Bloch Electrons in a Magnetic Field: Why Does Chaos Send Electrons the Hard Way?

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We find that a 2D periodic potential, with different modulation amplitudes in the x and y directions, and a perpendicular magnetic field may lead to a transition to electron transport along the direction of *stronger* modulation and to localization in the direction of *weaker* modulation. In the experimentally accessible regime we relate this new quantum transport phenomenon to avoided band crossings due to classical chaos.

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Theoretical studies of quantum chaos have recently included *quasiperiodic* systems concentrating so far on the kicked Harper model [1]. These studies have led to the discovery of interesting phenomena, e.g., surprising metal-insulator transitions [2] whose origins are avoided band crossings induced by classical chaos [3]. While the kicked Harper model is a numerically convenient toy model, we here focus on a prominent example [4] from solid state physics which nowadays is experimentally accessible [5]: an electron moving in a 2D periodic potential, a so-called Bloch electron, subjected to a magnetic field. It is classically chaotic [6], and it is quasiperiodic when the number of magnetic flux quanta per unit cell of the potential is irrational [4]. In this paper we present a new 2D quantum transport phenomenon: By changing the potential strength, keeping the ratio of the modulation strength in the x and y directions fixed, we find that transport exclusively along the direction of *weaker* potential modulation changes to transport exclusively along the direction of *stronger* modulation (Fig. 1).

Ballistic transport exclusively along the direction of *weaker* modulation is expected for the limiting cases of either a very small or a very large potential strength compared to the magnetic field strength. This is based on the properties of the Harper model [7–11] which approximates Bloch electrons in a magnetic field in both limiting cases. For transitions to ballistic transport along the direction of *stronger* modulation we find two distinct origins. In the regime where potential and magnetic field are of comparable strength [12] we show that there are many transitions which we relate to avoided band crossings induced by the classical nonintegrability. In the regime of large potential strength where the energy spectrum consists of separated bands, there are additional transitions in some of these bands which can be explained in the tight-binding approximation and which are not related to the classical limit. Experimentally, the transitions of the transport direction in the first regime should be observable with lateral surface superlattices on semiconductor heterojunctions.

