Universality and Saturation of Intermittency in Passive Scalar Turbulence

A. Celani,[1,2] A. Lanotte,[1] A. Mazzino,[3,4] and M. Vergassola[1]

[1] *CNRS, Observatoire de la Côte d'Azur, B.P. 4229, 06304 Nice Cedex 4, France*

[2] *Dipartimento di Fisica Generale, Università di Torino, and INFM Unità di Torino Università, I-10126 Torino, Italy*

[3] *INFM–Dipartimento di Fisica, Università di Genova, Via Dodecaneso, 33, I-16142 Genova, Italy*

[4] *The Niels Bohr Institute, Blegsdamvej, 17, Copenhagen, Denmark*

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The statistical properties of a scalar field advected by the nonintermittent Navier-Stokes flow arising from a two-dimensional inverse energy cascade are investigated. The universality properties of the scalar field are probed by comparing the results obtained with two types of injection mechanisms. Scaling properties are shown to be universal, even though anisotropies injected at large scales persist down to the smallest scales and local isotropy is not fully restored. Scalar statistics is strongly intermittent and scaling exponents saturate to a constant for sufficiently high orders. This is observed also for the advection by a velocity field rapidly changing in time, pointing to the genericity of the phenomenon.

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Ramp-and-cliff structures are a characteristic feature of fields, like dye concentration or temperature, obeying the passive scalar equation (see, e.g., Refs. [1,2])

$$\partial_t T(\boldsymbol{r},t) + \boldsymbol{v}(\boldsymbol{r},t)\cdot\nabla T(\boldsymbol{r},t) = \kappa\Delta T(\boldsymbol{r},t), \qquad (1)$$

i.e., advected by the velocity $\boldsymbol{v}$ and smeared out by the molecular diffusivity κ. Scalar gradients tend indeed to concentrate in sharp fronts separated by large regions of weak gradients (see Fig. 1). The experimental evidence for ramps and cliffs is long standing and massive [3–6]. Furthermore, numerical simulations indicate that scalar structures are not mere footprints of those in $\boldsymbol{v}$ and appear also for synthetic flow [7,8]. The presence of ramp-and-cliff structures raises some important issues about scalar turbulence and its intermittency properties. Following Kolmogorov's 1941 theory, it is indeed usually assumed that turbulence restores universality, i.e., independence of the large-scale injection mechanisms, and isotropy at small scales (see Ref. [2]). The evidence for scalar turbulence is, however, that anisotropies find their way down to the small scales, manifesting in the scalar gradient skewness of $O(1)$, independently of the Péclet number [3–8]. This is due to the preferential alignment of ramp-and-cliff structures with large-scale scalar gradients, present in most experimental situations. For structure functions $S_n(\boldsymbol{r}) = \langle (T(\boldsymbol{r}) - T(\boldsymbol{0}))^n\rangle$, this persistence is revealed by normalized odd orders $S_{2n+1}/S_2^{n+1/2}$ decaying more slowly than the expected $r^{2/3}$ (see Ref. [1]). Is this experimentally observed behavior signaling that small scales are fully imprinted by the large scales and that the universality framework should be discarded altogether? This is the first issue, raised in Refs. [1,5], that we shall investigate in this Letter. The second is about the consequences of cliffs for high-order intermittency. Their strength candidates them for the dominant contributions to strong event statistics, and the issue raised in Ref. [9] is whether structure function scaling exponents are then saturating to a constant for high orders n.

Numerical simulations are an ideal tool to analyze the previous questions, allowing one to probe universality, by comparing the results obtained with two different types of injection, and saturation, by gathering enough statistics to capture strong events. Here, we shall take for $\boldsymbol{v}$ a 2D flow generated by a Navier-Stokes inverse energy cascade [10]. Universality is then understood as dependence of scalar properties on the injection mechanisms for this fixed $\boldsymbol{v}$ statistics. The scalar is injected at large scales, comparable to those where the inverse cascade is stopped by friction effects, and its properties are investigated in the energy inertial range (see Ref. [11] for details). There, the velocity is isotropic, scale invariant with exponent $1/3$ (no intermittency corrections to Kolmogorov scaling [11,12]), and has dynamical correlation times (finite and free of synthetic flow pathologies discussed in Ref. [7]).

