Collapse Dynamics of Liquid Bridges Investigated by Time-Varying Magnetic Levitation

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Using a novel technique that facilitates temporal control over the total body force on a liquid, an unexpected scaling relationship was discovered for the collapse time of a liquid bridge. A paramagnetic liquid was suspended between the tips of two collinear rods in a strong magnetic field gradient that was adjusted to compensate gravity. A sudden change of the magnet current, corresponding to a change of Bond number, resulted in a deformation and ultimate collapse of the liquid bridge. The collapse time was found to be independent of the bridge length when other parameters were held constant.

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Liquid bridges, in which fluid is held captive between two or more solid supports, have diverse applications in fields from zone refining to porous media. The shape and stability of liquid bridges are strongly influenced by gravity, as well as surface tension and the nature of the liquid-support contact. A bridge that is stable is zero gravity will deform and may collapse in Earth's gravity. Although bridge stability has received considerable attention for over a century [1–10], experimental studies of *dynamics* have been hampered by the inability to effect large, rapid change in the total body acceleration [8,11,12]. Here we show that the dynamics of collapsing bridges may be studied by using an inhomogeneous, time-varying magnetic field. Cylindrical bridges consisting of a viscous paramagnetic liquid were initially subjected to zero net body force by having the magnetic force exactly compensate gravity. The magnetic force was then rapidly reduced and the collapse time measured. An unexpected result from these experiments was that the collapse time was largely independent of the length of the bridge when other parameters were held constant.

Rayleigh and Plateau demonstrated [1] that in zero gravity a cylindrical bridge remains stable against radial fluctuations so long as its length-to-diameter ratio Λ, known as the slenderness ratio, is less than π. The bridge environment is typically characterized by a dimensionless number B known as the Bond number, and is equal to $g_{\text{eff}}\rho R^2/\sigma$; where g_{eff} is the effective body acceleration due to gravity and other external forces, ρ is the density of the liquid, R is the radius of the bridge, and σ is the surface tension. Because of the difficulty in controlling g_{eff}, experiments have generally been limited to those performed in a Plateau tank which utilizes buoyancy forces, [7] or in space [8]. The neutral buoyancy technique uses two immiscible fluids and permits a wide range of values of g_{eff}, but is severely limited when studying dynamics. Although space-borne experiments provide zero or near-zero gravity conditions, they are not suitable when larger body forces are required. Because of this, previous investigations of bridge dynamics have been limited to a narrow range of experimental conditions. For example, very small bridge oscillations [8,11,12] have been simulated by a mechanical vibration of the column's support rods. To date, the only reported experimental study of the dynamics of full collapse involved a Plateau tank experiment in which the concentration of the bath mixture changed slowly with time by evaporation [13]; rapid changes of Bond number were not possible. As a consequence of such experimental constraints, the study of bridge collapse has been limited mostly to theoretical and numerical analyses of slender bridges [13–15]. In order to circumvent many of these experimental difficulties, we have used magnetic levitation as a powerful technique to study fluid bridges under conditions ranging between $g_{\text{eff}} = 0$ and $g_{\text{eff}} = g$, where g is the earth's gravitational field [16,17]. This method, which can be applied to a wide variety of fluid problems, facilitates large and rapid changes of the effective body acceleration and can easily be effected *in situ*, with no external mechanical motion.

Magnetic compensation of gravity requires a spatially inhomogeneous magnetic field [16]. For a material of volumetric magnetic susceptibility χ in a magnetic field H, the energy per unit volume is given by $U = -\frac{1}{2}\chi H^2$, and the force per unit volume is $-\nabla U$. To compensate gravity it is required that $\frac{1}{2}\chi\nabla H^2_{\text{comp}} \approx \rho g$, where H_{comp} corresponds to the magnetic field whose gradient just compensates gravity. For ∇H^2 larger or smaller than $2\rho g/\chi$, the liquid column will rise or sag, ultimately collapsing if ∇H^2 deviates too significantly from its gravity-compensating value. Thus the effective body acceleration $[g_{\text{eff}} \equiv g - \frac{1}{2}\chi\nabla(H^2)]$ may be controlled by varying the current in the magnet.

