Dispersion Properties of a Dusty Plasma Containing Nonspherical Rotating Dust Grains

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The dispersion relation for a dusty plasma containing elongated and rotating charged dust grains is obtained by assuming that the dipole moments of the dust particulates are nonzero. The longitudinal waves with frequency close to the angular rotation frequency of the dust grains are found to be unstable. The results should be relevant to enhanced fluctuations in astrophysical environments.

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It is well known that the dust grains in astrophysical as well as in terrestrial plasmas do not have a spherical form; the grains are typically elongated or flattened [1,2]. The grains also acquire a rotational motion due to their interaction with photons and particles of the surrounding gas. The angular frequency of such rotation can reach a rather large value $\approx 10^4 - 10^6$ sec^{-1} for thermal dust grains and $\approx 10^9$ sec^{-1} for suprathermal ones [1], and there is a different kind of orientation involving preferred direction (relative to the galactic disk) of the dust grain angular momentum vector.

During the last eight years, a large number of papers [3–6] have been written dealing with the dispersion properties of a dusty plasma. In all of these papers, dust grains were, however, assumed to be point charges or have a spherical shape. Consequently, the previous investigations of dusty plasma waves and instabilities have not studied the influence of dust grain elongation.

The nonspherical charged dust grains would have, in general, a nonzero dipole moment. Hence, the dusty plasma with elongated and rotating dust grains would acquire a new characteristic frequency, which equals the angular rotation frequency of the dust grains. In this Letter, it is shown that the dispersion properties of such a dusty plasma can be drastically modified. Specifically, the electrostatic waves with frequencies close to the dust rotational frequency are found to be unstable. In order to demonstrate this, a kinetic equation for charged, rotating dust grains is constructed, by assuming that the dust particles have a nonzero dipole moment. The dispersion relation for a dusty plasma is then derived, by supposing that elongated dust grains rotate with some preferred angular frequency. In conclusion, an indirect way is suggested for detecting the dust grain rotation by scattering of the radiation off the exited fluctuations in a dusty plasma.

Let us suppose that dust grains are elongated, and neglect the spin around the axis of the elongation. The angular moments of rotating dust grains are assumed to be directed along some direction, say along the z axis. The dust grain system can then be described by the one-particle distribution function $f_d(\mathbf{R}, \mathbf{v}, \Omega, \varphi, t)$ [7], which enumerates the particulates according to the coordinate $\mathbf{R}$ and the velocity $\mathbf{v}$ of the center of mass, and the angular frequency Ω. The azimuthal angle φ determines the direction of the dust grain elongation axis.

The charge density $\rho(\mathbf{r}, t)$ associated with the dust grains is defined in terms of the grains distribution function. We have

$$\rho(\mathbf{r}, t) = \int d\mathbf{R}\, d\Gamma\, \hat{\rho}(\mathbf{r} - \mathbf{R}, \varphi) f_d(\mathbf{R}, \mathbf{v}, \Omega, \varphi, t), \quad (1)$$

where $d\Gamma = d\mathbf{v}\, d\Omega\, d\varphi$. The integrand $\hat{\rho}(\mathbf{r} - \mathbf{R}, \varphi)$ describes the charge distribution on a single grain. It depends on the shape of the grain and the azimuthal orientation of the grain's elongation axis. Outside of the grain's volume, we have $\hat{\rho} = 0$.