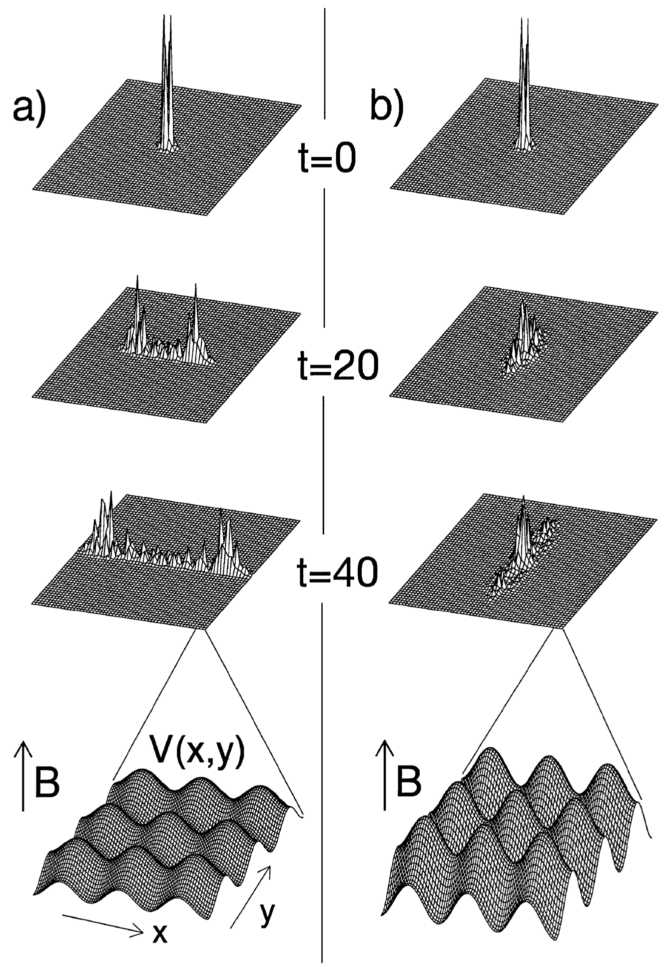

FIG. 1. Time evolution for initially localized electron wave packets [14] spreading in 2D periodic potentials with different modulation amplitude in the x and y directions (bottom) and a perpendicular magnetic field. (a) For $K = 5$ the wave packet spreads ballistically in the direction of *weaker* modulation only, as expected for $K \to 0$ and $K \to \infty$. (b) For $K = 10$ the wave packet spreads ballistically in the direction of *stronger* modulation only. Shown are 55×55 unit cells of the potential; times are in units of the cyclotron period $2\pi/\omega_c$, $V_y/V_x = 1.25$, and $\Phi/\Phi_0 = 89/55$.

The one-particle Hamilton operator for an electron with charge $-e$ and mass m in a magnetic field and in the simplest 2D periodic potential has the form

$$H = \frac{1}{2m}(\mathbf{p} + e\mathbf{A})^2 + V_x \cos\left(\frac{2\pi x}{a}\right) + V_y \cos\left(\frac{2\pi y}{a}\right). \quad (1)$$

The magnetic field $\mathbf{B} = \mathrm{rot}\mathbf{A}$, where $\mathbf{A}$ is the vector potential, is taken to be homogeneous and perpendicular to the xy plane. The properties of the system depend on the following three dimensionless parameters: the potential strength

$$K = (V_x + V_y)4\pi m a^2/h^2, \quad (2)$$

the ratio V_y/V_x, and the ratio of the magnetic flux penetrating a unit cell of the potential divided by the magnetic flux quantum, $\Phi/\Phi_0 = a^2B/(h/e)$ (see, e.g., Ref. [13]). The ratio of the potential strength to magnetic field strength is given in terms of these parameters by $2(V_x + V_y)/(\hbar\omega_c) = K\Phi_0/\Phi$, with the cyclotron frequency $\omega_c = eB/m$. In the following, we will vary only K, while the other parameters will be kept fixed: $V_y/V_x = 1.25$ and $\Phi/\Phi_0 = (\sqrt{5} - 1)/2$, the golden mean. Figure 1a shows the time evolution of an initially localized wave packet for $K = 5$ [14]. It spreads ballistically in the direction of weaker potential modulation (x direction) and localizes in the direction of stronger modulation (y direction) as expected for $K \to 0$ and $K \to \infty$. In contrast, for $K = 10$ (Fig. 1b), localization occurs in the direction of *weaker* modulation and one finds ballistic spreading in the direction of *stronger* modulation. In other words, as a function of potential strength (keeping the ratio V_y/V_x constant), Fig. 1 shows that the system undergoes a metal-insulator transition in the x direction and an insulator-metal transition in the y direction.

We demonstrate that this change in the direction of transport is related to changes of spectrum and eigenfunctions of operator (1). Numerically, one has to study rational approximants $\Phi/\Phi_0 = q/p$. Then, operator (1) is periodic in the x and y directions and its eigenenergies depend on the two phases k_x and k_y of the magnetic Brillouin zone. In Fig. 2 [top (bottom)], by increasing K from 5 to 10, one finds for the energy subspectrum with only k_x (k_y) varied a transition from wide to extremely narrow (narrow to wide) minibands while eigenfunctions change from extended to localized (localized to extended) in the x direction (y direction) [15]. These wide (extremely narrow) minibands correspond to an absolutely continuous (pure point) spectrum in the irrational case and we will abbreviate them in the following by "bands" ("levels"). Figure 2 shows that eigenfunctions in the y direction and the subspectrum under variation of k_y are dual to the corresponding behaviors in the x direction. The origin of this duality, which we have found for all parameters studied, seems to be related to the origin of the Aubry duality [9] of the Harper model and remains to be explored. By using the duality we concentrate in the following on the properties in the x direction. In Fig. 3, one can see the subspectrum under variation of k_x on larger scales than in Fig. 2 (top). For $K = 0$, one has Landau levels which for small K broaden linearly with K forming Landau bands which have a fine-structure described by the Harper model (see below). When increasing K, the spectrum becomes very complex: First of all, there are many transitions from "bands" to "levels" and vice versa. Second, one observes many avoided crossings of spectral regions, e.g., there is one in the box corresponding to Fig. 2a. One notices that the spectral transitions happen in the range of avoided crossings.