As most ordinary fluids are weakly diamagnetic and are incapable of being levitated except in the strongest magnets, we have chosen to study glycerol doped with highly paramagnetic manganese chloride tetrahydrate, $MnCl_2 \cdot 4H_2O$. For a solution that is 60% by weight $MnCl_2 \cdot 4H_2O$, we have determined the volumetric magnetic susceptibility to be $(5.8 \pm 0.1) \times 10^{-5}$ cgs by measuring the value of ∇H^2 at which $g_{\text{eff}} = 0$. By weighing a known volume of solution, the density of the solution was determined to be 1.55 g cm^{-3}. Additionally, using a

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cone-and-plate rheometer, we measured the viscosity to be $\eta = 25 \pm 1$ P at 25 °C, the approximate temperature at which the experiment was performed. The viscosity was found to have a temperature variation of approximately -2 $P/$°C in this region. Finally, by measuring the maximum allowed Bond number B_s vs Λ for which a bridge of slenderness ratio Λ remains stable [2], we have determined the surface tension to be 65 ± 5 ergs cm^{-2}; this value is close to the value for pure glycerol, indicting that the $MnCl_2 \cdot 4H_2O$ has a very small influence on the surface tension.

A pair of identical coaxial cylindrical rods with conical wetting barriers was inserted into an electromagnet equipped with Faraday pole pieces that provide a uniform upward magnetic *force*. One rod was mounted on a micrometer that permitted fine control over the separation between the rods, and therefore control over the length of the fluid column (Fig. 1). Experiments were performed using three such pairs of rods, having radii $R = 0.155$, 0.115, and 0.079 cm, respectively. This allowed us to study the influence of the bridge radius R on the collapse time. The liquid was injected from the side using a butterfly syringe and hypodermic needle. We began the experiment with a cylindrical bridge of a given radius and slenderness ratio in an environment of $g_{\mathrm{eff}} = 0$, corresponding to Bond number $B = 0$; this corresponded to a value of $H\nabla H = (2.63 \pm 0.06) \times 10^7$ G^2 cm^{-1}. We then effected a rapid change ΔB of the Bond number by suddenly decreasing the magnet current. Owing to the large inductance and small electrical resistance of the magnet, the magnetic *force* (and therefore the Bond number) would relax to its new set point in 200 ms [18]. Because $\tau \sim 3$ s for the fastest collapse time in our data set, the magnet's response approximates a discontinuity of the Bond number. The deformation and breaking of the bridge was video-recorded using a boroscope and CCD camera. The collapse time τ is defined as the time taken by the bridge to break into two disconnected fluid zones, measured from the time the Bond number is changed. For each bridge, τ was extracted from the videotape.

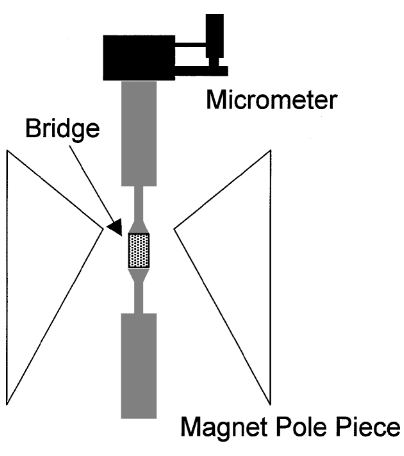

FIG. 1. Schematic representation of experimental setup.

Bridges were studied as a function of slenderness ratio for each of the three radii. Figure 2 shows a typical collapse sequence for a bridge of radius $R = 0.155$ cm, a slenderness ratio $\Lambda = 2.6$, and $\Delta B = 0.057$. The first five frames of the sequence are spaced in 5 s intervals; the final frame follows the fifth frame by 4 s. A thin filament separating the lower and upper regions is seen in the last frame; detailed studies on filament breakage have been performed in a Plateau tank [19]. Shortly thereafter the filament brake, and the upper and lower filament segments retracted upward and downward, respectively. This behavior has previously been observed qualitatively [9].

Analysis of the data for collapse time τ vs ΔB indicates that, for bridges of a given radius, τ depends only on the *excess* Bond number change $\delta(\Delta B)$ above the bridge's slenderness ratio-dependent stability limit B_s [2], i.e., $\delta(\Delta B) = \Delta B - B_s$. This indicates that we should plot our results as a function of $\delta(\Delta B)$; the quantity τ^{-1} vs $\delta(\Delta B)$ is thus shown in Fig. 3(a). [Note that for $\Delta B < B_s$ the bridge deforms but remains stable indefinitely, and that B_s is smaller for longer bridges (larger Λ) and vanishes [1] when $\Lambda = \pi$.] In Fig. 3(a) we see that the data lie along three distinct curves, one for each of the three bridge radii R, and have no explicit dependence on Λ.

