Effective Restoration of the $U_A(1)$ Symmetry in the SU(3) Linear σ Model

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The effective restoration of the chiral $U_A(1)$ symmetry in strong interactions is studied using the linear chiral $SU(3) \times SU(3)$ model at finite temperatures. We find that the disappearance of the chiral anomaly causes a considerable change in the meson mass spectrum. We propose several signals for detecting this chiral phase in ultrarelativistic heavy-ion collisions: The η/π^0 ratio is enhanced by an order of magnitude, the a_0 is suppressed in the $K\bar{K}$ mass spectrum, and the scalar κ meson appears as a peak just below the $K^*(892)$ in the invariant πK mass spectrum.

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Probing the phase transition of strongly interacting matter at finite temperature is one of the primary goals of ultrarelativistic heavy-ion collisions [1]. At BNL's Relativistic Heavy-Ion Collider (RHIC) the central collision zone of the two gold ions may create temporarily hot matter, presumably similar to what is studied numerically in lattice gauge calculations. Present simulations indicate that the phase transition seen in hot quantum chromodynamics (QCD) coincides with the chiral phase transition [2]. But the fundamental question of which chiral symmetry is restored in hot QCD is not settled yet. There are two scenarios according to Shuryak [3]: only flavor SU(3) chiral symmetry is restored or both SU(3) and $U_A(1)$ symmetries are restored. The latter scenario is particularly interesting, as it might lead to a first order phase transition for two-flavor QCD and drastic effects for the spectrum of pseudoscalar mesons [4].

The $U_A(1)$ symmetry is broken by nonperturbative effects [5] which gives the η' a finite mass in the chiral limit. The topological susceptibility in pure color SU(3) gauge theory can be linked with the finite η' mass via the Witten-Veneziano formula [6,7]. The restoration of the $U_A(1)$ symmetry should then be visible in the behavior of the topological susceptibility around the critical temperature. There exist some theoretical estimates in the literature of how the chiral anomaly behaves at finite temperatures. Changes in the topological susceptibility should be small for low temperatures [8]. In the plasma phase above T_c it should drop exponentially due to instanton effects [9]. More dramatically, it was shown in [10] that it even vanishes at T_c in the large N limit.

The topological susceptibility has been studied on the lattice, both in pure color SU(3) gauge theory [11,12] as well as for the unquenched case [13,14]. In both cases, a sharp drop by an order of magnitude is observed at the deconfining temperature showing the apparent restoration of the $U_A(1)$ symmetry. Other approaches utilize chiral susceptibilities, where simulations indicate that the $U_A(1)$ symmetry is not completely restored [15] but its effects are at or below the 15% level [16]. The screening mass difference of the pion and the $a_0(\delta)$ have also been studied as a measure of $U_A(1)$ breaking effects. Using staggered fermions it has been found that the anomalous chiral symmetry is probably not fully restored at T_c but that the $a_0 - \pi$ mass splitting drops drastically [17–19]. Simulations with domain wall fermions demonstrate that anomalous chiral symmetry breaking effects are at or below the 5% level above but close to T_c [20]. Note that all the above results consistently indicate that effects from the $U_A(1)$ breaking are strongly suppressed, i.e., an *effective* restoration of the $U_A(1)$ symmetry close to T_c.

Recently, the issue of finding signals for the restoration of chiral symmetry in ultrarelativistic heavy-ion collisions has received considerable attention. For example, signals for the restoration of the SU(2) chiral symmetry associated with the σ meson have been proposed in [21,22]. In particular, signals for the partial restoration of the $U_A(1)$ symmetry in connection with the η' meson have been invoked in [10,23–25]. Effects of the $U_A(1)$ anomaly on meson masses have also been studied within the SU(3) Nambu–Jona-Lasinio model by Kunihiro [26,27].

Here, we are going to study the full SU(3) linear σ model at finite temperature including effects from the effective restoration of the $U_A(1)$ symmetry. Our aim is to extract signals from the strong suppression of $U_A(1)$ breaking effects in a chirally restored phase. The additional scalar mesons in the SU(3) σ model compared to the standard SU(2) σ model provide novel signals for the effective restoration of the $U_A(1)$ symmetry: there will be an enhancement of the number of η mesons due to a feedback from the decay of a_0 mesons, its chiral partner, which will be suppressed then in the $K\bar{K}$ mass spectrum, and there appears a new scalar resonance in the invariant πK mass spectrum, stemming from the chiral partner of the kaon, the κ meson.

