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Experimental Investigation of Resonant Activation

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We experimentally investigate the escape from a metastable state over a fluctuating barrier of a physical system. The system is switching between two states under electronic control of a dichotomous noise. We measure the escape time and its probability density function as a function of the correlation rate of the dichotomous noise in a frequency interval spanning more than four frequency decades. We observe resonant activation, namely a minimum of the average escape time as a function of the correlation rate. We detect two regimes in the study of the shape of the escape time probability distribution: (i) a regime of exponential and (ii) a regime of nonexponential probability distribution.

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The thermal activated escape of a particle over a potential barrier in a metastable state is a classical problem since the seminal work of Kramers [1]. It occurs in a wide variety of physical, biological, and chemical systems [2]. When the height of the potential barrier is itself a random variable, the dynamics of the escape from the metastable state is affected by the statistical properties of the random variable controlling the barrier height. In the presence of barrier fluctuations rather counterintuitive phenomena may occur. A classical example is the phenomenon of resonant activation [3]. The signature of the resonant activation phenomenon is observed when the average escape time of a particle from the metastable state exhibits a minimum as a function of the parameters of the barrier fluctuations. The original work of Doering and Gadoua triggered a large amount of theoretical activity [4–15]. Analogic simulations of resonant activation have also been performed in a bistable system in the presence of multiplicative Gaussian or dichotomous noise [16,17].

This Letter aims to answer the following questions: (i) Is resonant activation experimentally observable in a physical system? (ii) What about the probability density function of escape time? We answer both questions by investigating the escape from a metastable state over a fluctuating barrier in a physical system. The physical system is a tunnel diode biased in a strongly asymmetric bistable state in the presence of two independent sources of electronic noise. Specifically we have a dichotomous noise source controlling the metastable potential and a Gaussian noise source mimicking the role of a thermal source. In this physical system we observe resonant activation, and we measure and characterize the probability density function (pdf) of escape time by finding two distinct regimes.

Our physical system is the series of a resistive network R with a tunnel diode in parallel to a capacitor (in our case the sum of diode capacitance and input capacitor of the measuring instrument). This system is rather versatile and allows one to investigate the dynamics of a bistable or metastable system in the presence of noise. It has been used to investigate stochastic resonance [18] and noise enhanced stability [19].

The equation of motion of such a system is

$$\frac{dv_d}{dt} = -\frac{dU(v_d)}{dv_d} + \frac{1}{RC}\, v_n(t) \tag{1}$$

with

$$U(v_d) = -\frac{V_B v_d}{RC} + \frac{v_d^2}{2RC} + \frac{1}{C}\int_0^{v_d} I(v)\, dv\,, \tag{2}$$

