Comment on "Singularities and Pseudogaps in the Density of States of Peierls Chains"

In a recent Letter [1] Bartosch and Kopietz (BK) presented what were claimed to be exact results for the zero-energy density of states of a particular model of electrons moving in a correlated random potential. In this Comment we show that BK have not calculated the density of states at all; rather they have determined some properties of the zero-energy wave function of the model they consider. The object of interest to BK is $\varrho(x,\omega) = \frac{1}{\pi}\,\mathrm{Im}G(x,x;\omega)$ where G is the Green function of a one-electron problem in one spatial dimension. Their exact result concerns this quantity evaluated at the particular frequency ($\omega = 0$). One may see that $\rho(x,\omega = 0)$ is related to a wave function by writing the general expression for ϱ in terms of the exact eigenstates $\varphi_n(x)$ and energies E_n of the problem:

$$\rho(x,\omega) = \sum_n |\phi_n(x)|^2 \delta(\omega - E_n)\,. \tag{1}$$

In a finite system of length l and definite boundary conditions (e.g., periodic) there is a discrete set of energy levels and from Eq. (1) we see $\varrho(x,\omega)$ is a series of δ functions in ω. At each allowed energy it has spatial dependence determined by the eigenfunctions at that energy. BK consider an infinite translation-invariant system, in which a disordered region of size L is embedded. In this case, eigenvalues exist at all ω but for fixed ω $\varrho(x,\omega)$ still gives the square of the amplitude of the wave function of energy ω. Evaluation of $\rho(x,\omega)$ at one frequency (as done by BK) provides no information about the density of states; this information requires discussion of the variation of ρ over a range of ω.

We now solve the specific problem considered by BK. This is specified by the equation

$$[\partial_x - \Delta(x)\sigma_2]\psi(x) = i\omega\sigma_3\psi(x) \tag{2}$$

with $\psi(x)$ a two-component spinor representing right and left moving components of the electron wave function, $\sigma_{2,3}$ the usual Pauli matrices, and $\Delta(x)$ a real random function different from 0 on the interval $0 \le x \le L$. BK considered the case in which the region $0 \le x \le L$ is embedded in an infinite translation-invariant medium. For convenience we assume instead that it is embedded in a ring of radius $R + L$ with $R \gg L$. The boundary condition is $\psi(0) = \psi(L + R)$. For ω very near 0 and $0 \le x \le L$ we may neglect the ω on the right-hand side of Eq. (2); the resulting equation may be integrated, yielding $[A(x) = \int_0^x dy\Delta(y)]$

$$\psi(x) = e^{A(x)\sigma_2}\psi(0)\,. \tag{3}$$

On the interval $L \le x \le R + L$, $\Delta(x) = 0$ and the equation may again be solved. We define $C = A(L)$ and write

$$\psi(R + L) = e^{i\omega\sigma_3 R}e^{C\sigma_2}\psi(0)\,. \tag{4}$$

The periodic boundary condition may now be implemented; it implies the matrix $e^{i\omega\sigma_3 R}e^{C\sigma_2}$ has unit eigenvalue; the wave function is then the corresponding eigenvector. Allowed solutions correspond to $\cos\omega R = \frac{1}{\cosh C}$; for $R \gg L$, there exist allowed $\omega \sim \pi/2R$ small enough that the ω term can be neglected in the range $0 < x < L$ justifying the approximation used to construct Eq. (3). If ω is chosen to satisfy the eigenvalue condition then

$$\psi(x) = \frac{1}{\sqrt{2}}\, e^{A(x)\sigma_2}\begin{bmatrix} 1 \\ \frac{1 \mp i\,\sinh C}{i\,\cosh C} \end{bmatrix}. \tag{5}$$

Squaring Eq. (5) and performing some algebra leads to

$$|\psi(x)|^2 = \frac{\cosh[A(x) - B(x)]}{\cosh C} \tag{6}$$

with $B(x) = C - A(x)$. This is precisely Eq. (17) of BK, which they asserted represented the local density of states. Their subsequent results, which involve averages of this quantity over distributions of $\Delta(x)$, are therefore seen to pertain to wave function statistics rather than to the density of states. For example, the disorder average of $|\psi(x)|^2$ yields $\ln\rho_{\mathrm{BK}}(x,\omega = 0) \sim x(L - x)$; the exponential variation of ρ_{BK} with x and its divergence with system size are not physically sensible for a density of states but are expected for wave functions in disordered systems.

The physically relevant density of states is obtained by averaging Eq. (1) over a range of energies large compared to L^{-1} but small compared to intrinsic scales. We have determined this behavior at all frequencies, using a mixture of analytical and numerical techniques, for $\Delta(x)$ a Gaussian random variable with spatial correlation length ξ. At sufficiently low energy one may use the techniques outlined above to map Eq. (2) onto the fluctuating (delta-correlated) gap model solved by Ovchinnikov and Erikhman [2]; the result, confirmed by numerics, is a density of states that diverges as $1/\omega$ with logarithmic corrections for all ξ. The critical value of ξ proposed in Eq. (23) of BK is found not to exist. A complete report of our results will be published elsewhere.

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