As for scalar injection, a first choice is naturally suggested by experiments, where it usually takes place via a large-scale gradient. We assume then, as in Refs. [7,8], that the average $\langle T\rangle = \boldsymbol{g}\cdot\boldsymbol{r}$ and we integrate the equation for the fluctuations $\theta = T - \boldsymbol{g}\cdot\boldsymbol{r}$, i.e., (1) with a source term $-\boldsymbol{v}\cdot\boldsymbol{g}$ on the right-hand side. A snapshot of the θ field is shown in Fig. 1. The presence of the gradient $\boldsymbol{g}$ breaks isotropy and allows for asymmetries and nonvanishing odd-order moments in the scalar statistics. The second choice is a more artificial random forcing $f(\boldsymbol{r},t)$ added to (1). Its motivation is to produce an isotropic statistics, e.g., by taking f Gaussian, with zero average and correlation function $\langle f(\boldsymbol{r},t)f(\boldsymbol{0},0)\rangle = \delta(t)\chi(r/L)$. The scale L where the injection is concentrated is taken comparable to the velocity integral scale. The equations for the scalar are integrated in parallel to the 2D Navier-Stokes equation for about 100 eddy turnover times by a standard pseudospectral code on a 2048^2 grid. In the runs presented in the following, the diffusive term is replaced by a bi-Laplacian, but it was checked by another series of simulations that using a Laplacian gives consistent results, although on less extended scaling ranges.

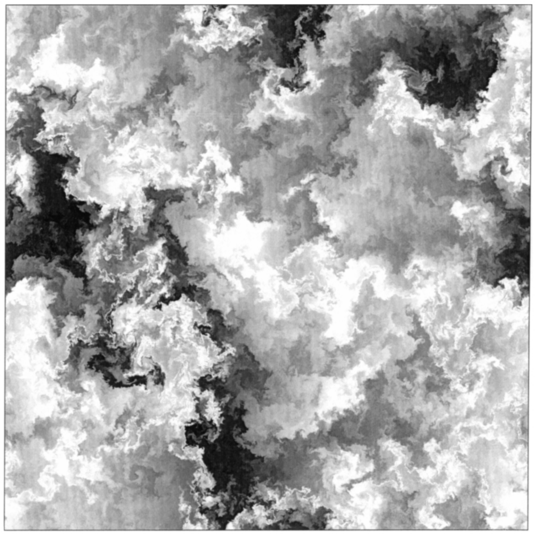

FIG. 1. A snapshot of the scalar fluctuation field for the injection by a mean gradient. Grey scale is coded according to the deviation from the average value (from white to black).

Let us first show that the persistence of anisotropies observed in experiments occurs also in our case. Odd-order structure functions vanish in the randomly forced case. In the shear case they do not, except for separations $\boldsymbol{r} \perp \boldsymbol{g}$. For nonorthogonal $\boldsymbol{r}$'s, the scaling exponents do not depend on the direction $\boldsymbol{r}$ and in Fig. 2 we present the parallel structure functions, i.e., $\boldsymbol{r}$ aligned with $\boldsymbol{g}$. The resulting third-order skewness $S_3/S_2^{3/2}$ scales as $r^{0.25}$, the second-order exponent being $\simeq 2/3$ (see Fig. 4). As in the experiments, the skewness decay is slower than the expected $r^{2/3}$. Furthermore, here enough statistics is accumulated to give access also to the fifth order. The persistence effect is now dramatic as $S_5/S_2^{5/2} \sim r^{-0.2}$ increases at small scales. Intermittency generates, of course, an ambiguity in the normalization, e.g., $S_5/S_4^{5/4}$ is decaying, albeit very slowly. This reflects the fact that scalar increment probability distribution functions (PDF's) change shape with r, and one should then be specific about which part of it is sampled and the choice of the observable representative of the anisotropy degree. It is, however, unambiguously clear that local isotropy is not fully restored at small scales and the quality of the scaling laws found here indicates that this is a genuine effect, not related to finite Péclet numbers.