The inclusion of the strangeness degree of freedom, i.e., going from SU(2) to SU(3), finds its justification in the recent findings that the strange quark mass is about $m_s = 100–150$ MeV [28], close to half the expected temperatures at RHIC of $T \approx 200–300$ MeV in the initial stage of the collision. Therefore, strangeness has to be included in the linear σ model for studying hot matter relevant to physics at RHIC.

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The SU(3) linear σ model has been known for a long time [29]. Only recently, more than thirty years later, there has been a renaissance of this chiral Lagrangian. The resurrection of the σ meson [30] and the finding of the $\kappa(900)$ resonance in the πK scattering data [31–33] lead to the conclusion that there is a low-mass scalar nonet [34]. Jaffe predicted this scalar nonet long ago [35] as being built out of $(q\bar{q}q\bar{q})$ states with an inverted mass spectrum compared to the pseudoscalar meson spectrum. Recent lattice simulations are supporting this picture [36]. It is interesting to note that the inversion of the mass spectrum is implemented in the SU(3) linear σ model by the anomaly term which has the opposite sign for the pseudoscalar and scalar masses. The model Lagrangian gives a surprisingly good agreement with the data already at tree level [37,38]. The model was also extended to finite temperatures [39,40] without implementing effects from the effective $U_A(1)$ symmetry restoration.

Let us now write down the SU(3) $\times$ SU(3) chiral Lagrangian:

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi) - \lambda\,\mathrm{Tr}(\Phi^\dagger \Phi)^2 - \lambda'(\mathrm{Tr}\Phi^\dagger \Phi)^2 + c \cdot (\det\Phi + \det\Phi^\dagger) + \epsilon \cdot \sigma + \epsilon' \cdot \zeta\,, \tag{1}$$

where Φ is a 3×3 complex matrix describing the pseudoscalar and scalar nonet. The term proportional to the determinant breaks the $U_A(1)$ symmetry. The last two terms break chiral symmetry explicitly, which are simulating effects from a finite light quark and a strange quark mass, respectively. There are two order parameters corresponding to the light quark condensate σ and the strange quark condensate ζ. Chiral symmetry is restored at a certain temperature as the σ order parameter drops towards zero (and effects from the explicit symmetry breaking term $\epsilon \cdot \sigma$ can be ignored). Then the masses of the pion and the sigma mesons and the masses of the η and the a_0 meson are the same separately. Their mass gap is proportional to the coefficient c of the anomaly term times the strange order parameter ζ:

$$m_\pi = m_\sigma < m_{a_0} = m_\eta, \qquad \Delta m = 4c \cdot \zeta\,. \tag{2}$$

If the chiral $U_A(1)$ symmetry is effectively restored, then this mass gap vanishes and all four meson masses are the same irrespective of the value of ζ:

$$m_\pi = m_\sigma \approx m_{a_0} = m_\eta \quad \text{for } c \approx 0\,. \tag{3}$$

Note that this is only the case for a (effective) restoration of the $U_A(1)$ symmetry ($c \approx 0$) as the strange order parameter ζ will not decrease strongly at T_c due to the finite strange quark mass [41].

We assume now that the coefficient of the anomaly term c drops as a function of temperature. As a guideline we take the temperature effects to be proportional to the topological susceptibility in pure glue theory as suggested by the Witten-Veneziano formula [6,7]. We use the lattice data as published in [12] so that the coefficient is nearly constant until T_c and drops then by an order of magnitude (but is not vanishing). We take the critical temperature to be $T_c = 150$ MeV as deduced from recent investigations on the lattice (see [2] for a summary). Thermal excitations for all pseudoscalar and scalar fields are taken into account in a Hartree scheme in a self-consistent way. The gap equations for the two order parameters are solved together with the expressions for the meson masses and the thermal excitations iteratively until convergence is achieved.

The meson masses as a function of temperature are shown in Figs. 1 and 2. The most striking feature compared to a calculation with a constant anomaly coefficient is that the phase transition is shifted to lower temperatures. For a constant coefficient, the phase transition is much less pronounced and happens at $T_c \approx 210$ MeV [40]. Here, the chiral phase transition happens precisely at the same temperature at which the anomaly coefficient drops down, i.e., at $T_c = 150$ MeV. Remarkably, the restoration of $U_A(1)$ symmetry shifts the value of the critical temperature and strengthens the chiral phase transition which was also seen within the Nambu–Jona-Lasinio model [26]. The $U_A(1)$ symmetry is restored at $T \approx 250$ MeV which can be read off of Fig. 1 by the degeneracy of the chiral partners $\pi - \sigma$ and $\eta - a_0$. The overall meson mass spectrum changes significantly across the phase transition. The mass of the η' drops down considerably to $m_{\eta'} \approx 650$ MeV and is then approximately degenerate with the kaon mass above T_c. The pseudoscalar mixing angle is ideal above T_c due to the smallness of the anomaly term. This means in principle that the η', as the chiral partner of the a_0, is a purely light quark system while the η becomes purely strange. Nevertheless, it is apparent from Fig. 1 that a level crossing of the η and the η' masses occurs around T_c (see also

