Bartosch and Kopietz Reply: The main result of our Letter [1] is that the average density of states (DOS) $\langle\rho(\omega)\rangle$ of the fluctuating gap model (FGM) exhibits a singularity at frequency $\omega = 0$ for any finite value of the correlation length ξ [2] if the fluctuating order parameter field $\Delta(x)$ is real and its average $\langle\Delta(x)\rangle$ is sufficiently small. We detected this singularity by exactly calculating the *local* DOS

$$\rho(x,\omega) = -\frac{1}{\pi}\,\mathrm{Im}G_R(x,x,\omega) = \langle x|\delta(\omega - \hat{H})|x\rangle \quad (1)$$

at $\omega = 0$ with special boundary conditions *such that the spectrum is continuous.* Here G_R is the retarded Green function and $\hat{H}$ is the Hamiltonian of the FGM. It is very important that in Ref. [1] we have considered an *infinite system* where the disorder potential vanishes outside a finite interval $[0,L]$. That the considered boundary conditions imply a continuous spectrum can be seen most easily with the help of the formalism developed in Ref. [3]. Since in Ref. [1] we have calculated the right-hand side of Eq. (1) for $\omega = 0$, we have *by definition* [4] calculated the exact local DOS at $\omega = 0$. In their Comment [5] Millis and Monien (MM) claim that the evaluation of $\rho(x,\omega)$ "*at one frequency provides no information about the DOS.*" Below we shall refer to this claim as C.

MM justify C by showing that our result for $\rho(x,0)$ can be written as $\rho(x,0) = |\psi_0(x)|^2$, where $\psi_0(x)$ is the zero energy solution of the two-dimensional random Dirac equation that determines the eigenfunctions of the FGM. MM compare this result with the usual expression for the local DOS in a finite system with a discrete spectrum E_n and normalizable wave functions $\phi_n(x)$,

$$\rho(x,\omega) = \rho_d(x,\omega) \equiv \sum_n |\phi_n(x)|^2\delta(\omega - E_n)\,. \quad (2)$$

However, Eq. (2) is not valid if the spectrum is continuous. In this case the general definition (1) can be written as

$$\rho(x,\omega) = |\psi_\omega(x)|^2, \quad (3)$$

where $\psi_\omega(x) = (\langle x\mid\omega+\rangle, \langle x\mid\omega-\rangle)^T$. Here $|\omega\pm\rangle$ are the usual scattering states corresponding to incoming waves from the left and the right of the interval $[0,L]$. Note that the scattering states are eigenstates of the Hamiltonian and can be chosen to satisfy $\langle\omega\alpha\mid\omega'\alpha'\rangle = \delta_{\alpha\alpha'}\delta(\omega - \omega')$. Equation (3) means that *for a continuous spectrum the local DOS can be identified with the absolute square of the wave function.* It seems to us that this has not been realized by MM, who do not carefully distinguish between a continuous and a discrete spectrum [6]: they work with a finite system, but their derivation of the relation $\rho(x,0) = |\psi_0(x)|^2$ implicitly assumes that the spectrum is continuous. The observation by MM that we have analyzed the wave-function statistics is correct—but we have chosen the boundary conditions such that *this is exactly the same as the statistics of the local DOS.* This obviously invalidates the claim C by MM.

We find it also not surprising that for our choice of boundary conditions $\ln\langle\rho(x,\omega)\rangle \propto x(L - x)$: The local DOS is finite at the boundaries of the interval $[0,L]$ and increases or decreases exponentially as we move into the disordered region. For $L \to \infty$, the local DOS either approaches ∞ or 0 in the bulk, indicating a singularity or a pseudogap in the DOS.

Finally, we would like to take this opportunity to emphasize two subtleties: (i) In Ref. [1] we have implicitly assumed that for x sufficiently far away from the boundaries the quantity $\lim_{L\to\infty}\langle\rho(x,0)\rangle$ calculated by us can be identified with $\lim_{L\to\infty}\langle\rho_d(x,0)\rangle$. This assumption is based on the expectation that in the thermodynamic limit bulk properties should be independent of boundary conditions. (ii) As we have already pointed out in our Letter [1], the limits $\omega \to 0$ and $L \to \infty$ do not commute, so that with the method used in Ref. [1] we cannot calculate the ξ-dependent critical value of $\langle\Delta(x)\rangle$ above which we expect for small but positive ω a pseudogap instead of a singularity in the DOS. Controlled numerical calculations of $\langle\rho_d(\omega)\rangle$ for arbitrary ω are given in Ref. [3], where the central result of Ref. [1] is confirmed: there exists a Dyson singularity in the DOS of the FGM for any finite correlation length ξ if $\Delta(x)$ is real and $\langle\Delta(x)\rangle = 0$. Such a singularity was previously known only to exist in the white-noise limit, and it was believed that for large ξ the pseudogap in the DOS would persist even at $\omega = 0$. It turns out that for $\langle\Delta(x)\rangle = 0$ this is true only for complex $\Delta(x)$, corresponding to incommensurate fluctuations [3].

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[1] L. Bartosch and P. Kopietz, Phys. Rev. Lett. **82**, 988 (1999).
[2] We never proposed a critical ξ as was claimed in Ref. [5].
[3] L. Bartosch and P. Kopietz, Phys. Rev. B **60**, 15 488 (1999).
[4] See, for example, C. Itzykson and J.-M. Drouffe, *Statistical Field Theory* (Cambridge University Press, New York, 1989), Vol. 2, p. 648.
[5] A. J. Millis and H. Monien, preceding Comment, Phys. Rev. Lett. **84**, 2546 (2000).
[6] Because MM consider a finite system of length $R + L$, their $\psi_0(x)$ should satisfy $\int_0^{R+L} dx|\psi_0(x)|^2 = 1$. It is easy to show that this is not the case.