where R is the biasing resistor of the network, C is the capacitance in parallel to the diode, V_{rms} is the amplitude of the Gaussian noise $v_n(t)$, V_B is the biasing voltage, and $I(v)$ is the nonlinear current-voltage characteristic of the tunnel diode. The effective associated potential is a bistable potential. We control the parameters of our electronic network (V_B and R) in such a way that one of the two wells is much deeper than the other. By setting such strong asymmetry between the two wells we essentially deal with a metastable state. In fact, the probability that the system once escaped goes back into the metastable state during the experimental time is negligible. By using a digital electronic switch we vary the value of the series resistance of our metastable system between the two values R_+ and R_-. When the system switches from R_+ to R_-, two effects take place. One concerns the variation of the height of the barrier of the metastable state, whereas the other consists in a slight change of the value of intrinsic time scale of the system [the RC term of Eqs. (1) and (2)]. In our experiment we use a germanium tunnel diode 1N3149A, which has nominal peak current $I_p = 10.0$ mA, peak voltage $V_p = 60$ mV, valley current $I_v = 1.3$ mA, valley voltage $V_v = 350$ mV, and a rather short switching time (less than 50 nsec). We perform our experiments by choosing $V_b = 8.95$ V, $R_+ = 1100\ \Omega$, $R_- = 1080\ \Omega$, and $C = 45$ pF. The noise $v_n(t)$ is obtained starting from a digital pseudorandom generator. The noise voltage is a stochastic Gaussian noise synthesized by a commercial source (the noise generator DS345 of Stanford Instruments). The Gaussian noise $v_n(t)$ is added to the bias signal V_B by an electronic adder based on a low-noise wide band operational amplifier. At the output of the operational amplifier we measure the noise $v_n(t)$. It is a Gaussian noise characterized by a spectral density which is flat at low frequency ($f < 1$ MHz), having a moderate (approximately 7 dB) increase in the region ($1 < f < 3$ MHz) and quickly decreasing after the cutoff frequency ($f_{\mathrm{cutoff}} =$ 4.6 MHz). By defining the correlation time of the Gaussian noise as the time at which the normalized autocorrelation function assumes the value $1/e$, we measure $\tau_n =$ 68 ns. In addition to the Gaussian noise source, a dichotomous noise source is also present. The dichotomous noise source is obtained by using the commercial chip MM5437 of National semiconductor. An external clock drives the dichotomous noise source and allows us to control the noise correlation time over a wide range. The normalized autocorrelation function of this pseudorandom noise is linearly decreasing from 1 to zero in one clock period $1/f_c$. After this time the autocorrelation function is equal to zero within the experimental errors. By defining the correlation time as before, we have that $\gamma \equiv \tau_C^{-1} \simeq 1.3 \times f_c$. In our experiments the correlation rate γ is varied in the range from $\gamma_{\min} = 13$ Hz to $\gamma_{\max} = 411\,096$ Hz. This is an experimental interval consisting of more than four frequency decades. To investigate so wide a range of frequency we have to overcome two experimental conflicting constraints: (i) we are forced to set the time constant of our system $\tau_s \equiv RC$ to a low value satisfying the inequality $\gamma_{\max} \ll 1/\tau_s$, and (ii) we need to use a high value of τ_s to maintain the ratio τ_n/τ_s as low as possible to conduct our experiments in the "white noise" limit of $v_n(t)$. The best compromise we find is to set $\tau_s \leq R_+C = 49.5$ ns. With this choice $1/\tau_s \approx 50\gamma_{\max}$ and $\tau_n/\tau_s \approx 1.40$. In other words, we guarantee the investigation of the resonant activation phenomenon in a wide range of frequency by performing our experiments in a regime of moderately colored noise.

Under computer control, we measure, for each value of the correlation time of the binary noise, 5×10^3 escape times from the metastable fluctuating state. In each statistical realization, the system is put into the metastable state by an electronic switch. We define the escape time T as the time interval measured between the setting of the system in the metastable state ($t = 0$) and the crossing of v_d of a voltage threshold. The selected voltage threshold ($v_d = 0.4$ V) ensures that the system is quite far from the starting well. The exact value of the threshold is not a determinant parameter because the escape from the well is fast (of the order of τ_s). From the set of measured escape times T, we determine the average value $\langle T \rangle$ and the distribution $P(T)$. The first measurement is devoted to determine the Kramers rates as a function of the noise amplitude V_{rms} for the two metastable states in the *absence* of the dichotomous noise modulation. In our experiments we vary V_{rms} in the interval from 0.816 to 1.00 V, and we observe that the logarithm of the average escape time is a linear function of the inverse of the noise intensity $1/V_{\mathrm{rms}}^2$ for the two states "+" and "−" of the metastable state. In other words, the measured values lay in a straight line verifying the Kramers law in both cases. The two metastable states are characterized by different slopes of the two lines fitting experimental data. These experimental observations confirm that the shape of the metastable potential is different in the two states "+" and "−." The observation of a Kramers law is in agreement with the theoretical prediction of the escape from a metastable state both in the absence and in the presence of colored noise. Specifically, in a regime of colored noise $v_n(t)$, the expected functional form is still Kramers-like [20,21] and can be written as $\langle T \rangle = C \exp[\Delta U_m \tau_s^2/V_{\mathrm{rms}}^2 \tau_n]$ [22]. Here C is a prefactor and ΔU_m is the measured potential barrier height associated with a system characterized by a time constant τ_s and evolving in the presence of a colored noise with correlation time τ_n. Theoretical studies predict that ΔU_m is a function of τ_n/τ_s [20–22]. We verify that we are in a regime of colored noise by performing an experimental test [23].

