: $\beta = 1$ for the pure potential disorder and $\beta = 4$ in the presence of a spin-orbit scattering with the characteristic length $L_{so} \ll L$. Thus, the important differences between the equilibrium persistent (current) and the rectified dc currents are (1) the magnitude and (2) the sign if $L_{so} \gg L$. In this orthogonal case the ensemble-averaged dc current $\langle I_E(\phi)\rangle = I_E \sin(4\pi e\phi/hc)$ is *diamagnetic*, i.e., $\partial\langle I_E(\phi)\rangle/\partial\phi < 0$ at small ϕ.

This is the central result of the Letter. The relationship Eq. (1) holds regardless of the nature of the noise, since no single parameter of the system and environment enters in the right-hand side of Eq. (1). The noise could be extrinsic or intrinsic and need not be electronic in character (e.g., the phonon wind). The only important condition is that the noise must be *nonequilibrium*. This implies (see also Ref. [19]) that the two puzzles, an anomalously large persistent current and an anomalously large temperature-independent dephasing rate, may be closely related. Instead of trying to identify the unique source of noise which could lead to both anomalies we suggest here a universal relationship Eq. (1) which should hold for any type of nonequilibrium noise under not too restrictive conditions specified below.

Actually, the relationship Eq. (1) can be understood by the dimensional analysis. As an example of a noise source, consider a monochromatic ac electric field with a frequency ω and an amplitude E_ω. Given the diffusion constant D one can construct a dimensionless combination:

$$\alpha = \frac{D}{\omega^3}\left(\frac{e}{h}E_\omega\right)^2 = \left(\frac{L_\omega}{L_E}\right)^2, \tag{2}$$

where $L_\omega = \sqrt{D/\omega}$, and the characteristic length L_E is determined by the equation $eE_\omega L_E = h\omega$. One can estimate the dephasing rate in *long* wires at $T = 0$ as

$$\frac{1}{\tau_\varphi} = \omega f_\varphi(\alpha). \tag{3}$$

Evaluation of $f_\varphi(\alpha)$ goes beyond the dimension analysis.

As for the nonlinear dc current in *mesoscopic* rings, its amplitude depends on *two* parameters:

$$I_E = e\omega f_I(\alpha,\gamma), \tag{4}$$

the parameter α [Eq. (2)] and the "mesoscopic" parameter

$$\gamma = \omega\tau_D = \left(\frac{L}{L_\omega}\right)^2. \tag{5}$$

In the weak-field limit $\alpha \to 0$ both the dc current and the dephasing rate are quadratic in E_ω, i.e., linear in α:

$$I_E = e\omega\alpha f_I(\gamma), \qquad \tau_\varphi^{-1} = \omega f'_\varphi(0)\alpha, \tag{6}$$

where $f_I(\gamma)$ is still an unknown function of γ. Provided that this function has a nonzero limit $f_I(\infty)$ at $\gamma \gg 1$, we immediately obtain

$$I_E\tau_\varphi = eC_\beta, \qquad C_\beta = f_I(\infty)/f'_\varphi(0), \tag{7}$$

where C_β is a constant of the order of 1.

This is essentially Eq. (1). The above analysis suggests that Eq. (1) is valid when α is small, and γ is large. According to Eqs. (2)–(5), $L_\varphi \sim L_\omega\alpha^{-1/2} \sim L(\alpha\gamma)^{-1/2}$. Thus, the mesoscopic condition $L \ll L_\varphi$ is equivalent to $\alpha\gamma \ll 1$, and the above consideration is valid when

$$1 \ll \gamma \ll \alpha^{-1}, \qquad L_\omega \ll L \ll L_\varphi. \tag{8}$$

One can also write Eq. (8) as $\omega \gg \max\{\tau_D^{-1}, eE_\omega L/\hbar\}$.

Another condition concerns the spatial correlation of the field E_ω which was neglected in the previous argument. This can be done [18] if at the length scale of L_ω the field is strongly correlated.