Dimensional analysis suggests an alternative form in which to present the data. The bridge collapse time can depend only on the viscosity η, surface tension σ, radius R of the support rod, slenderness ratio Λ, Bond number change ΔB, and density ρ. In the overdamped limit of viscous flow, we must have $\tau = \eta R/\sigma \times f(\Lambda, \Delta B)$, where $f(\Lambda, \Delta B)$ is a to-be-determined function of Λ and ΔB. When τ is not too large, the surface tension will be unimportant and then we expect $1/\tau$ to be proportional to the driving force, and therefore linear in ΔB. At the stability limit, where $\Delta B = B_s$, $\tau \to \infty$ and therefore the quantity $\eta R/\sigma\tau$ $[= 1/f(\Lambda, \Delta B)]$ must vanish. This implies the approximate relationship $\eta R/\sigma\tau = \delta(\Delta B) \times h(\Lambda)$, where $h(\Lambda)$ is a function of only the slenderness ratio.

Accordingly, in Fig. 3(b) we show the dimensionless quantity $\eta R/\sigma\tau$ vs $\delta(\Delta B)$, and see that *all* the data approximately fall on the same curve. Thus, we find the surprising experimental result

$$\eta R/\sigma\tau = M\delta(\Delta B),$$

where M is a universal dimensionless constant that is independent of the slenderness ratio, and has a numerical value of order 0.1.

The implication of this result is that the dominant flow processes must occur in a segment of the liquid bridge that is independent of the actual length of the bridge. To see whether this is the case, and to develop a theoretical prediction for the value of M, we turn to numerical analysis, and employ a one-dimensional model due to Meseguer

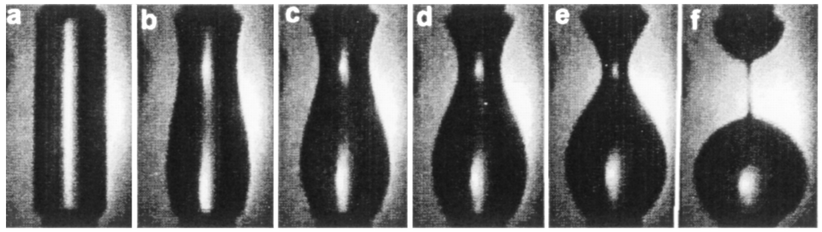

FIG. 2. Collapse sequence for a bridge of radius $R = 0.155$ cm and slenderness ratio $\Lambda = 2.6$ after changing the Bond number from zero to 0.057. Panel (a) shows the bridge at time $t = 0$. Panels (b) through (f) show the bridge at times $t = 5$, 10, 15, 20, and 24 sec, respectively.

[13] in which the radial component of fluid momentum is neglected. In this model the liquid bridge is held vertically and is subjected to axial gravity. The liquid is assumed to be an isothermal incompressible Newtonian fluid. The following additional assumptions are made: (i) Internal fluid motion is caused *only* by capillary pressure gradients due to interface deformation in response to acceleration. (ii) The effect of the atmosphere around the bridge is negligible. (iii) Only the axisymmetric response is considered. This approach is a simplification of the full three-dimensional set of equations for mass and momentum transfer in the liquid, the balance of normal and tangential force components at the surface, the kinematic boundary condition, the conditions at the support rods, the requirement of axisymmetry, and the initial conditions of a cylindrical cylinder and zero fluid momentum; the full equation set is given by Eqs. (1)–(10) in Ref. [20]. These equations define an unsteady free boundary problem since the location (and hence the shape) of the interface is *a priori* unknown and must be determined along with the velocities and pressure as part of the solution. The 1D Meseguer approximation [13] allows us to reduce the complexity of the problem and to facilitate an examination of a relatively wide range of parameters without excessive computation time. It has been used in several previous numerical studies of liquid bridge dynamics [13,20] and has been shown to be valid provided the slenderness ratio Λ is large, typically $\Lambda > 2$. It is based on a one-dimensional model for fluid jets [21] in which the axial velocity component is assumed to depend only on the z coordinate and time. The approximations and the numerical method for the solution follow the approach used by Zhang and Alexander [20].