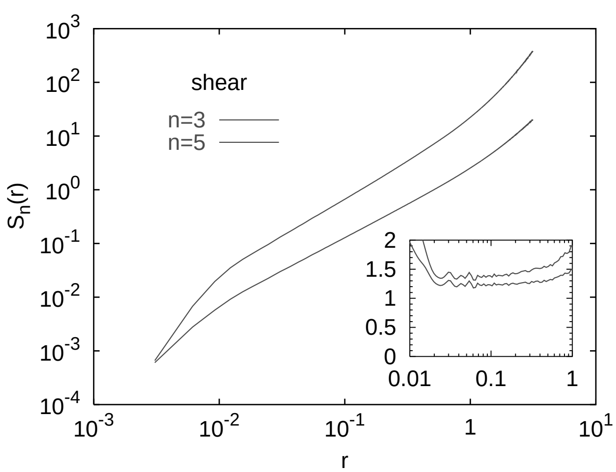

FIG. 2. The third and the fifth-order parallel structure functions for the injection by a mean gradient. In the inset, local scaling exponents $dS_n(r)/d\log r$. The measured exponents are $\zeta_3 = 1.25 \pm 0.04$, $\zeta_5 = 1.38 \pm 0.07$, with error bars estimated from rms fluctuations of local scaling exponents.

More insight into this breaking of full universality is gained by analyzing scalar increment PDF's and moments of even order, which are nonvanishing for both types of forcing. Figure 3 shows that the PDF's for the two types of injections do not have the same shape (the same holding when symmetric parts are taken). In the shear case, the separations $\boldsymbol{r}$ have been taken along the diagonal directions, at angles $\phi = \pi/4$ with respect to $\boldsymbol{g}$. This choice is motivated by the application of the procedure developed in Refs. [13,14] to the 2D case and permits removal of the first subleading anisotropic contribution $\propto \cos 2\phi$ to even-order moments. The fact that the PDF's have different shapes implies that the adimensionalized constants C_n in structure functions $S_n(r) = C_n(\epsilon r)^{n/3}(L/r)^{n/3-\zeta_n}$ are not universal, as it was also explicitly checked by direct comparison. Conversely, in Fig. 4 it is shown that scaling exponents of even-order moments are the same for the two types of forcing. For the PDF's this means that, although having different shapes, the curves are rescaling with r in the same way.

The picture emerging from these results is as follows: structure function exponents ζ_n are universal, while constants, and thus the PDF's of scalar increments, are not. The difference between isotropic and anisotropic situations is that the nonuniversal constants C_{2n+1} in odd-order structure functions vanish by symmetry for the former case, while they generically do not for the latter. Structure functions present anomalous scaling and there is no full

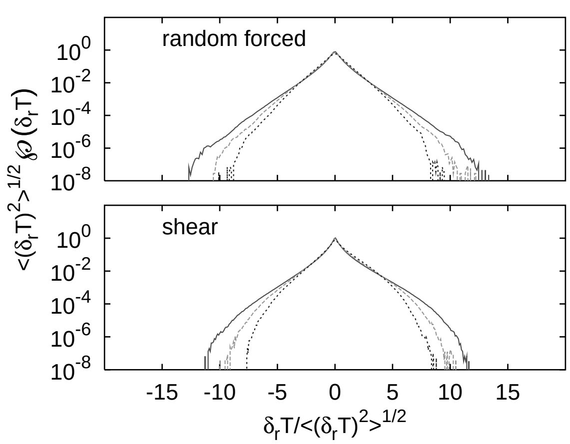

FIG. 3. PDF's of scalar increments normalized by their standard deviations for three separations $r = 2.5 \times 10^{-2}$, 5×10^{-2}, 10^{-1} in the inertial range.

