Multi-TeV Scalars are Natural in Minimal Supergravity

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For a top quark mass fixed to its measured value, we find natural regions of minimal supergravity parameter space where all squarks, sleptons, and heavy Higgs scalars have masses far above 1 TeV and are possibly beyond the reach of the Large Hadron Collider at CERN. This result is simply understood in terms of "focus point" renormalization group behavior and holds in any supergravity theory with a universal scalar mass that is large relative to other supersymmetry breaking parameters. We highlight the importance of the choice of fundamental parameters for this conclusion and for naturalness discussions in general.

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The standard model with a fundamental Higgs boson suffers from a large and unexplained hierarchy between the weak and Planck scales [1]. Because supersymmetric theories are free of quadratic divergences, however, this hierarchy is stabilized in supersymmetric extensions of the standard model when the scale of superpartner masses is roughly of the order of the weak scale $M_{\rm weak}$ [2]. The promise of providing a natural solution to the gauge hierarchy problem is the primary phenomenological motivation for supersymmetry.

Because the requirement of naturalness places upper bounds on superpartner masses, this criterion has important experimental implications. In a model-independent analysis, naturalness constraints are weak for some superpartners, e.g., the squarks and sleptons of the first two generations [3]. However, in widely studied scenarios where the scalar masses are unified at some high scale, such as minimal supergravity, it is commonly assumed that squark and slepton masses must all be $\lesssim$1 TeV. This bound places all scalar superpartners within the reach of present and near future colliders, and is a source of optimism in the search for supersymmetry at the high-energy and high precision frontiers. We show here, however, that this assumption is invalid, and, in fact, it is precisely in supergravity theories with a universal scalar mass that *all* squark and slepton masses may naturally be far above 1 TeV.

Supersymmetric theories are considered natural if the weak scale is not unusually sensitive to small variations in the fundamental parameters. Although the criterion of naturalness is inherently subjective, its importance for supersymmetry has motivated several groups to provide quantitative definitions of naturalness [4–12]. In this analysis, we adopt the following prescription.

(i) We consider the minimal supergravity framework with its 4 + 1 input parameters

$$\{P_{\rm input}\} = \{m_0, M_{1/2}, A_0, \tan\beta, {\rm sgn}(\mu)\}, \qquad (1)$$

where m_0, $M_{1/2}$, and A_0 are the universal scalar mass, gaugino mass, and trilinear coupling, respectively, $\tan\beta = \langle H_u^0\rangle/\langle H_d^0\rangle$ is the ratio of Higgs expectation values, and μ is the Higgsino mass parameter. The first three parameters are at the grand unified theory (GUT) scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV, i.e., the scale where the $U(1)_Y$ and SU(2) coupling constants meet.

(ii) The naturalness of each point $\mathcal{P} \in \{P_{\rm input}\}$ is then calculated by first determining all the parameters of the theory (Yukawa couplings, soft supersymmetry breaking masses, etc.), consistent with low-energy constraints. Renormalization group (RG) equations are used to relate high- and low-energy boundary conditions. In particular, at the weak scale, proper electroweak symmetry breaking requires

$$\frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \equiv f(m_{H_d}^2, m_{H_u}^2, \tan\beta) - \mu^2, \qquad (2)$$

$$2B\mu = \sin 2\beta (m_{H_d}^2 + m_{H_u}^2 + 2\mu^2), \qquad (3)$$

where $m_{H_u}^2$ and $m_{H_d}^2$ are the soft scalar Higgs masses, and $B\mu$ is the bilinear scalar Higgs coupling. (The tree-level conditions are displayed here for clarity of presentation. In all numerical results presented below, we use the full one-loop Higgs potential [13], minimized at the scale $m_0/2$, approximately where one-loop corrections are smallest, as well as two-loop RG equations [14], including all low-energy thresholds [13,15].)

(iii) We choose to consider the following set of (GUT scale) parameters to be free, independent, and fundamental:

$$\{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\}. \qquad (4)$$

(iv) All observables, including the Z boson mass, are then reinterpreted as functions of the fundamental parameters a_i, and the sensitivity of the weak scale to small fractional variations in these parameters is measured by the sensitivity coefficients [4,5]

$$c_i \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|. \qquad (5)$$

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(v) Finally, we form the fine-tuning parameter

$$c = \max\{c_i\}, \tag{6}$$

which is taken as a measure of the naturalness of point $\mathcal{P}$, with large c corresponding to large fine tuning.