In Fig. 4 we replot three sets of experimental data for the quantity $\eta R/\sigma\tau$ vs ΔB at different radii. The solid curves in Fig. 4 represent numerical computations of the breaking times calculated from the one-dimensional model, *with no free parameters*. The agreement between experiment and theory is clearly good, indicating that neglect of the radial component of momentum is an acceptable procedure. The slight underestimate of the inverse collapse time τ^{-1} found in the simulations is likely a consequence of the absence of active temperature control in the experiment and the resulting variation of viscosity. The curvature of the

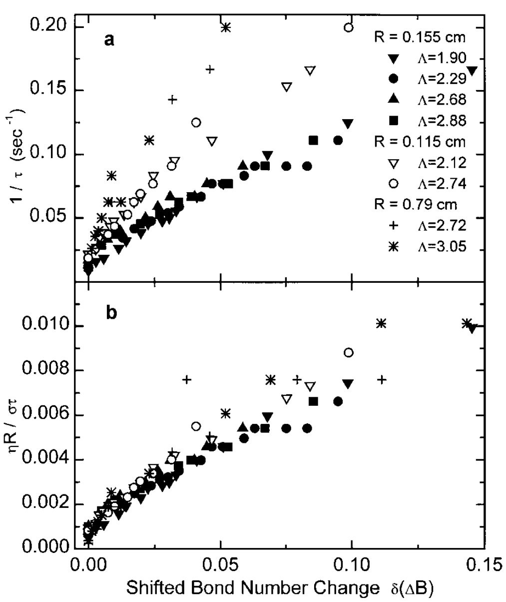

FIG. 3. Collapse data vs shifted Bond number change $\delta(\Delta B)$. (a) The actual inverse collapse time $1/\tau$ vs $\delta(\Delta B)$. (b) The dimensionless ratio of radius R to collapse time τ, formed by multiplying R/τ by the ratio of the viscosity η to surface tension σ, is shown as a function of $\delta(\Delta B)$ for three different support-rod diameters.

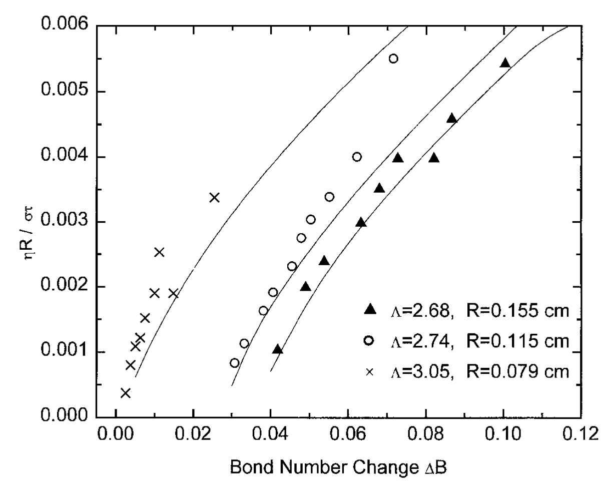

FIG. 4. The same dimensionless ratio $\eta R/\sigma\tau$ that is shown in Fig. 3(b) as a function of unshifted Bond number change ΔB. The solid lines are predictions of numerical computations based on a one-dimensional hydrodynamic model.

theoretical results prevents a unique determination of the value of M, but it is nevertheless clear that a good qualitative prediction can be made by using $M = 0.1 \pm 0.03$, consistent with experimental results.

These results show that the use of time-varying magnetic levitation is a powerful tool for the study of liquid bridges. In particular, it has led to the discovery of the unexpected independence of the collapse time rate $\eta R/\sigma\tau$ from variations in the slenderness ratio Λ. In addition, time-varying magnetic levitation provides control that facilitates a wide variety of dynamics experiments that are not otherwise possible either in space-borne experiments or using the neutral buoyancy technique. Moreover, the applicable range of Bond number is far greater than could otherwise be accomplished with forced mechanical vibrations, and avoids the additional complication of physical motion. Phenomena as diverse as rapid acceleration or deceleration, gravitational jitter, and earthquake vibration may be simulated *in situ* with the use of this technique.

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