We perform our investigation of the average escape time as a function of the correlation rate of the dichotomous noise for two different values of the amplitude of the Gaussian noise. The chosen values are $V_{\mathrm{rms}} = 0.816$ V and $V_{\mathrm{rms}} = 0.852$ V. In the absence of switching between the two states, the average escape times from state "+" and "−" are $\langle T_+ \rangle = 0.030$ s and $\langle T_- \rangle = 0.0021$ s when $V_{\mathrm{rms}} = 0.816$ V and $\langle T_+ \rangle = 0.0082$ s and $\langle T_- \rangle =$ 0.00081 s when $V_{\mathrm{rms}} = 0.852$ V. In the presence of switching between the two states the behavior is different. In Fig. 1 we show the average escape time as a function of γ in a log-log plot. We also indicate the value of the Kramers rates $\mu_+ = 1/\langle T_+ \rangle$ and $\mu_- = 1/\langle T_- \rangle$ for the

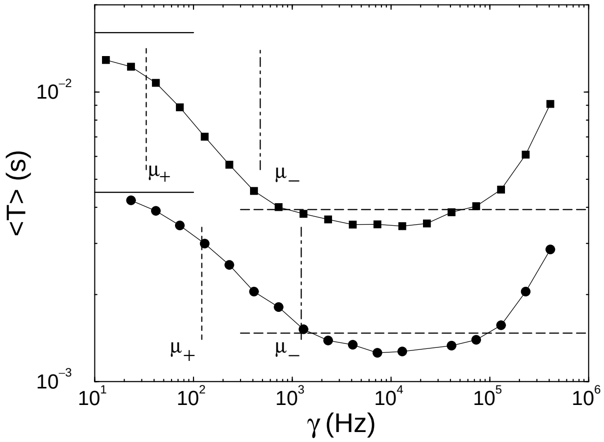

FIG. 1. Average escape time as a function of the correlation rate of the dichotomous noise. The amplitude of the Gaussian noise is set to $V_{\mathrm{rms}} = 0.816$ V (squares) and $V_{\mathrm{rms}} = 0.852$ V (circles). For both sets of data, dashed and dot-dashed lines indicate the Kramers rate of the two states "+" and "−," respectively. Three regimes are clearly seen: (i) the regime of slow dynamics between states ($\gamma < \mu_+$); (ii) the regime of resonant activation ($10^3 < \gamma < 10^5$ Hz); (iii) the regime of fast dynamics between states ($\gamma > 10^5$ Hz). The solid lines indicate the asymptotic behavior expected for slow dynamics, whereas the long-dashed lines indicate the asymptotic value expected from the kinetic approximation. The regime of resonant activation is broader for the value $V_{\mathrm{rms}} = 0.816$ V (squares).

two sets of experiments. In both cases the distinctive characteristics of resonant activation are observed [3], the average escape time initially decreases, reaches a minimum value, and then again increases as a function of γ. Specifically for values of the correlation rate γ less than μ_+ the average escape time approaches the value $\langle T\rangle = (\langle T_-\rangle + \langle T_+\rangle)/2$ predicted by resonant activation theory [3,6,13]. We address this regime as the regime of slow dynamics of the barrier height. For values of the correlation rate approximately 10 times higher than μ_-, the minimum value of the average escape time is observed. Here resonant activation is fully effective. In this regime, the value of the average escape time is in both cases approximately 10% less than the one expected from resonant activation theories in the kinetic approximation [6,13], namely $\langle T\rangle = [(\mu_+ + \mu_-)/2]^{-1}$. We have not a definitive explanation for this discrepancy. One possibility is that this effect reflects the fact that in our experimental set-up our system switches between two states in a regime of colored Gaussian noise rather than in the regime of white noise. A second one is that this behavior is related to the fact that our system switches between two states which in addition to a different potential barrier height are also characterized by a different shape of the potential and a different time constant. For values of γ higher than 10 kHz the average escape time starts to increase as a function of γ and approaches the value associated with an effective potential characterized by an average barrier at the highest value of γ. This is the regime of fast dynamics of the barrier height. A comparison of the two sets of experiments of Fig. 1 also shows that the region of resonant activation becomes flatter when the noise intensity decreases. For our experiments the barrier heights are kept constant in the two series of experiments. This observation is complementary to the theoretical observation that the resonant activation region becomes flatter when the barrier height is increased while the noise intensity is kept constant [12,13].