As is clear from the description above, several subjective choices have been made, as they must be in any definition of naturalness. The choice of minimal supergravity in step (i), and particularly the assumption of a universal scalar mass, plays a crucial role. Deviations from this assumption will be considered below.

The choice of fundamental parameters in step (iii) is also important and varies throughout the literature. An appealingly simple choice (see, e.g., Ref. [11]) is $\{a_i\} = \{\mu\}$, where μ is to be evaluated at the weak scale. This is equivalent to using μ^2 as a fine-tuning measure, since Eqs. (2) and (5) imply $c_\mu = 4\mu^2/m_Z^2$. While generally adequate, this definition is insensitive to large fine tunings in the function f of Eq. (2), as we will see below; such fine tunings are accounted for in the more sophisticated choice of Eq. (4).

The top quark Yukawa Y_t (sometimes along with other standard model parameters, such as the strong coupling) is included among the fundamental parameters in some studies [6–8] and not in others [4,9,10]. This choice typically attracts little comment, and attitudes toward it are at best ambivalent [5]. This ambiguity reflects, perhaps, a diversity of prejudices concerning the fundamental theory of flavor. It is important to note, however, that, unlike the parameters of Eq. (4), Y_t is not expected to be related to supersymmetry breaking and is, in some sense, now measured, as it is strongly correlated with the top quark mass m_t. For these reasons, we find it reasonable to assume that in some more fundamental theory Y_t is fixed to its measured value in a flavor sector separate from the supersymmetry breaking sector, and we therefore do not include it among the a_i. This choice is critical for our conclusions, as will be discussed below.

In step (v), various other choices are also possible. For example, the c_i may be combined linearly or in quadrature; we follow the most popular convention. In other prescriptions, the c_i are combined after first dividing them by some suitably defined average $\overline{c}_i$ to remove artificial appearances of fine tuning [7,8]. We have not done this, but note that such a normalization procedure typically reduces the fine-tuning measure and would only strengthen our conclusions.

Given the prescription for measuring naturalness described above, we may now present our results. In Fig. 1, contours of constant c, along with squark mass contours, are presented for $\tan\beta = 10$. Moving from low to high m_0, the contours are determined successively by c_{μ_0}, $c_{M_{1/2}}$, and c_{m_0}. The naturalness requirement $c < 25$ ($c < 50$) allows regions of parameter space with $m_0 \approx 2$ TeV (2.4 TeV). More importantly, regions with $m_0 \approx 2$ TeV, where all squarks and sleptons have masses well above 1 TeV, are as natural as the region with $(m_0, M_{1/2}) \lesssim$ (1000 GeV, 400 GeV), where squark masses are below 1 TeV.

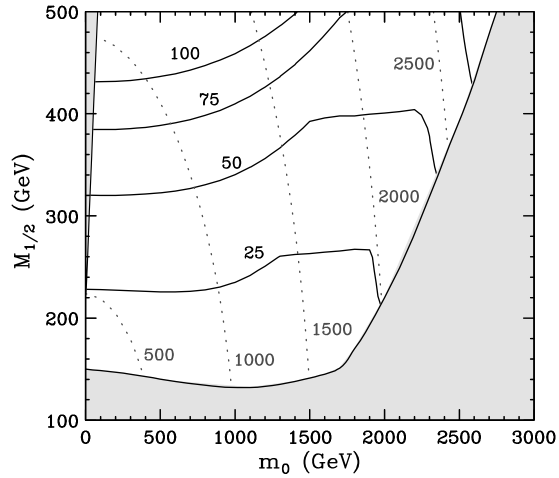

FIG. 1. Contours of constant fine tuning c (solid) and $m_{\tilde{u}_L}$ in GeV (dotted) in the $(m_0, M_{1/2})$ plane for $\tan\beta = 10$, $A_0 = 0$, and $\mu > 0$. The shaded regions are excluded by the requirement that the lightest supersymmetric particle be neutral (top left) and by the chargino mass limit of 90 GeV (bottom and right).

The naturalness of multi-TeV m_0, though perhaps surprising, may be simply understood as a consequence of a focus point in the RG behavior of $m_{H_u}^2$ [16], which renders its value at $M_{\rm weak}$ highly insensitive to its value in the ultraviolet. This insensitivity has been noticed previously in a different language (see, e.g., Ref. [5]), but the conclusion that multi-TeV scalars are therefore natural has not been drawn. Note, however, that, for moderate and large $\tan\beta$, Eq. (2) implies that m_Z^2 is insensitive to $m_{H_d}^2$, and so if $m_{H_u}^2$ is insensitive to ultraviolet boundary conditions, so is m_Z^2.