For point particulates, we have $\hat{\rho}(\mathbf{r} - \mathbf{R}, \phi) = q\delta(\mathbf{r} - \mathbf{R})$, where q is the total charge of the dust grain and $\delta(x)$ is the Dirac-delta function, and the relation (1) gives the well-known expression for the charge density. The dust grains are identical. Then, we can partly determine the dependence of $\hat{\rho}$ on the azimuthal angle φ. Every given direction of the grain elongation axis, determined by the angle φ, can be considered as a final position of the turning of the axis (and simultaneously of the whole grain) from the direction with $\varphi = 0$. This allows us to write

$$\hat{\rho}(\mathbf{r} - \mathbf{R}, \varphi) \equiv \hat{\rho}(\bar{A}(\varphi)(\mathbf{r} - \mathbf{R}), 0) \equiv \hat{\rho}(\bar{A}(\mathbf{r} - \mathbf{R})), \quad (2)$$

where $\bar{A}(\varphi)$ is the tensor of turning by the angle φ, *viz.*

$$\bar{A}(\varphi) = \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{pmatrix}. \quad (3)$$

Furthermore, we assume that the dust grain size is smaller than the scale length of the plasma inhomogeneity,

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viz. $a \ll l$. Substituting (2) into (1) and expanding the distribution function f_d around the point $\mathbf{r}$, we obtain

$$\rho(\mathbf{r},t) = q\int d\Gamma f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t) - \int d\Gamma(\mathbf{d}\cdot\nabla)f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t), \quad (4)$$

where

$$q = \int d\mathbf{r}\hat{\rho}(\mathbf{r}) \quad (5)$$

and

$$\mathbf{d} = \bar{A}^{-1}(\varphi)\int d\mathbf{r}\,\mathbf{r}\hat{\rho}(\mathbf{r}) \quad (6)$$

are the total charge and the dipole moment of the dust grain, respectively. Furthermore, $\bar{A}^{-1}(\varphi)$ is the inverse of the tensor $\bar{A}(\varphi)$. Quite analogous calculations lead to the following expression for the dust current density

$$\mathbf{J}(\mathbf{r},t) = q\int d\Gamma\mathbf{v}f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t) - \int d\Gamma\mathbf{v}(\mathbf{d}\cdot\nabla)f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t) + \int d\Gamma[\mathbf{\Omega}\times\mathbf{d}]f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t), \quad (7)$$

where $\mathbf{\Omega} = (0,0,\Omega)$. The first two terms on the right-hand side of (7) describe the transfer of the charge (4), and the third term describes the current arising from the dust grain rotation. Below we shall show that the expressions (4) and (7) are connected with the continuity equation, as it should be. The motion of the charged dust grain in the electromagnetic fields $[\mathbf{E} = -\nabla\phi - (1/c)\partial_t\mathbf{A}$ and $\mathbf{B} = \nabla A]$ is described by the Hamiltonian

$$H = \frac{1}{2m_d}\left\{\mathbf{P} - \frac{q}{c}\mathbf{A} + \frac{1}{c}[\mathbf{d}\times\mathbf{B}]\right\}^2 + \frac{P_\phi^2}{2I} + q\phi - (\mathbf{d}\cdot\mathbf{E}), \quad (8)$$

where m_d is the mass of the dust grain, $\mathbf{P}$ the generalized momentum, $P_\phi = I\Omega$ the angular momentum, I the moment of inertia of the elongated grain, c the speed of light, and ϕ and $\mathbf{A}$ are the scalar and vector potentials of the electromagnetic fields, respectively. By means of the Hamiltonian (8), the kinetic equation for the dust grain distribution function can be written as

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial\mathbf{r}} + \frac{1}{m_d}\left\{[q + (\mathbf{d}\cdot\nabla)]\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right) + \frac{1}{c}[[\mathbf{\Omega}\times\mathbf{d}]\times\mathbf{B}]\right\}\cdot\frac{\partial f_d}{\partial\mathbf{v}} + \Omega\frac{\partial f_d}{\partial\varphi} + \left[\mathbf{d}\times\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\right]_z\frac{\partial f_d}{\partial p_\phi} = 0. \quad (9)$$

From (9) it can be easily shown that the dust grain density and the current, defined by (4) and (7), satisfy the continuity equation. Equation (9) and the kinetic equations for the electrons and the ions, namely,

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial\mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right\}\cdot\frac{\partial f_\alpha}{\partial\mathbf{v}} = 0, \quad (10)$$

supplemented by the Maxwell equations, allow us to deduce the dispersion properties of the dusty plasma containing rotating charged dust grains. Here, α equals e for the electrons and i for the ions.