For each value of the noise intensity V_{rms}^2 and for each value of the correlation rate we measure $P(T)$, the pdf of the escape time. In Fig. 2 we show all the pdfs measured with $V_{\mathrm{rms}} = 0.816$ V in a three-dimensional semilogarithmic plot. The regimes of slow dynamics, resonant activation, and fast dynamics of the dichotomous noise are clearly observed moving from the left to the right. Concerning the shape of the pdf, our results show that two distinct regimes are present. In particular, for values of γ greater than μ_-, $P(T)$ is well described by an exponential decay function $P(T) = \exp[-T/\langle T\rangle]/\langle T\rangle$, whereas in the opposite regime the pdf is nonexponential. Hence, in the resonant activation regime, the pdf has an exponential shape. This experimental observation is in agreement with the numerical observation that the system escapes preferentially through the state with the lowest barrier at the resonant activation [3]. In other words, at the resonant activation the system approximately experiences a single potential barrier and this ends up into an exponential pdf. Exponential pdfs are also observed for higher values of γ in the fast dynamics regime. The experimental detection of an exponential pdf for high values of γ is in agreement with the simple description that the system experiences an effective potential with an average barrier for the highest values of the correlation rate.

The final investigation of our study concerns the degree of nonexponential behavior observed when $\gamma < \mu_-$. To quantify the difference between the real pdf and an exponential pdf we measure the standard deviation of the

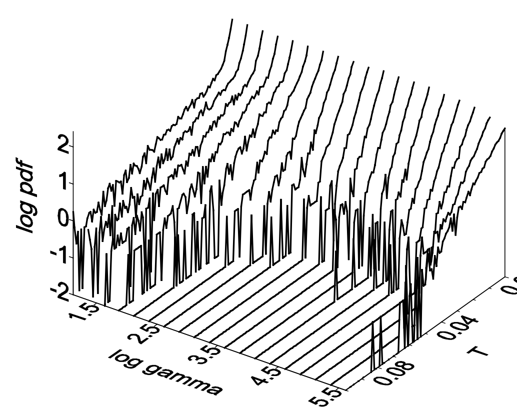

FIG. 2. Overall summary of the probability density functions of escape time measured by setting $V_{\mathrm{rms}} = 0.816$ V. The z axis is logarithmic. From left to right the regimes of slow dynamics, resonant activation, and fast dynamics are clearly seen. In the regimes of resonant activation and fast dynamics the probability density function is well described by an exponential function, whereas in the slow dynamics regime a nonexponential shape emerges.

average escape time. For an exponential pdf the two observables coincide; hence a difference in these observables manifests a nonexponential behavior of the pdf. In Fig. 3 we show the average escape time together with its standard deviation as a function of γ measured by setting $V_{\mathrm{rms}} = 0.816$ V. The two regimes are clearly seen. For high values of γ the pdf is well described by an exponential function; here the average escape time and its standard deviation essentially coincide. When the correlation rate is less than μ_-, a deviation from the exponential form starts to emerge and becomes more pronounced for lowest values of γ. In particular, for very low values of the correlation rate ($\gamma < \mu_+ < \mu_-$) the pdf assumes the form of a double exponential,

$$P(T) = \frac{1}{2\langle T_+\rangle} \exp[-T/\langle T_+\rangle] + \frac{1}{2\langle T_-\rangle} \exp[-T/\langle T_-\rangle]. \tag{3}$$

The inset of Fig. 3 shows $P(T)$ measured when $\gamma = 13$ Hz in a semilogarithmic plot. The shape is clearly nonexponential. It is well approximated by Eq. (3) when the measured values of $\langle T_+\rangle$ and $\langle T_-\rangle$ are used (solid line in the inset). In this regime the correlation rate is smaller than Kramers rates of both states. This implies that the system essentially starts and ends each realization of the stochastic process in one of the two states with probability $1/2$ and the overall pdf is given by Eq. (3).

