# Self-Amplification of Coherent Spontaneous Emission in a Cherenkov Free-Electron Maser

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Ultrashort pulses of microwave radiation have been produced in a dielectric-lined Cherenkov free-electron maser (FEM) amplifier. An intense initial seed pulse, due to coherent spontaneous emission (CSE), arises at the leading edge of the electron pulse. There is evidence to show that 3–4 cycle spikes are produced through the amplification of these seed pulses. A strong dependence of the start-up power on the rise time of the electron pulse has been found. The experimental results are verified by a theoretical analysis. Our study shows that amplification in a FEM amplifier is always initiated by CSE arising from the edge of the electron pulse when the rise time is comparable to the electromagnetic wave period.

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Ultrashort pulses of coherent electromagnetic radiation, a few cycles in duration, can be produced with high efficiency in the free-electron maser (FEM) through a nonlinear process, which is responsible for the saturation of the amplifier. Amplification in these devices relies upon the collective interaction of electrons through a common radiation field. The interaction between the electromagnetic field and electrons in the FEM is governed by a ponderomotive wave which gives rise to bunching of the electrons with a wavelength periodicity [1]. This initially results in exponential growth of the field such that $E(z) \propto \exp(\sqrt{3}\, z/l_g)$, where $l_g$ is the gain length. The nonlinear process, known as superradiance, is characterized by the development of a solitonlike self-similar pulse growing quadratically in intensity while shrinking in duration due to synchrotron oscillations under the influence of the ponderomotive potential [2–4]. Superradiance usually occurs in the latter stages of amplification in free-electron lasers (FELs) based on self-amplification of spontaneous emission (SASE) [5–7] and short pulse oscillators [2,8,9]. Single-pass SASE FEL amplifiers avoid the need for cavity mirrors and thus are good candidates for x-ray FELs. In the SASE FEL amplifier operating at short wavelengths, the initial seed that is amplified occurs from incoherent spontaneous emission arising from the shot noise of the electron beam. However, superradiance in long wavelength FEMs can be initiated by both incoherent spontaneous emission, due to shot noise, and coherent spontaneous emission (CSE) arising from the longitudinal spectral components of the electron pulse shape [10–13]. Recent studies [14,15] have shown that when amplification evolves from CSE there can be a very large enhancement of the start-up power as compared with that evolving from shot noise. The logarithmic dependence of the saturation length on start-up intensity gives scope for reducing the length of amplifiers. This may have important implications for FELs operating in the SASE regime and the more general problem of developing sources in the vacuum ultraviolet and x-ray regions, where undulator lengths are long and expensive. CSE is also important in FEM amplifiers and oscillators that are driven by prebunched beams or electron pulses with lengths comparable to the wavelength of the amplified pulses [16,17].

The dielectric Cherenkov FEM is a source of coherent, tunable, high-power microwave radiation [18,19]. Attractive features are the relatively simple geometry and insensitivity to beam thermal spread. In the experiments reported in this Letter, a single-pass amplifier has been used to study self-amplification of CSE (SACSE) originating from the leading edge of the electron pulse. This study contrasts with an earlier preliminary study of Cherenkov superradiance [20] that did not discuss the start-up mechanism of the emission process. To study the start-up mechanism we have used a Cherenkov FEM in which the slowly guided electromagnetic wave in a dielectric lined waveguide couples with the slow space-charge mode of a pulsed annular relativistic electron beam. Synchronism between wave and electrons occurs when the angular frequency $\omega = k_z v_0 - \omega_p/\gamma_0$, where $k_z$ is the axial wave number of the radiation, $\omega_p$ is the relativistic plasma frequency, $v_0$ is the axial velocity of the electrons, and $\gamma_0$ is the relativistic factor. The dispersion relation for the $TM_{01}$ mode is

$$\frac{YI_0(Yf)}{I_1(Yf)} = \frac{X}{\varepsilon}\left[\frac{N_0(Xf)J_0(X) - N_0(X)J_0(Xf)}{N_1(Xf)J_0(X) - N_0(X)J_1(Xf)}\right], \quad (1)$$

where $\varepsilon$ is the dielectric constant of the dielectric liner, $X = r_0(\varepsilon\omega^2/c^2 - k_z^2)^{1/2}$, $Y = r_0(k_z^2 - \omega^2/c^2)^{1/2}$, $c$ is the speed of light, $r_0$ is the outer radius of the dielectric, $f = r_i/r_0$ is the filling factor, $r_i$ is the inner radius of the dielectric, and $I_n$, $J_n$, and $N_n$ are modified, first, and second kind Bessel functions of order $n$. The gain parameter $\rho$ is defined as

$$\rho = \frac{1}{\gamma_0\omega}\left(\frac{2\pi^2 e I_c^2 I}{\varepsilon_0 m_e k_z \alpha^2 A^2 Q}\right)^{1/3}, \quad (2)$$