To determine the dielectric permittivity, we must introduce a small perturbation on the equilibrium distribution functions f_{d0} (for the dust particles) and $f_{\alpha0}$ (for the electrons and the ions). Such deviations occur due to small fluctuations of the electric and magnetic fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, which in turn are produced by the perturbation of the equilibrium. We write the perturbed distribution functions as

$$f_d = f_{d0} + \delta f_d \quad (11)$$

and

$$f_{\alpha0} = f_{\alpha0} + \delta f_\alpha, \quad (12)$$

and suppose that $\delta f_d \ll f_{d0}$ and $\delta f_\alpha \ll f_{\alpha0}$. Assuming that the perturbed quantities are proportional to $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where ω and $\mathbf{k}$ are the wave frequency and the wave vector, we linearize (10) and obtain

$$\delta f_\alpha = -i\frac{e_\alpha}{m_\alpha}(\omega - \mathbf{k}\cdot\mathbf{v})^{-1}\mathbf{E}\cdot\frac{\partial f_{\alpha0}}{\partial\mathbf{v}}. \quad (13)$$

In the following, we consider the case where the dust grain size is smaller than the wavelength of the perturbation, *viz.* $ka \ll 1$, and that the phase velocity of the wave exceeds the thermal velocity of the dust particles. Accordingly, (9) can be simplified, and after linearization we obtain

$$\delta f_d = -i\frac{q}{m_d}(\omega - \mathbf{k}\cdot\mathbf{v})^{-1}\mathbf{E}\cdot\frac{\partial f_{d0}}{\partial\mathbf{p}} + \frac{d}{2}\frac{\partial f_{d0}}{\partial p_\varphi}\left\{\frac{E_x - iE_y}{\omega - \mathbf{k}\cdot\mathbf{v} - \Omega}e^{i\varphi} - \frac{E_x + iE_y}{\omega - \mathbf{k}\cdot\mathbf{v} + \Omega}e^{-i\varphi}\right\}, \quad (14)$$

where we have introduced the following relations for the components of the dust grain's dipole moment:

$$d_x = d\cos(\varphi), \qquad d_y = d\sin(\varphi). \tag{15}$$

In the unperturbed state, we choose the Maxwellian distribution functions for the electrons and the ions with the temperature T_α and the number density $n_{\alpha 0}$. On the other hand, for the unperturbed distribution function of the rotating dust grains, we can use [8]

$$f_{d0} = n_{d0}(2\pi m_d T_d)^{-3/2}(2\pi I T_d)^{-1/2} \times \exp\left\{-\frac{p^2}{2m_d T_d} - \frac{(p_\varphi - p_{\varphi 0})^2}{2IT_d}\right\}, \tag{16}$$

where n_{d0} and T_d are the unperturbed number density and the temperature of the dust grains, respectively. We assumed the dust grains to rotate with the preferred angular frequency Ω_0, so that $p_{\varphi 0} = I\Omega_0$. The unperturbed quasineutral dusty plasma does not have equilibrium charge and current densities. However, the charge and current densities are induced due to the perturbed electromagnetic fields. Expressing the induced current density through the perturbations of the distribution functions (13) and (14), we can find the dielectric tensor for the dusty plasma following the standard method [9]. The result is

$$\epsilon_{ij}(\omega,\mathbf{k}) = \frac{k_i k_j}{k^2}\,\epsilon^l(\omega,\mathbf{k}) + \left\{\delta_{ij} - \frac{k_i k_j}{k^2}\right\}\epsilon^t(\omega,\mathbf{k}) + \left\{\delta_{ij} - \frac{\Omega_i\Omega_j}{\Omega^2}\right\}\epsilon^d(\omega,\mathbf{k}), \tag{17}$$