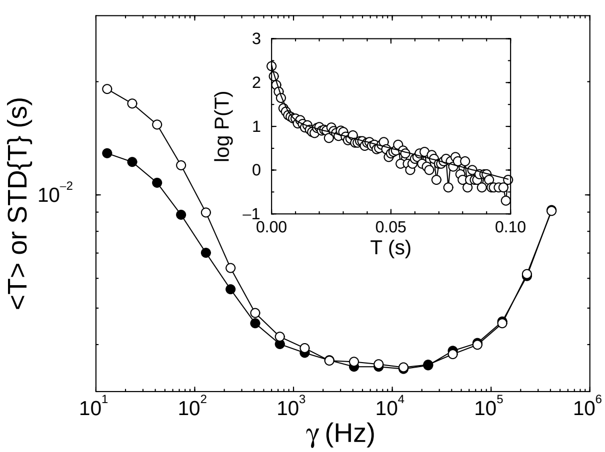

FIG. 3. Average escape time (solid circles) and its standard deviation (open circles) as a function of the correlation rate of the dichotomous noise. The amplitude of the Gaussian noise is set to $V_{\mathrm{rms}} = 0.816$ V. The difference between the two observables quantifies the degree of nonexponential behavior of the pdf. Two regimes are observed: (i) the regime of values of $\gamma > \mu_-$ where an exponential pdf is observed within experimental errors; (ii) the opposite regime ($\gamma < \mu_-$) where a nonexponential behavior is observed. The degree of nonexponential behavior is maximal in the regime of slow dynamics. In the inset we show the probability density function (circles) of the escape time obtained by setting $\gamma = 13$ Hz. The pdf is well approximated by Eq. (3) with $\langle T_+\rangle$ and $\langle T_-\rangle$ measured in the absence of switching. This function is shown as a solid line in the inset.

In this Letter we present an experimental study detecting resonant activation and we verify some theoretical results obtained in model systems for limit regimes of the correlation rate. Resonant activation is observed in spite of the fact that the Gaussian noise mimicking the presence of temperature is colored instead of being “white.” In our opinion, our result shows that the resonant activation phenomenon is pretty robust and may be observed in a variety of physical systems. By investigating the pdf of the average escape time, we detect a nonexponential shape of the pdf in the regime of low values of γ. This finding suggests that systems characterized by a nonexponential escape time probability distribution could be simply modeled in terms of metastable system with a fluctuating barrier.

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[1] H. A. Kramers, Physica **7**, 284 (1940).
[2] P. Hänggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. **62**, 251 (1990).
[3] C. R. Doering and J. C. Gadoua, Phys. Rev. Lett. **69**, 2318 (1992).
[4] U. Zürcher and C. R. Doering, Phys. Rev. E **47**, 3862 (1993).
[5] C. Van den Broeck, Phys. Rev. E **47**, 4579 (1993).
[6] M. Bier and R. D. Astumian, Phys. Rev. Lett. **71**, 1649 (1993); Phys. Lett. A **247**, 385 (1998).
[7] J. J. Brey and J. Casado-Pascual, Phys. Rev. E **50**, 116 (1994).
[8] P. Pechukas and P. Hänggi, Phys. Rev. Lett. **73**, 2772 (1994).
[9] A. J. R. Madureira *et al.*, Phys. Rev. E **51**, 3849 (1995).
[10] P. Reimann, Phys. Rev. Lett. **74**, 4576 (1995); Phys. Rev. E **52**, 1579 (1995).
[11] J. Iwaniszewski, Phys. Rev. E **54**, 3173 (1996).
[12] P. Reimann and T. C. Elston, Phys. Rev. Lett. **77**, 5328 (1996).
[13] M. Boguñá *et al.*, Phys. Rev. E **57**, 3990 (1998).
[14] P. Reimann, R. Bartussek, and P. Hänggi, Chem. Phys. **235**, 11 (1998).
[15] R. N. Mantegna and B. Spagnolo, J. Phys. IV (France) **8**, 247 (1998).
[16] F. Marchesoni and L. Gammaitoni, Phys. Lett. A **201**, 275 (1995).
[17] M. Marchi *et al.*, Phys. Rev. E **54**, 3479 (1996).
[18] R. N. Mantegna and B. Spagnolo, Phys. Rev. E **49**, R1792 (1994); Nuovo Cimento Soc. Ital. Fis. D **17**, 873 (1995).
[19] R. N. Mantegna and B. Spagnolo, Phys. Rev. Lett. **76**, 563 (1996).
[20] A. J. Bray and A. J. McKane, Phys. Rev. Lett. **62**, 493 (1989).
[21] M. I. Dykman, Phys. Rev. A **42**, 2020 (1990).
[22] P. Hänggi *et al.*, Phys. Rev. A **32**, 695 (1985).
[23] In our test we measure $\langle T\rangle$ as a function of $1/V_{\mathrm{rms}}^2$ for different values of τ_s. The test confirms that our system is in a regime of colored noise because we observe that $\Delta U_m \tau_s$ decreases when τ_s increases and approximates to a constant value for $\tau_n/\tau_s < 0.01$.