

**Hellberg and Manousakis Reply:** In our Letter [1] we concluded that the ground state of the  $t$ - $J$  model does not have stripes at  $J/t = 0.35$ . In the preceding Comment [2], White and Scalapino (WS) raise several objections to our findings. We refute all of their points and argue that our analysis is indeed correct.

The physical mechanism cited by WS explains why *if a striped state were the ground state of the  $t$ - $J$  model*, the domain boundaries would prefer a  $\pi$ -phase shift in the antiferromagnetic background. However, this mechanism does not favor the stripe state over other alternative candidates for the ground state of the  $t$ - $J$  model in the low hole-doping region. One serious candidate is a state of electronic phase separation [3,4] where an antiferromagnetic region is separated from the hole rich region by only one (as opposed to infinitely many as is the case for the striped state) energetically costly interface. When a *finite-size* system is in a region of the parameter space for which the infinite system would phase separate, its energy will be best minimized if the two components in which the finite-system tends to phase-separate respect the geometry imposed by the boundaries. Thus an instability or near instability to phase separate may cause domain walls or other structures to form in a *finite-size* system. The argument cited by WS explains why if one has stripes one has to have a  $\pi$ -phase shift in the antiferromagnetic order parameter to accommodate hole motion. It does not explain why stripes are formed as opposed to a phase separated state.

WS view their boundary condition as a symmetry breaking field whose strength can be taken to zero. However, this procedure requires taking the thermodynamic limit and studying whether or not the stripes remain. By studying WS's results obtained for cylindrical boundary conditions as a function of the number of legs in the cylinder, it seems that the stripes are strongly influenced by finite-size effects. Depending on the cylinder's width, WS find stripes with different linear hole densities along the stripe. In six-leg ladders, the optimum linear density is  $\rho_6 = 2/3$  [5], and in eight-leg ladders,  $\rho_8 = 1/2$  [6].

WS find that the addition of a  $t'$  term destroys stripes in their calculations. A next-nearest-neighbor hopping  $t'$  inhibits phase separation in the  $t$ - $J$  model. If in a particular finite geometry, the near instability to phase separate is manifested by stripe formation, adding a  $t'$  hopping will destabilize the stripes.

WS are incorrect in stating that our conclusions would have been different if we had excluded the clusters which they believe are too one dimensional. The (2,2) translation vectors are four lattice spacings in distance, just as the (0,4) translation vectors are. However, irrespective of which clusters we keep or exclude, our conclusions are unchanged. Even if we restrict ourselves to cluster No. 2, the lowest energy state of this cluster has energy  $\mathcal{E}_0 = -0.6397t$  and is not striped. State (*e*), the lowest energy vertical stripe state, has energy  $\mathcal{E}_e = -0.6353t$ .

Thus even if we had examined only the cluster in which we found the vertical stripes, we would still conclude that these stripes are excited states.

WS believe a system with at least four holes is required to study stripe formation. If many-hole correlations are important, one needs to explain why stripes are seen in mean-field studies. Prelovšek and Zotos [7] found stripe correlations only for large  $J/t$  and did not study stripe correlations in two-hole clusters. The degenerate excited states (e) and (f) in our periodic cluster No. 2 are nearly identical to the stripes seen by WS [6]. The charge density wave amplitude and the spin structure (including the  $\pi$ -phase shift) are the same. And, as shown in Fig. 3 of our Letter [1], the use of open boundary conditions in one direction breaks the degeneracy and stabilizes these states as the ground state.

WS argue that our results might be a finite-size effect (FSE); however, our main reason for performing a small cluster exact calculation was to study the role of FSEs on the formation of stripes. Thus, FSEs are *welcome* in our calculation. Since stripes are periodic structures, calculations on small periodic clusters that are commensurate with the stripe order including the  $\pi$ -phase shift between stripes, such as our cluster No. 2, *favor* the formation of stripes. We do find stripes in cluster No. 2 with exactly the same structure as those found by WS but only as excited states. The fact that these stripes are not the ground state even of the cluster most favorable for their formation indicates that the ground state of the infinite system is not striped.

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