An assumption that $f_I(\gamma)$ has a finite limit at $\gamma \to \infty$ is anything but trivial. It implies that the quadratic in E_ω dc current flows coherently even at $L_\omega \ll L$. It was first mentioned in Ref. [17] and further discussed in [18,20] that the nonlinear dc current is not destroyed at $L_\omega \ll L$. Note that this conclusion applies only to the dc current and is *incorrect* for the ensemble-averaged second harmonic current [18].

It is intuitively clear that for dc current I_E to flow, the environment and the electrons *should be out of the thermal equilibrium*. Indeed, I_E vanishes identically for the equilibrium electric noise [20]. On the other hand, even the equilibrium electric noise causes dephasing [12]. Therefore, at finite temperatures, I_E is *smaller* than $C_\beta e/\tau_\varphi$ even at $L \ll L_\varphi$, due to the contribution from the equilibrium part of the noise to $1/\tau_\varphi$. At $T \to 0$ this contribution vanishes and I_E converges with $C_\beta e/\tau_\varphi$ as required by Eq. (1). This may explain the experimental fact [4,6] that the persistent current changes substantially in the low-temperature region where $1/\tau_\varphi$ is nearly constant.

Of course, the arguments presented above cannot substitute an analytic derivation which we now present. Consider a quasi-1D system of noninteracting electrons with an external ac field $E(t) = -\frac{1}{c}\frac{\partial A_t}{\partial t}$ where A_t is a time-dependent tangential vector potential with the zero mean value $\overline{A_t} = 0$. Here the bar means the time averaging. In contrast with Ref. [18] the field A_t represents a noise with short-range time correlations rather than a strictly monochromatic field [21]. The correlation function $\overline{A_t A_{t'}}$ is supposed to decrease at $|t - t'| > t_c \sim \omega^{-1}$. For simplicity we consider this field to be constant along a ring though the actual requirement [18] for the scale r_c of space variation is much weaker $r_c \gg L_\omega$.

We consider two different geometries: a long wire with the length $\mathcal{L} \gg L_\varphi$ and a ring with the circumference $L \ll L_\varphi$. In the latter case we study a dc current that flows when a *time-independent* flux ϕ threads the ring.

The weak localization correction $\langle I_{\rm wl}(t)\rangle$ to the disorder-averaged current in such a system is given by the well-known cooperon contribution [12]:

$$\langle I_{wl}(t)\rangle = \frac{C_\beta e^2 D}{2\hbar L}\int_0^\infty d\tau\, C_{t-\tau/2}\left(\frac{\tau}{2},-\frac{\tau}{2}\right)E(t-\tau). \tag{9}$$

Here $C_t(\tau,\tau')=\sum_q C_t(q,\tau,\tau')$ is a cooperon at coincident space points, and $C_t(q,\tau,\tau')$ is determined by [12]

$$\frac{\partial C_t}{\partial\tau} + D\left(q - \frac{e}{\hbar c}(A_{t+\tau}+A_{t-\tau})\right)^2 C_t = \delta(\tau-\tau'), \tag{10}$$

where q is a momentum. It is continuous if the wire is long; for a ring $q=(2\pi/L)(m-2e\phi/hc)$, $m=0,\pm1,\pm2,\ldots$. Equations (9) and (10) are valid if the conductance of the system is large, $g\gg1$, and the field is weak enough, $(e/\hbar c)A_t l \ll 1$ (l is the mean free path of the electrons).