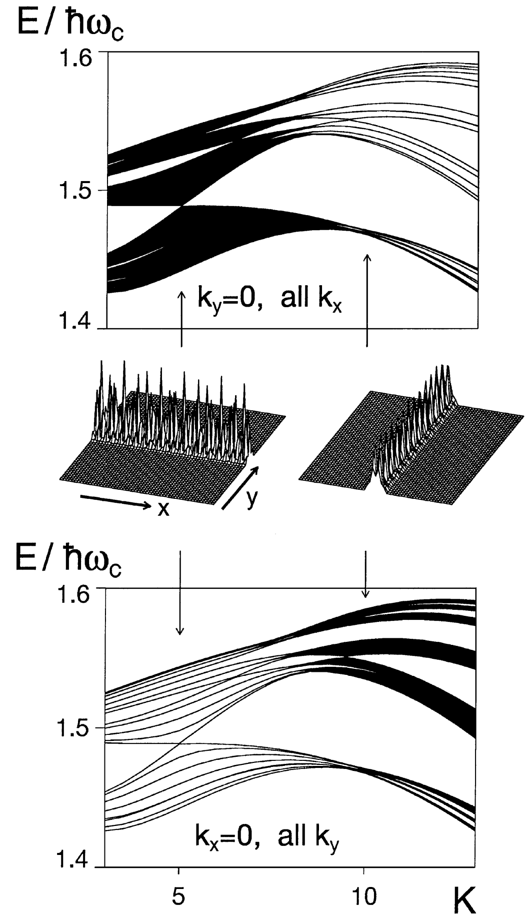

FIG. 2. Subspectra for $k_y = 0$ and all k_x (top), $k_x = 0$ and all k_y (bottom) vs K, and typical eigenfunctions [15] for $K = 5$ and $K = 10$ ($\Phi/\Phi_0 = 89/55$).

We propose that these spectral transitions for Bloch electrons in a magnetic field are in fact due to the *avoided band crossings*: These are analogous to avoided level crossings in classical chaotic, bounded quantum systems with a discrete spectrum, and they occur in extended systems with a classically nonintegrable Hamiltonian. In Ref. [3], consequences of avoided band crossings were analyzed by studying a three-band model, in which each band was described by a tight-binding Hamiltonian, e.g., by a Harper model. In the range where the bands avoid crossing, a perturbation calculation predicted that the parameters of their tight-binding Hamiltonian effectively change. In