  

where $I$ is the peak current, $I_c$ and $Q$ are mode-dependent constants, $A$ is the cross-sectional area of the annular beam, and $\alpha = (k_z^2 - \omega^2/c^2)^{1/2}$ is the transverse wave number in vacuum [15]. The cooperation length $l_c$ is then given by

$$l_c = \frac{\lambda}{2\pi\rho} \frac{v_0 - v_g}{v_g}, \quad (3)$$

where $\lambda$ is the wavelength of the radiation and $v_g$ is the group velocity, and the gain length $l_g$ is given by

$$l_g = \frac{\lambda}{2\pi\rho}. \quad (4)$$

In our experiments, the measured microwave frequency of 30 GHz, which was 7% greater than the theoretical value, peak current of 0.74 kA, and estimated electron energy of 280 keV, gives a $\rho = 0.102$. This indicates that the interaction is in the low efficiency limit, $\rho \ll 1$, where changes in the electron energies are small during the interaction. The cooperation length and gain length are estimated to be $l_c = 1.4$ cm and $l_g = 1.6$ cm, respectively. The incoherent start-up power can be estimated from the expected steady-state saturated power, $P_{\text{sat}}$, as

$$P_{\text{inc}} \sim P_{\text{sat}}/N_c, \quad (5)$$

which depends on the number of electrons in a cooperation length, $N_c$, given by $N_c = Q_c/e$, where $Q_c$ is the pulse charge in a cooperation length [2]. The saturation power can be calculated from $\rho$ and geometrical coupling factors [15]. The total start-up radiation power, including both incoherent and coherent contributions, is given by

$$P(\omega) = P_0(\omega)N_c + P_0(\omega)N_c^2|f(\omega)|^2, \quad (6)$$

where $P_0 = P_{\text{inc}}/N_c$ is the power radiated by a single electron and $f(\omega)$ is a form factor given by the Fourier transform of the longitudinal electron density [16]. The first term in Eq. (6) is the contribution from shot noise while the second term is that from CSE.

A table-top RADAN 303B accelerator and subnanosecond slicer have been used to provide a subnanosecond accelerating potential to produce 500–800 ps, 0.6–1.0 kA, 250–300 keV electron pulses from an explosive emission cathode in a magnetically insulated coaxial diode [21]. The fast rising electron beam current pulses and accelerating voltage pulses were measured using a Faraday cage strip-line current probe and an inline capacitive voltage probe, respectively. The microwave radiation pulses were measured using a hot-carrier germanium crystal detector. The current probe, voltage probe, and microwave detector had response times of 400, 150, and 110 ps, respectively, and the signals were recorded using a 7 GHz (50 ps rise time) Tektronix 7250 digitizing oscilloscope.

In the experiments the high current electron pulses are transported through a 9.8-mm-diam, 30-cm-long, cylindrical stainless steel waveguide sheathed on the inside with a dielectric lining. The waveguide is immersed in a 5 T solenoidal guiding magnetic field. The dielectric liner consists of a quartz tube ($\varepsilon = 3.8$) with a 5.7 and 9.7 mm inner and outer diameters, respectively. The tube was lined on the outside by a 0.05-mm-thick Mylar film ($\varepsilon = 3.0$). The 3.6-mm-diam cathode produces an annular electron pulse with the outer diameter measured to be 4.0 mm at the entrance of the interaction region. The close proximity of the beam to the dielectric surface ensures good coupling between the wave and electron beam. A schematic of the experimental setup is shown in Fig. 1. An example of a typical electron current pulse wave form is shown in Fig. 2(a). However, the measured current pulse is convoluted by the relatively slow Faraday probe response function.

Total measured peak output power for a 20-cm dielectric liner was found to be 2.8 MW. This provides a lower estimate of the maser power, as not all of the radiation is coupled out of the interaction region because of the abrupt end of the dielectric medium. The measured far-field mode pattern indicates the presence of predominantly a $TM_{01}$ mode and a small fraction of the $HE_{11}$ mode. An example of a typical output pulse is shown in Fig. 2(b).

The emitted microwave pulses were not perfectly resolved by the detection system due to a limited bandwidth. However, there is reasonable evidence to show that a series of (~100 ps) spikes, roughly $l_c$ in length, are produced by the amplification of CSE. The short spikes become apparent when the pulse is deconvoluted by the instrument response function, as shown in Fig. 2(b). Instantaneous spectral measurements giving a relative 3-dB bandwidth of 12% provide further evidence of 3–4 cycle duration spikes. The spikes occur as individual amplified CSE pulses arising from the substructure in the density and energy/phase distribution of the electron pulse. Frequency tuning in the range 28–32 GHz was achieved by varying the anode-cathode separation distance which in turn varies the electron energy. However, the current varies inversely with gap distance because of the diode characteristics.