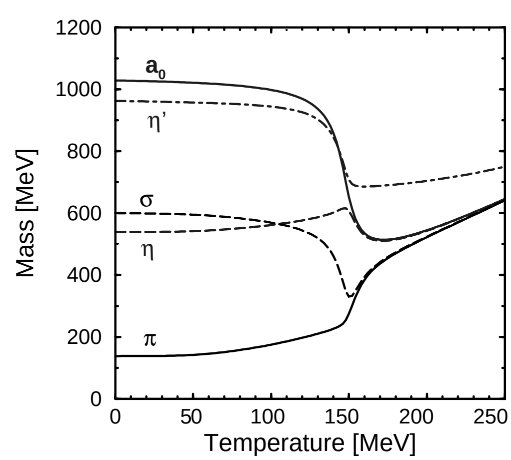

FIG. 1. Meson masses versus temperature for the π, σ, η, η', and a_0. There is a level crossing of the η and η' states. The decay $a_0 \to \pi + \eta$ is blocked just below the chiral phase transition.

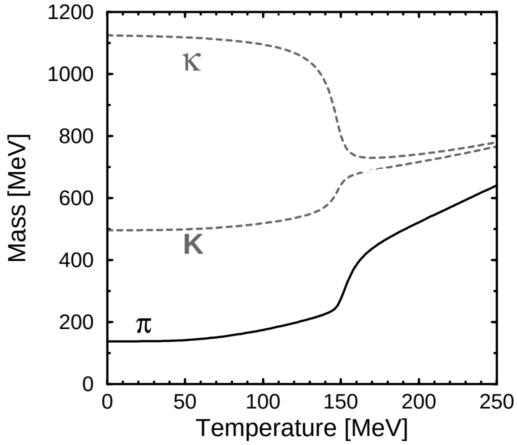

FIG. 2. Meson masses versus temperature for the pion, kaon, and κ. The decay $\kappa \to \pi + K$ is blocked just below T_c.

[4,26]). Hence, the η and η' are switching their identity at T_c. The η is now the chiral partner of the a_0 and is the nonstrange state while the η' is the pure strange quark state. Surprisingly, the η' mass is even slightly smaller than the single strange quark state, the kaon, which is an interesting nontrivial effect originating from the different thermal contributions for the kaon and the η_s masses. The number of η' mesons will then be enhanced in the hot medium. In return, the number of η mesons will increase then by the decay $\eta' \to \eta\pi\pi$ at freeze-out which will be seen by a modified slope parameter in the low transverse momentum spectra.

A more dramatic effect is associated with the scalar isovector meson a_0. The free a_0 decays mainly to $\eta + \pi$ and to two kaons. The decay to two pions is forbidden as it violates isospin. As evident from Fig. 1, the a_0 mass decreases strongly with temperature, as is also seen on the lattice [17–20]. As chiral SU(3) symmetry is restored at T_c, its mass gets degenerate with the η mass, its chiral partner. In addition, as $U_A(1)$ is effectively restored, the a_0 and η masses will be close to the pion mass just above T_c (as seen in Fig. 1). Hence, already *below* T_c the decay $a_0 \to \eta + \pi$ must be blocked by phase space just when the mass difference of the a_0 and the η equals the thermal pion mass. Also, the decay matrix element of $a_0 \to \eta + \pi$ is considerably reduced above T_c as it is proportional to the σ order parameter and the anomaly term. The decay to two kaons is heavily suppressed as the a_0 is actually lighter than one kaon alone if $U_A(1)$ is effectively restored (see Fig. 1 compared with Fig. 2). Hence, the inelastic channels for the a_0 are closed above T_c. The elastic channels are still large as they are proportional to the coupling constant λ which is of the order of 10. We conclude that there is chemical equilibrium between the a_0, η, σ, and pion in the chiral phase as it is $U(2) \times U(2)$ symmetric. This argument is supported by the work of Song and Koch [21] who find that the σ and pion mesons are in chemical equilibrium in the SU(2) linear σ model.