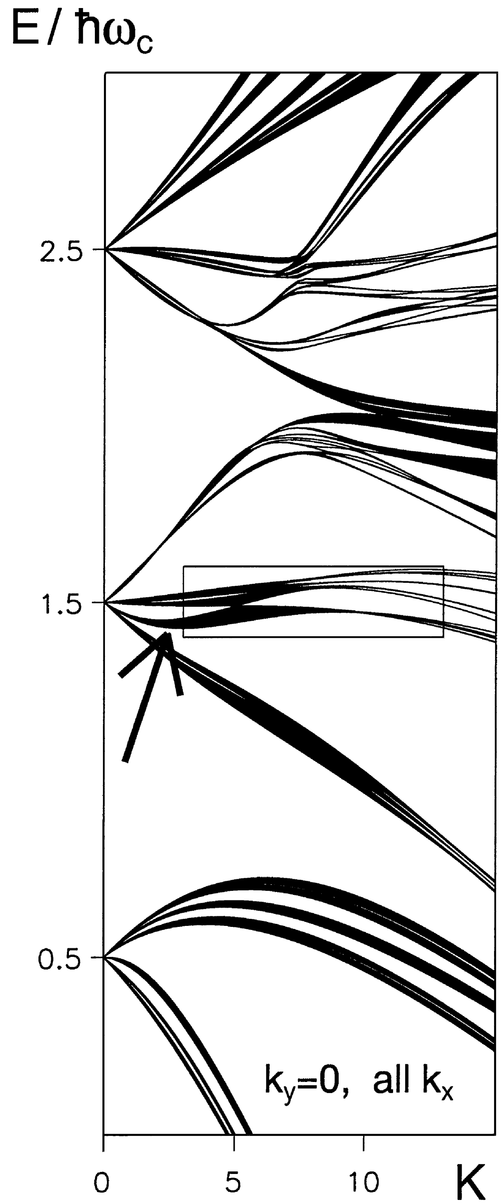

FIG. 3. The subspectrum for $k_y = 0$ and all k_x vs K ($\Phi/\Phi_0 = 34/21$). Several avoided crossings can be observed which are accompanied by transitions from energy "bands" to energy "levels" or vice versa. The box indicates the region shown in Fig. 2 and the arrow points to the spectral region that avoids crossing the one above.

particular, when one of these tight-binding Hamiltonians is a quasiperiodic Harper model and if the change is large enough this leads to transitions from "bands" to "levels" or vice versa [3]. In our case of Bloch electrons in a magnetic field, such a perturbation analysis for isolated avoided crossings cannot be repeated as the energy spectrum is much too complex (Fig. 3). There is, however, a strong analogy: There are spectral regions performing avoided crossings as this extended system is classically nonintegrable [6]. Before the crossings (small K), these spectral regions may be described by Harper-like models [16]. Finally, we also find transitions from "bands" to "levels" or vice versa in the range of the avoided crossings. This analogy strongly suggests that, also for Bloch electrons in a magnetic field, avoided band crossings due to classical chaos give rise to the spectral transitions in Fig. 3, thereby inducing the change of transport properties in the x direction in Fig. 1. Thus, finally, the change in transport is linked to the classical nonintegrability of the Hamiltonian (1). It is important to stress that the classical dynamics shows no corresponding change in transport for these parameters. Instead, as usual in the field of quantum chaos, the classical dynamics enters via its nonintegrability that here leads to the avoidance of band crossings.

There are exceptional spectral transitions for large K which have a different origin: For strong potential, $K \gg \Phi/\Phi_0$, and for sufficiently low energy the spectrum of Bloch electrons in a magnetic field consists of well-separated bands. In the tight-binding approximation each of them has a fine-structure described by Harper's equation [7]

$$\psi_{n+1} + \psi_{n-1} + 2\lambda\cos(2\pi\sigma n + \nu)\psi_n = \tilde{E}\psi_n\,, \quad (3)$$