Measurements of the output power as a function of the accelerating voltage pulse shape revealed a strong dependence of the power on the voltage pulse rise time. A linear dependence of the current on the voltage allowed us to estimate the current pulse shape from the measured voltage pulse profile. Figure 3 shows the total peak power

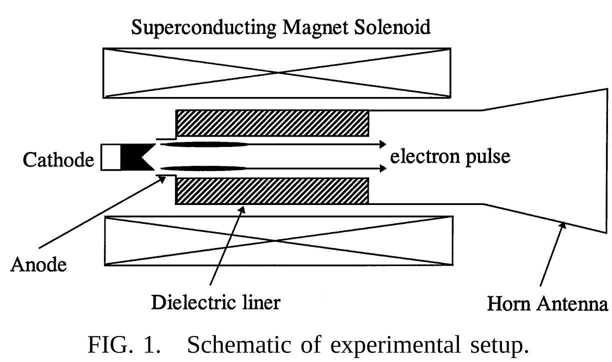


FIG. 1. Schematic of experimental setup.

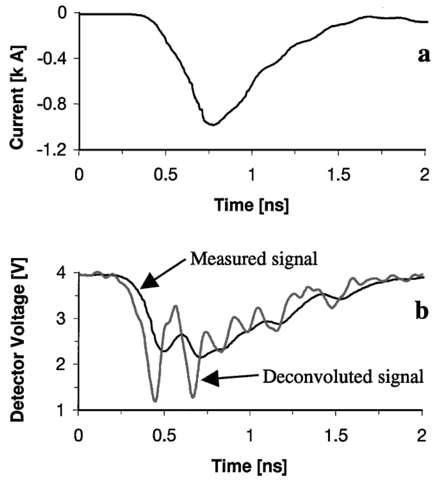


FIG. 2. Typical (a) electron current pulse and (b) microwave output pulse.

measured as a function of the electron current pulse rise time for a 20-cm-long dielectric liner. The microwave signal was found to decrease sharply as the rise time increased, i.e., as the slope of the leading edge of the profile decreased. This suggests that the main source of the amplified signal is coherent spontaneous emission from electrons at the leading edge of the current pulse. The electron pulse sweeps over the CSE radiation pulse, because of the Cherenkov phase matching condition, leading to exponential amplification. A slower rise time results in a longer electron pulse duration and therefore, in the absence of CSE, we would expect the peak power to increase because of the increase of the total charge. However, our measurements, presented in Fig. 3, show that this is not the case, clearly indicating that CSE is acting as a strong initial seed in the amplification process.

A numerical simulation of the Cherenkov FEM experiment was carried out using a model similar to that of [15] but extended to include electron velocity spread and longitudinal space charge. These simulations agree with the observed experimental behavior of the peak intensity as a function of current rise time, as shown in Fig. 4.

An exponential growth of the radiation power as a function of interaction length was measured for lengths up to 16 cm. This confirms that there are at least 2 orders of magnitude of exponential growth still in the linear regime

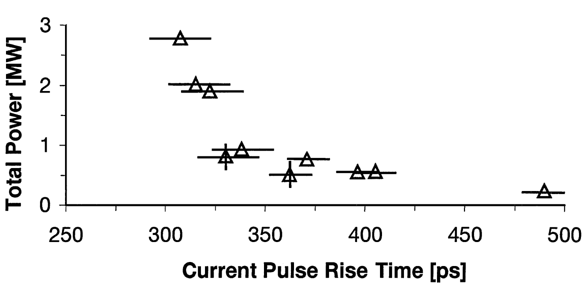


FIG. 3. Power dependence on current pulse rise time.

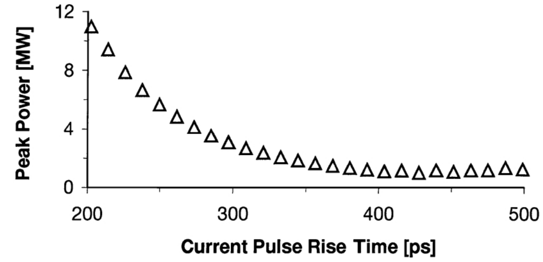


FIG. 4. Simulations of the peak power as a function of current pulse rise time for a parabolic-shaped electron pulse.