For detecting this chiral symmetric phase in ultrarelativistic heavy-ion collisions, the expansion stage of the hot and chemically equilibrated matter must be short and/or out of equilibrium. If the expansion is adiabatically, the system adjusts itself, freezes out like a free gas, and no effect will be visible. A signal will show up, if either the system expands from the chiral phase above T_c until the freeze-out temperature T_f faster than the lifetime of the a_0 of about 2–4 fm or if the chemical freeze-out happens before the thermal freeze-out [24]. Then the number of a_0's is approximately conserved during the expansion and the numbers of a_0's at freeze-out will be approximately equal to the numbers of pions and 3 times the number of η mesons (due to isospin counting). The a_0 will then mainly decay to $\eta + \pi$ increasing the numbers of the η mesons drastically. Taking into account the decays $a_0 \to \eta + \pi$ and $\sigma \to 2\pi$, we get the ratio $\eta/\pi^0 = (3n_{a_0} + n_\eta)/(n_{a_0} + 2/3n_\sigma + n_{\pi^0}) = 3/2$ as a signal of the formation of an $U(2) \times U(2)$ symmetric phase. Hence, it is even possible to produce more η mesons than π^0's. The two-kaon decay channel is suppressed at finite temperature due to the larger mass of the kaon and the smaller mass of the a_0 at T_f [42]. This decay is a subthreshold process even at $T = 0$ so that a slight change in the masses will reduce the branching ratio.

We now discuss another observable related to the effective restoration of the $U_A(1)$ which is associated with the $\kappa(900)$ resonance. The κ meson is very broad similar to the σ meson [31–33] and decays to a pion and a kaon. Its mass depends strongly on the $U_A(1)$ anomaly and is decreasing with temperature as shown in Fig. 2. The decay width depends on the coefficient of the anomaly term and decreases therefore in the chiral $U_A(1)$ phase. We find at tree level that the width changes from values around $\Gamma \approx 0.8$ GeV to $\Gamma \approx 0.2$ GeV when setting the contribution from the anomaly term to zero. Hence, the barely visible broad resonance gets a much smaller width in the chiral $U_A(1)$ phase and can possibly be seen. As indicated by Fig. 2, the mass of the κ approaches that of the kaon towards chiral symmetry restoration so that the strong decay $\kappa \to K + \pi$ is blocked by phase space. This happens already below T_c similar as for the a_0. The κ meson can then be visible in the invariant πK mass spectrum, if the system freezes out dominantly around T_c. Chiku and Hatsuda have demonstrated this effect in connection with the σ meson appearing in the $\pi\pi$ channel [22]. The decay channel $\kappa \to \pi + K$ opens just below T_c so that there will be a pronounced cusp structure in the corresponding spectral function as depicted in Fig. 3. The κ resonance will then emerge in the πK invariant mass spectrum around 850 MeV. For the πK mass spectrum, there is only one background in that mass region, which is from the vector kaon, the $K^*(892)$. Vector meson masses go up with temperature if studied in a SU(2) gauged chiral Lagrangian [43] as they have to be degenerate with their heavier axial vector chiral partners. A study in a SU(3) linear σ

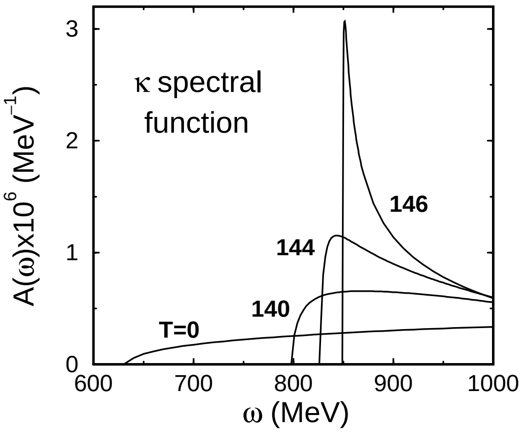

FIG. 3. The spectral function for the κ meson for temperatures of $T = 0$, 140, 144, and 146 MeV. A pronounced threshold enhancement appears for temperatures close to T_c.

model with vector mesons shows that the $K^*(892)$ mass stays approximately constant until T_c and effectively rises then for higher temperatures [44]. Hence, the background from $K^*(892)$ should be comparably low.

The appearance of the κ in the πK spectra can be detected by two particle correlation at BNL's RHIC. Here, the same techniques employed for reconstructing the ρ in the $\pi\pi$ mass spectrum [45] can be utilized. The STAR group, as well as BRAHMS and PHOBOS, will reconstruct ϕ mesons in the $K\bar{K}$ spectra [46] where the a_0 will be seen, too. The η meson will be measured in the diphoton spectra at the PHENIX detector [47] and the enhancement proposed here can be checked.