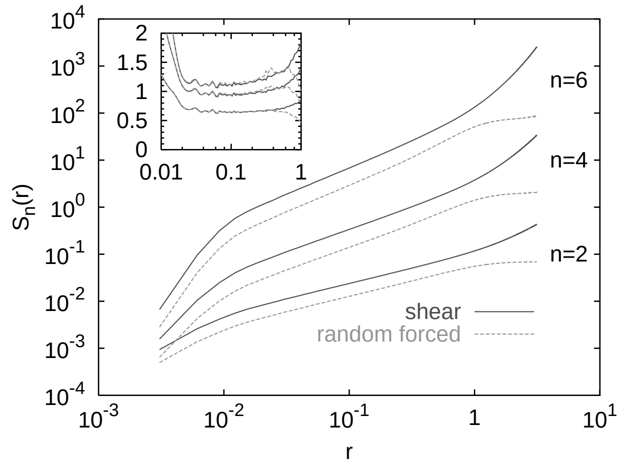

FIG. 4. Low-order even structure functions. Local scaling exponents are shown in the inset. The measured exponents are $\zeta_2 = 0.66 \pm 0.03$, $\zeta_4 = 0.95 \pm 0.04$, and $\zeta_6 = 1.11 \pm 0.04$.

restoration of isotropy while going toward small scales. This picture of universality is weaker than in Kolmogorov's 1941 theory, but coincides with the one emerged for intermittency in the Kraichnan passive scalar model [15–17] (see also Ref. [1]). The velocity used here has finite correlation times; scalar correlation functions do not obey closed equations, yet the universality properties are the same. This points to a broader validity of the mechanisms identified for the Kraichnan model, and it is likely that the same universality framework generically applies to passive scalar turbulence.

Let us now discuss the consequences of cliffs for the intermittency at high orders. Their singularity strength suggests that the scaling exponents might saturate at large orders, i.e., ζ_n tends to a constant ζ_∞ for large enough n. Physical self-consistency for the survival of steepening strong fronts is demonstrated in Ref. [18], where saturation is shown to imply that dissipation preferentially spares the cliffs with the largest jumps. High-order structure functions in our simulations are shown in Fig. 5, together with the ζ_n vs n curve, compatible with saturation. The same holds for ratios of two moments vs r or one moment vs the other. Note that, for any finite-size field, there are orders where the moments start to be spoiled and some strongest single structure having a dissipation width will plausibly dominate the statistics, as in Burgers' equation [19]. The convergence of the moments was inspected by the usual test of checking that $(\delta_r T)^{14}\mathcal{P}$ decays before the PDF $\mathcal{P}(\delta_r T)$ of the scalar increments $\delta_r T \equiv T(r) - T(0)$ becomes noisy. An alternative observable more reliable than moments (as less sensitive to the extreme tails) is looking, for fixed $\delta_r T$, at how $\mathcal{P}$ varies with the separation r. Saturation is equivalent to the PDF taking the form $\mathcal{P}(\delta_r T) = r^{\zeta_\infty}\mathcal{Q}(\delta_r T/T_{\rm rms})$ for $\delta_r T$ sufficiently larger than $T_{\rm rms} = \langle (T - \langle T\rangle)^2\rangle^{1/2}$. The collapse of the curves $r^{-\zeta_\infty}\mathcal{P}(\delta_r T)$ in Fig. 6 is therefore a signature of saturation and gives the unknown function $\mathcal{Q}$. In

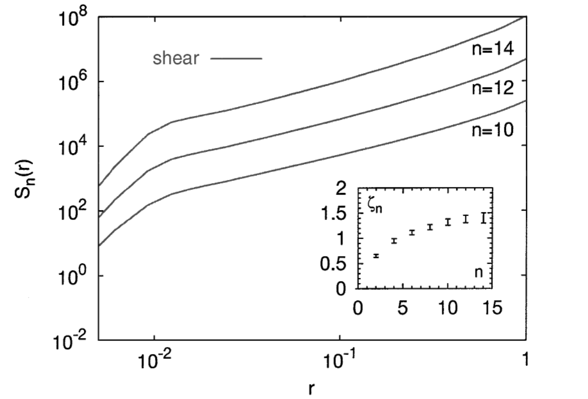

FIG. 5. Scalar structure functions of orders 10, 12, and 14. The scaling exponents are shown in the inset.

Fig. 7, we plot the cumulated probabilities $\int_{\delta_r T}^{\infty} \mathcal{P}$ vs r for various $\delta_r T$ and the parallelism of the curves is again the footprint of saturation. Explicit evidence for the universality of ζ_∞ is provided in Fig. 6.