The dc component of the current $\langle I_E(\phi)\rangle$ is given by the time average of Eq. (9). Since this average does not depend on the reference point we can shift $t\to t+\tau/2$ and express the nth flux harmonic $I_E^{(n)}$ of the current,

$$\langle I_E(\phi)\rangle = \sum_{n=1}^{\infty} I_E^{(n)}\sin\left(4\pi n\frac{e\phi}{hc}\right), \tag{11}$$

through the nth flux harmonic $C_t^{(n)}(\tau)$ of the cooperon,

$$I_E^{(n)} = -\frac{iC_\beta e^2 D}{\hbar c L}\int_0^\infty d\tau\,\overline{C_t^{(n)}(\tau)\frac{\partial A_{t-\tau/2}}{\partial t}}. \tag{12}$$

Solving Eq. (10) and using the Poisson summation formula, one can find an exact expression for $C_t^{(n)}(\tau)$:

$$C_t^{(n)}(\tau) = \sqrt{\frac{\tau_D}{4\pi\tau}}\,e^{-(n^2\tau_D/4\tau)}e^{inS_1[A]}e^{-\tau S_2[A]}. \tag{13}$$

Here

$$S_1[A] = \frac{2eL}{\hbar c}\left[\frac{1}{\tau}\int_{t-\tau/2}^{t+\tau/2}A_{t_1}\,dt_1\right] \equiv \frac{2eL}{\hbar c}\langle A_{t_1}\rangle_{t;\tau}, \tag{14}$$

$$S_2[A] = \frac{2e^2D}{\hbar^2c^2}\left[\langle A_{t_1}^2\rangle_{t;\tau} + \langle A_{t_1}A_{2t-t_1}\rangle_{t;\tau} - 2(\langle A_{t_1}\rangle_{t;\tau})^2\right]. \tag{15}$$

According to Eqs. (9) and (10) the weak-localization correction to the conductance of a long wire equals

$$\delta\sigma = C_\beta\frac{\sqrt{\pi D}\,e^2}{2h}\int_0^\infty\frac{d\tau}{\sqrt{\tau}}\,\overline{\exp\{-\tau S_2[A]\}} \tag{16}$$

[we substitute a dc field E_0 for $E(t-\tau)$ and used the definition Eq. (15)]. The form of Eqs. (13) and (16) suggests that $S_2[A]$ is related with the dephasing rate, while $S_1[A]$ is responsible for the nonlinear dc current.

Now we assume that the correlation time of the ac field is shorter than the relevant time scale τ_0 in the integrals Eqs. (12) and (16). For the problem of dephasing in a long wire, Eq. (16), $\tau_0^{(\rm deph)}\sim\tau_\varphi$, while for the problem of dc current in a ring, Eq. (12), $\tau_0^{(\rm dc)}\sim\tau_D$. Under these assumptions one can neglect the second and third terms in Eq. (15) and identify the average $\langle A_{t_1}^2\rangle_{t;\tau}$ defined in Eq. (14) with the true time average $\overline{A_{t_1}^2}$. As a result, $S_2[A]=2D(e^2/\hbar^2c^2)\overline{A_t^2}$ becomes independent of t and τ. Using Eq. (16), we identify S_2[A] with the noise-induced dephasing rate:

$$\frac{1}{\tau_\varphi} = 2D(e^2/\hbar^2c^2)\overline{A_t^2}. \tag{17}$$

In order to compute the amplitude $I_E^{(n)}$ of the dc current we have to evaluate the time average in Eq. (12):

$$\overline{\frac{\partial A_{t-\tau/2}}{\partial t}\exp\left\{\frac{in}{\tau}(2eL/\hbar c)\int_{t-\tau/2}^{t+\tau/2}A_{t_1}\,dt_1\right\}} = \frac{in}{\tau}(2eL/\hbar c)\overline{A_t^2}. \tag{18}$$

Since the time average of the total time derivative is zero, we can transfer the differentiation to the exponent. In the limit $(L/L_\varphi)^2(t_c/\tau_D)\ll1$, one can differentiate only the lower limit of the integral and set $\langle A_{t_1}\rangle_{t,\tau}=\overline{A_t}=0$ in the exponent *after* the differentiation. Substituting the result in Eq. (12) we arrive at an integral over τ, which can be evaluated exactly. Finally, we use Eq. (17) to express $\overline{A_t^2}$ in terms of the dephasing rate and obtain the amplitude of the nth flux harmonic of the dc current averaged over the ensemble of mesoscopic rings, Eq. (11):

$$I_E^{(n)} = C_\beta\left(\frac{e}{\tau_\varphi}\right)\exp\left[-n\frac{L}{L_\varphi}\right]. \tag{19}$$

Equation (1) for the principal $h/2e$-periodic component $I_E^{(1)}$ is just in the limit of Eq. (19) at $L\ll L_\varphi$.