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[1] Proceedings of Quark Matter '99, Torino, Italy [Nucl. Phys. A (to be published)].
[2] E. Laermann, Nucl. Phys. (Proc. Suppl.) **63**, 114 (1998).
[3] E. Shuryak, Comments Nucl. Part. Phys. **21**, 235 (1994).
[4] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
[5] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).
[6] E. Witten, Nucl. Phys. **B156**, 269 (1979).
[7] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).
[8] E. Shuryak and M. Velkovsky, Phys. Rev. D **50**, 3323 (1994).
[9] R. D. Pisarski and L. G. Yaffe, Phys. Lett. **97B**, 110 (1980).
[10] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Phys. Rev. Lett. **81**, 512 (1998).
[11] M. C. Chu and S. Schramm, Phys. Rev. D **51**, 4580 (1995).
[12] B. Allés, M. D'Elia, and A. D. Giacomo, Nucl. Phys. **B494**, 281 (1997).
[13] M. C. Chu, S. M. Ouellette, S. Schramm, and R. Seki, hep-lat/9712023.
[14] B. Alles, M. D'Elia, A. D. Giacomo, and P. W. Stephenson, Nucl. Phys. (Proc. Suppl.) **73**, 518 (1999).
[15] C. Bernard *et al.*, Phys. Rev. Lett. **78**, 598 (1997).
[16] S. Chandrasekharan *et al.*, Phys. Rev. Lett. **82**, 2463 (1999).
[17] E. Laermann, Nucl. Phys. **A610**, 1c (1996).
[18] S. Gottlieb *et al.*, Phys. Rev. D **55**, 6852 (1997).
[19] J. B. Kogut, J. F. Lagae, and D. K. Sinclair, Phys. Rev. D **58**, 054504 (1998).
[20] P. Chen *et al.*, hep-lat/9812011.
[21] C. Song and V. Koch, Phys. Lett. B **404**, 1 (1997).
[22] S. Chiku and T. Hatsuda, Phys. Rev. D **57**, 6 (1998).
[23] J. Kapusta, D. Kharzeev, and L. McLerran, Phys. Rev. D **53**, 5028 (1996).
[24] Z. Huang and X.-N. Wang, Phys. Rev. D **53**, 5034 (1996).
[25] S. E. Vance, T. Csörgő, and D. Kharzeev, Phys. Rev. Lett. **81**, 2205 (1998).
[26] T. Kunihiro, Phys. Lett. B **219**, 363 (1989).
[27] T. Kunihiro, Nucl. Phys. **B351**, 593 (1991).
[28] T. Blum, A. Soni, and M. Wingate, Phys. Rev. D **60**, 114507 (1999).
[29] M. Levy, Nuovo Cimento A **52**, 23 (1967).
[30] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).
[31] S. Ishida, M. Ishida, T. Ishida, K. Takamatsu, and T. Tsuru, Prog. Theor. Phys. **98**, 621 (1997).
[32] D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **58**, 054012 (1998).
[33] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999).
[34] D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **59**, 074026 (1999).
[35] R. Jaffe, Phys. Rev. D **15**, 267 (1977).
[36] M. Alford and R. L. Jaffe, hep-lat/0001023.
[37] M. Y. Ishida, Nucl. Phys. **A629**, 148c (1998).
[38] N. A. Törnqvist, hep-ph/9905282.
[39] H. Meyer-Ortmanns and B.-J. Schäfer, Phys. Rev. D **53**, 6586 (1996).
[40] J. Schaffner-Bielich and J. Randrup, Phys. Rev. C **59**, 3329 (1999).
[41] J. B. Kogut, D. K. Sinclair, and K. C. Wang, Phys. Lett. B **263**, 101 (1991).
[42] We are grateful to Horst Stöcker for bringing this to our attention.
[43] R. D. Pisarski, Phys. Rev. D **52**, R3773 (1995).
[44] D. Zschiesche, P. Papazoglou, S. Schramm, C. Beckmann, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, "Springer Tracts in Modern Physics" (Springer, Heidelberg, to be published).
[45] A. Chikanian and J. Sandweiss (private communication).
[46] F. Wang, R. Bossingham, Q. Li, I. Sakrejda, and N. Xu, "STAR Note" (1998), Vol. 383.
[47] M. Tannenbaum (private communication).