where $\epsilon^l(\omega,\mathbf{k})$ and $\epsilon^t(\omega,\mathbf{k})$ are the usual longitudinal and transverse dielectric permittivities, respectively. They are given by

$$\epsilon^l(\omega,\mathbf{k}) = 1 + \sum_\beta \frac{\omega_{p\beta}^2}{k^2 V_{T\beta}^2}\left\{1 - I_+\left(\frac{\omega}{kV_{T\beta}}\right)\right\} \tag{18}$$

and

$$\epsilon^t(\omega,\mathbf{k}) = 1 - \sum_\beta \frac{\omega_{p\beta}^2}{\omega^2}\, I_+\left(\frac{\omega}{kV_{T\beta}}\right), \tag{19}$$

where $\beta = e,\ i,\ d,\ \omega_{p\beta}$, and $V_{T\beta}$ are the plasma frequency and the thermal velocity of the particle specie β. The function $I_+(x)$ is [9]

$$I_+(x) = \frac{x}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{dz\,\exp(-z^2/2)}{x - z}. \tag{20}$$

The asymptotic forms of (20) are

$$I_+(x) \approx 1 + \tfrac{1}{x^2} + \ldots - i\sqrt{\pi/2}\,x\exp(-x^2/2)$$

for $|x| \gg 1$, $|\mathrm{Re}x| \gg |\mathrm{Im}x|$, $\mathrm{Im}x < 0$, and

$$I_+(x) \approx -i\sqrt{\pi/2}\,x \tag{21}$$

for $|x| \ll 1$. The influence of the dust grain rotation is described by $\epsilon^d(\omega,\mathbf{k})$, which equals

$$\epsilon^d(\omega,\mathbf{k}) = -\frac{\Omega_r^2}{\omega^2}\frac{k^2}{\bar{k}^2}\frac{\omega}{\omega - \Omega_0} I_+\left(\frac{\omega - \Omega_0}{\bar{k}V_{Td}}\right) + \frac{\Omega_r^2}{\bar{k}^2 V_{Td}^2}\left(\frac{\kappa^2}{\bar{k}^2} + \frac{k^2}{\bar{k}^2}\frac{\Omega_0}{\omega}\right) \times \left\{1 - I_+\left(\frac{\omega - \Omega_0}{\bar{k}V_{Td}}\right)\right\}, \tag{22}$$

where $\Omega_r = [4\pi d^2 n_{d0}/2I]^{1/2}$, $\kappa^2 = m_d/I$, and $\bar{k} = [k^2 + \kappa^2]^{1/2} \approx \kappa$.

Let us now consider the frequency regimes

$$kV_{Td} \ll \omega \ll V_{Ti}, kV_{Te}$$
$$\omega - \Omega_0 \gg \bar{k}V_{Td}. \tag{23}$$

The participation of the charged dust gas is decisive only in the wave motion when the frequency and wave vector satisfy the relation (23). Without loss of generality, we may assume that the vector $\mathbf{k}$ lies in the (x,z) plane, i.e., $\mathbf{k} = (k_\perp, 0, k_z)$. Then, from the general dispersion relation

$$\left|k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\,\epsilon_{ij}(\omega,\mathbf{k})\right| = 0, \tag{24}$$

and according to (17) to (22) we find (a) for the transversal waves, which are polarized along the y axis so that $\mathbf{E} = (0,E,0)$, the following dispersion relation:

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{pd}^2}{\omega^2} - \frac{\Omega_r^2}{(\omega - \Omega_0)^2}. \tag{25}$$

For $\Omega_0 = 0$ the influence of the dust grain rotation disappears. However, for waves with frequencies close to Ω_0 ($\omega_{pd} \approx \Omega_r$), we obtain

$$\omega = \Omega_0\left\{1 \pm i\,\frac{\Omega_r}{(k^2c^2 + \omega_{pd}^2)^{1/2}}\right\}. \tag{26}$$