[2]. The circles in Fig. 5 correspond to measurements made using a 26-dB amplifier with a response time of 88 ps to amplify the signal. An offset due to a noisy background has been subtracted for these points. To estimate the start-up power, the exponential growth has been extrapolated back to the beginning of the interaction region. This gives an estimated start-up power of the order of 1 W. The gain, estimated from the slope of Fig. 5, is $\sim 3.7\ \mathrm{dB\,cm^{-1}}$, which gives a gain length of $\sim 1.76$ cm. Because of the sensitivity of the microwave output power to the shape of the accelerating voltage pulse, only microwave pulse data with the same voltage pulse shape have been selected. This was made possible by simultaneously capturing the voltage and microwave pulses on the high bandwidth oscilloscope. This procedure improved the signal to noise ratio and the reproducibility of the measurements enormously.

The measured start-up power should be compared with that expected from incoherent spontaneous power, $P_{\mathrm{inc}}$, which is estimated to be $\sim 300\ \mu\mathrm{W}$. This is a few orders of magnitude less than the extrapolated start-up power from Fig. 5 and indicates that coherent spontaneous emission from the leading edge of the electron pulse is amplified as it propagates through the electron pulse and drives the interaction.

A numerical simulation of the exponential growth is shown in Fig. 6. This agrees with the measurements confirming self-amplification of the coherent spontaneous emission [12,15]. Both the measurements and the simulations show that the amplification is driven quite strongly by CSE but does not reach saturation. Discrepancies between experiment and simulations may be attributed to

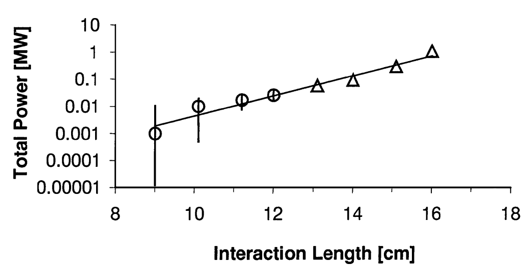


FIG. 5. Power dependence on interaction length. The circles represent measurements with an amplifier and triangles represent measurements without.

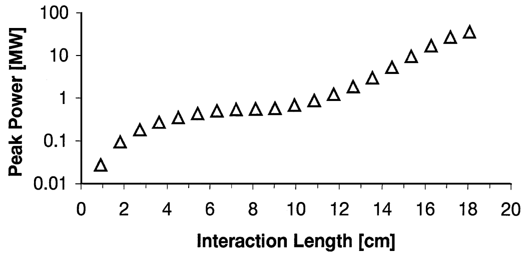


FIG. 6. Simulations of the peak power as a function of interaction length for a parabolic-shaped electron pulse.

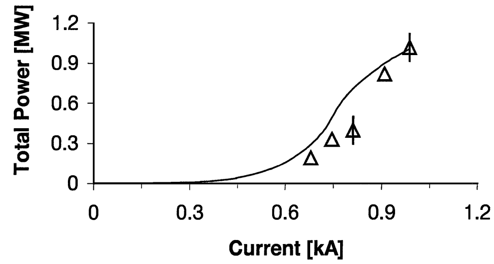


FIG. 7. Power dependence on electron pulse current. The solid curve is the theoretical power in arbitrary units.

the neglect of effects such as emittance, 3D effects, and an idealized electron pulse shape [12,15]. Longer interaction lengths have been used to reach the nonlinear regime, where the scaling laws characterizing superradiance have been observed and will be presented in a future publication.

The growth of the radiation pulse power, $P$, in the linear regime is described by the dependence

$$P(I) \propto \omega(I)k(I)I^2 \exp[2\sqrt{3}\,\pi\rho(I)z/\lambda], \qquad (7)$$

where $\omega$ and $k$ depend on the current. Figure 7 shows the exponential growth with current as determined experimentally for a dielectric liner of length 16 cm. The curved line depicts the theoretical dependence as described by Eq. (7). A similar $P(I)$ dependence on current is also found when neglecting the frequency and wave-number dependencies on current.

Intense spikes of $Ka$-band microwave radiation with megawatt power levels, appearing to be as short as $\sim$100 ps in duration, or roughly two cooperation lengths wide, have been generated in a dielectric Cherenkov FEM amplifier. Coherent spontaneous emission arising in the leading edge of the electron pulse significantly increases the start-up power. The nonlinear dependence of the power on current for a fixed length amplifying medium has been shown to result from SACSE. This dependence on current should not be confused with the quadratic scaling characterizing the self-similar superradiant pulses. There is great potential in the use of CSE to drive many varied electron-wave interactions from the microwave to ultraviolet regions of the spectrum. SACSE, arising from a prebunched beam with Fourier components close to the resonance frequency, may also provide a means of reducing the length and improving the stability of free-electron laser sources operating in the x-ray region.

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