Unlike other theories [8,9] of persistent current, the relationship Eq. (1) gives a correct magnitude of the dc current. Indeed, in a given sample at $T=0$ the current as a function of the noise intensity reaches its maximal value when L_φ becomes comparable to the sample size L (further increase of the intensity would suppress the dc current exponentially, $\sim\exp\{-L/L_\varphi\}$). This condition can be rewritten as $\tau_\varphi\sim\tau_D$. Using Eq. (1) we find [18] that the maximal value of the current is of the order

$$I_E^{\max}\sim e/\tau_D. \tag{20}$$

This is the order of magnitude of the current which was observed in all experiments.

The ensemble-averaged current observed in copper rings by Levy *et al.* [4] was about 0.3 nA. We can estimate $2e/\pi\tau_\varphi < 0.9$ nA. In Ref. [7] an ensemble of 10^5 GaAs/GaAlAs rings has been studied. In this case the estimation gives $4e/\pi\tau_\varphi < 1.2$ nA, while the observed ensemble-averaged current was about 1.5 nA. In both cases there is a great deal of uncertainty: the saturated value of τ_φ has not been measured, and for estimation we used values of τ_φ measured in similar structures at $T \approx 1$ K and $T \approx 50$ mK, respectively. Nevertheless the estimations of the dc current based on Eq. (1) are much closer to the experimental values than the predictions of the theories Ref. [8], which assume thermal equilibrium.

Recently Jariwala *et al.* measured the low-temperature dephasing and the "persistent" current in the similar quasi-1D conductors [6,19]. The dephasing time in long gold wires saturated at $\tau_\varphi \approx 4$ ns. The persistent current has been obtained from the magnetization of 30 gold rings fabricated in the same way as the wires. The amplitude of the $h/2e$ dc current was found to be $\sim$0.06 nA, while $2e/\pi\tau_\varphi \approx 0.03$ nA. Therefore, in all three experiments, Refs. [4,6,7,19], Eq. (1) was satisfied up to a factor of $\sim$2, though the magnitude of the persistent current varied within two decades.

The situation with the sign of the magnetization is not so clear. In Ref. [4] this sign was not measured. In GaAs/GaAlAs rings of Ref. [7] the sign of I_E was observed to be *diamagnetic*, in full agreement with Eq. (1) (the spin-orbit effects are negligible [22]). However, in Refs. [6,19] the sign of the $h/2e$ response was also *diamagnetic*. This contradicts Eq. (1): I_E should be *paramagnetic*, since spin-orbit effects in gold are strong. We believe that the contradiction can be explained in the following way. The number of rings $N = 30$ in Refs. [6,19] might not be sufficiently large to average out the random mesoscopic part in the $h/2e$-periodic component of the current. Indeed, there can be no h/e-periodic component in the *ensemble-averaged* current. However, in Refs. [6,19] the h/e magnetic response of 30 rings was approximately the same as the $h/2e$ one. This means that the ensemble-averaged dc current I_E and the $h/2e$-periodic current averaged over 30 rings can differ substantially and even have opposite signs. The prediction that the sign of the persistent current is determined by the strength of the spin-orbit interaction is the most critical for our theory and deserves a serious experimental check.

In conclusion, we have derived a relationship [Eq. (1)] between the averaged dc current generated by a nonequilibrium ac noise in an ensemble of mesoscopic rings and the dephasing rate caused by the same noise. It provides a much better fit for the magnitude of low-temperature ring magnetization than other existing theories, which assume the equilibrium. More experimental work is needed to confirm the role of the nonequilibrium noise. However, we have reasons to suspect that we are dealing here with significantly nonequilibrium mesoscopic systems.

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