Equation (26) reveals that the ordinary transversal waves become unstable. Let us note that in the plasma without the dust grains the low-frequency transverse oscillations decay aperiodically due to their collisionless absorption by the electrons [9]. (b) For the longitudinal waves ($\omega^2 \ll k^2c^2$), the modified dispersion relation for the dust-acoustic wave [3,4], which is deduced from (17), reads

$$1 + \frac{1}{k^2 r_D^2} - \frac{\omega_{pd}^2}{\omega^2} - \frac{k_\perp^2}{k^2}\frac{\Omega_r^2}{(\omega - \Omega_0)^2} = 0, \tag{27}$$

where

$$1/r_D^2 = \sum_{\alpha=e,i} \omega_{p\alpha}^2/V_{T\alpha}^2. \tag{28}$$

Equation (27) is formally similar to that dispersion relation which has been discussed in Ref. [10] in connection with the two-stream instability. From (27) it follows that the dust grain rotation gives the contribution only for waves with $k_\perp \neq 0$. For $\Omega_0 = 0$ that contribution is expressed in the change of the dust-acoustic frequency

$$\omega = \left[\omega_{pd}^2 + \frac{k_\perp^2}{k^2} \Omega_r^2 \right]^{1/2} \left[1 + \frac{1}{k^2 r_D^2} \right]^{-1/2}. \quad (29)$$

However, in the presence of the dust grain rotation, (27) admits complex solutions for any rotation frequency Ω_0, satisfying the condition

$$\Omega_0 < \omega_{pd} \left[1 + \frac{1}{k^2 r_D^2} \right]^{-1/2} \left[1 + \left(\frac{k_\perp^2}{k^2} \frac{\Omega_r^2}{\omega_{pd}^2} \right)^{1/3} \right]^{3/2}. \quad (30)$$

The equality of Ω_0 on the right-hand side of (30) defines the boundary of the stability of the dust-acoustic wave. Letting $\omega \simeq \Omega_0 + i\gamma$, where $\gamma \ll \Omega_0$, we obtain from (27) the growth rate for $\omega_{pd}^2/\Omega_0^2 \approx 1 + 1/k^2 r_D^2$,

$$\gamma = 3^{1/2} 2^{-4/3} \left[\frac{k_\perp^2}{k^2} \frac{\Omega_r^2}{\omega_{pd}^2} \right]^{1/3} \Omega_0 . \quad (31)$$

In summary, we have presented the dispersion relation for a dusty plasma containing elongated and rotating dust grains. The energy of the dust rotation can flow into plasma oscillations, driving them at nonthermal levels. The instability of longitudinal waves occurs only in the case when the wave vector lies in the plane of the dust grain rotation. In this case, there exists a coupling between the longitudinal electric field and charges that are placed on the dust grain surfaces and that rotate together with the dust grains. It is well known that the cross section of the scattering of transverse electromagnetic waves in a plasma has sharp maxima near the natural plasma low frequencies [9,11]. Since the thermal motion of the dust particles [see (23)] is typically neglected, the scattering would occur on the electrons and the ions only, and the form of the scattering line will be determined by their contribution to the spectral distribution of fluctuations. Therefore, for the dependence of the cross section on the frequency, we have

$$d\sigma \approx \delta[\mathrm{Re}\,\epsilon^l(\omega, \mathbf{k})] \frac{d\omega}{\omega} . \quad (32)$$

For the case considered above [see, Eq. (31)] the cross section (32) has the sharp maximum at $\omega \simeq \Omega_0$. When the dust grain rotation frequency Ω_0 approaches a critical value, defined by the right-hand side of (31), the fluctuations of longitudinal waves sharply increase and the scattering cross section must also sharply increase. Thus, the existence of a preferred frequency of the dust grain rotation can be found by means of the scattering of transverse electromagnetic waves off enhanced dust-acoustic fluctuations in a dusty plasma.

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