where λ is the important parameter in the following. While $\lambda > 1$ corresponds to localization in the x direction, $\lambda < 1$ corresponds to spreading in the x direction, for typical irrational Φ_0/Φ [10]. These separated bands follow the bands in the case *without* magnetic field in which the Hamiltonian separates in the x and y directions such that each band is the sum of an energy band in the x and one in the y direction. The n_αth band ($n_\alpha = 0, 1, 2, \ldots$) of the 1D potential $V_\alpha \cos(2\pi\alpha/a)$ has a dispersion amplitude $\epsilon_\alpha(n_\alpha)$, where $\alpha = x, y$, that is determined by tunneling. We will use the fact that it increases with index n_α and that it decreases exponentially with potential strength V_α. *With* magnetic field the transport properties of each band, enumerated by (n_x, n_y), are determined in the tight-binding approximation by Harper's equation with $\lambda = \epsilon_y(n_y)/\epsilon_x(n_x)$. While for $K \ll \Phi/\Phi_0$, each Landau band has a fine-structure again described by Harper's equation [Eq. (3)] [8] with a constant $\lambda = V_x/V_y$, here λ depends on the band indices n_x, n_y. There are two cases: For $n_y \leq n_x$ we find from the properties of $\epsilon_\alpha(n_\alpha)$ that $\epsilon_y(n_y) < \epsilon_x(n_x)$ (for $V_y > V_x$), resulting in $\lambda < 1$ as in the limit $K \to 0$. For $n_y > n_x$ we find that there exists a $K_c(n_x, n_y)$ where a transition occurs. For $K > K_c$, one still has $\epsilon_y(n_y) < \epsilon_x(n_x)$. In contrast, for $K < K_c$ the opposite is true, leading to $\lambda > 1$ and thus to localization in the x direction. These transitions within a tight-binding band are a consequence of the λ dependence of the Harper model and are not related to avoided band crossings or the classical dynamics. An example of such a transition is the tight-binding band $(0, 1)$ for which $K_c \approx 1000$ [17]. In the subspectrum with k_x varied it consists of "bands" for $K > K_c$, while for $K < K_c$ we find "levels" which appear (below the regime where the tight-binding approximation is valid) in Fig. 3 at $E/(\hbar\omega_c) \approx 1.5$, $K = 15$.

In order to complete the picture on spectral transitions for Bloch electrons in a magnetic field let us mention that, without magnetic field for large K, pairs of bands can make isolated crossings, e.g., the bands with $(0, 2)$ and $(1, 1)$ at $K \approx 50$. In the presence of a magnetic field each of these crossings becomes an avoided crossing of Harper models and may be well described by the models of Ref. [3] and may have many spectral transitions and corresponding changes in transport. Let us mention that the properties of the tight-binding bands before and after an isolated crossing are not affected.

By generalizing these results to arbitrary $V_y > V_x$ for Bloch electrons in a magnetic field [Eq. (1)] we find that there are spectral transitions which have two distinct origins: (i) In the regime where the tight-binding description is valid, the K dependence of the bandwidths $\epsilon_\alpha(n_\alpha)$ and the properties of the Harper model predict spectral transitions within tight-binding bands with $n_y > n_x$ that are unrelated to the classical limit. In the subspectrum with k_x varied these transitions are from "bands" to "levels" only, and not vice versa when decreasing K from ∞. In order to end up again with "bands" for $K \rightarrow 0$ there have to be further transitions from "levels" back to "bands." (ii) These transitions *and many more transitions in both ways* appear in the regime where tight-binding bands merge and form Landau bands (Fig. 3). We ascribe them to avoided crossings of spectral regions which in turn are induced by classical chaos. Corresponding to the spectral transitions in the subspectrum when k_x is varied, we find dual transitions when k_y is varied (Fig. 2). These transitions in both subspectra cause the observed change of transport from the direction of weaker to that of stronger modulation (Fig. 1). In general, transport in just one direction, as in Fig. 1, will occur whenever an initial wave packet excites an energy range with eigenfunctions of just one type, namely, either extended in the x or in the y direction. In the exceptional case that the small energy range of width kT around the Fermi energy contains both types of eigenfunctions, one finds a superposition of transport in each direction, namely, a crosslike spreading. For other 2D periodic potentials, e.g., the potential of Eq. (1) with different lattice constants or rectangular antidot potentials, we find the same changes of the transport direction in the regime of avoided band crossings [18].

These phenomena should be experimentally accessible using lateral superlattices on semiconductor heterojunctions [5] by measuring the ratio $\rho_{yy}/\rho_{xx} = \sigma_{xx}/\sigma_{yy}$. Weak disorder as present in the experimental probes will destroy the transport phenomenon in the regime of the exponentially narrow tight-binding bands. Instead, in the regime where tight-binding bands and Landau bands merge and avoided band crossing occurs, preliminary numerical studies show that the phenomenon should be observable.

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