The physical origin of cliffs resides in the Lagrangian structure of (1), i.e., in the fact that particles are passively transported by the velocity $\boldsymbol{v}$. In regions where velocity gradients are sufficiently persistent in space and time, widely spaced particles tend to approach and generate the observed abrupt variations of the scalar field. This suggests that even though quantitative aspects, such as the order of saturation or the value ζ_∞, depend on the choice of $\boldsymbol{v}$, the saturation phenomenon itself should occur for a wide class of random velocity fields. The Kraichnan model [9] is unfavorable for saturation because of the short velocity correlation time. Despite this, for large dimensionalities of space, saturation analytically follows from an instanton solution [20]. For the 3D case, saturation was phenomenologically suggested in Ref. [21] and inferred from an instantonic bound in Ref. [22]. Direct numerical evidence is provided by our 3D numerical simulations whose results

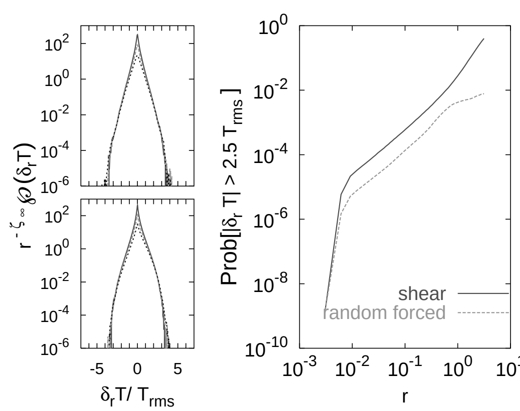

FIG. 6. On the left, the PDF's in Fig. 3 multiplied by $r^{-\zeta_\infty}$. The upper curves are for the random forcing and the lower ones for the shear case. On the right, cumulated probabilities for scalar fluctuations to exceed $\lambda T_{\rm rms}$ *vs* r, with $\lambda = 2.5$.

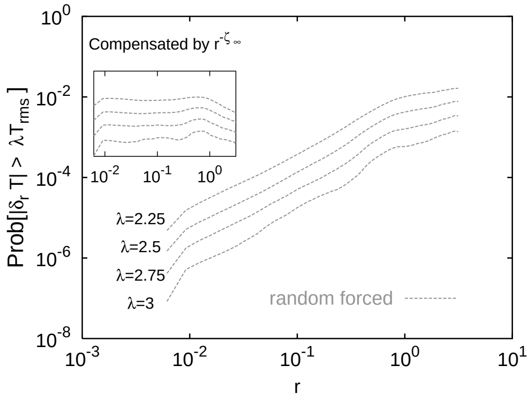

FIG. 7. As in Fig. 6, for different values of λ in the randomly forced case. The curves compensated by $r^{-\zeta_\infty}$, with $\zeta_\infty = 1.4$, are shown in the inset.

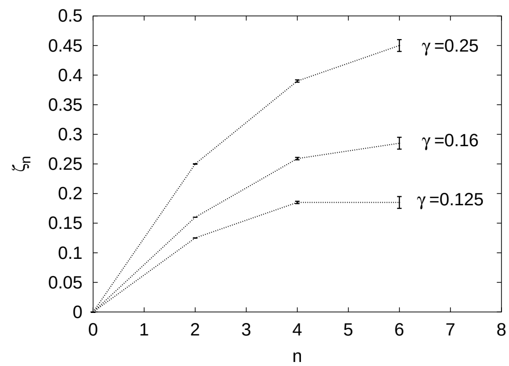

FIG. 8. Scaling exponents of the second-, fourth-, and sixth-order scalar structure functions for three different roughness exponents γ in the Kraichnan model.

are presented in Fig. 8. Scaling exponents have been measured using the Lagrangian method presented in Ref. [23] and $(2 - \gamma)/2$ is the spatial Hölder exponent of $\boldsymbol{v}$, as in Ref. [1]. The order of the moments needed to observe saturation is expected to diverge for $\gamma \to 2$, while for $\gamma \to 0$ the action of large-scale gradients should favor close approaches between particles. The order is thus expected to reduce with γ and for the smoothest velocity in Fig. 8 saturation is indeed occurring already at the fourth order and thus becomes observable. This confirms the physical picture of saturation due to the cliffs formed in the scalar field and the genericity of the phenomenon for scalar turbulence intermittency.

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