Consider any set of minimal supergravity input parameters. These generate a particular set of RG trajectories, $m_i^2|_{\rm p}(t), M_i|_{\rm p}(t), A_i|_{\rm p}(t), \ldots$, where $t \equiv \ln(Q/M_{\rm GUT})$ and Q is the renormalization scale. Now consider another set of boundary conditions that differs from the first by shifts in the scalar masses. The new scalar masses $m_i^2 = m_i^2|_{\rm p} + \delta m_i^2$ satisfy the RG equations,

$$\frac{d}{dt} m_i^2 \sim \frac{1}{16\pi^2}\left[-g^2 M_{1/2}^2 + Y^2 A^2 + \sum_j Y^2 m_j^2\right], \tag{7}$$

at one loop, where positive numerical coefficients have been omitted, and the sum is over all chiral fields ϕ_j interacting with ϕ_i through the Yukawa coupling Y. However, because the $m_i^2|_{\rm p}$ are already a particular solution to these RG equations, the deviations δm_i^2 obey the homogeneous equations

$$\frac{d}{dt}\,\delta m_i^2 \sim \frac{1}{16\pi^2} \sum_j Y^2\,\delta m_j^2\,. \tag{8}$$

Such equations are easily solved. Assume for the moment that the only large Yukawa coupling is Y_t, i.e., $\tan\beta$ is not extremely large. Then $\delta m_{H_u}^2$ is determined from

$$\frac{d}{dt}\begin{bmatrix}\delta m_{H_u}^2\\ \delta m_{U_3}^2\\ \delta m_{Q_3}^2\end{bmatrix} = \frac{Y_t^2}{8\pi^2}\begin{bmatrix}3 & 3 & 3\\ 2 & 2 & 2\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}\delta m_{H_u}^2\\ \delta m_{U_3}^2\\ \delta m_{Q_3}^2\end{bmatrix}, \quad (9)$$

where Q_3 and U_3 denote the third generation squark SU(2) doublet and up-type singlet representations, respectively. The solution corresponding to the universal initial condition $\delta m_0^2(1,1,1)^T$ is

$$\begin{bmatrix}\delta m_{H_u}^2\\ \delta m_{U_3}^2\\ \delta m_{Q_3}^2\end{bmatrix} = \frac{\delta m_0^2}{2}\left\{\begin{bmatrix}3\\ 2\\ 1\end{bmatrix}\exp\left[\int_0^t \frac{6Y_t^2}{8\pi^2}\,dt'\right] - \begin{bmatrix}1\\ 0\\ -1\end{bmatrix}\right\}. \quad (10)$$

For t and Y_t such that $\exp[(6/8\pi^2)\int_0^t Y_t^2\,dt'] = 1/3$, $\delta m_{H_u}^2 = 0$; i.e., $m_{H_u}^2$ is independent of δm_0^2.

The RG evolution of $m_{H_u}^2$ in minimal supergravity is shown for several values of m_0 in Fig. 2. As expected, the RG curves exhibit a focus (not a fixed) point, where $m_{H_u}^2$ is independent of its ultraviolet value. Remarkably, however, for the physical top mass of $m_t \approx 175$ GeV, the focus point is very near the weak scale. Thus, the weak scale value of $m_{H_u}^2$ and, with it, the fine-tuning parameter c are highly insensitive to m_0. If the particular solution is natural (say, with all input parameters near the weak scale), the new solution, even with very large m_0, is also natural.

We have also checked numerically that the focusing effect persists even for very large values of $\tan\beta$. Indeed, in the limit $Y_t = Y_b \gg Y_\tau$, Eq. (8) can be similarly solved analytically, and one finds that focusing occurs for $\exp[(7/8\pi^2)\int_0^t Y_t^2\,dt'] = 2/9$. For the experimentally preferred range of top masses, the focus point is again tantalizingly close to $M_{\rm weak}$ [17].

The naturalness of multi-TeV m_0 has important implications for collider searches. Although $m_{H_u}^2$ is focused to the weak scale, all other soft masses remain of order m_0. From Eqs. (8) and (10), we find that, for $m_0 \gg M_{1/2}, A_0$, the physical masses of squarks, sleptons, and heavy Higgs scalars are well approximated by

$$\begin{aligned}&\tilde{t}_R : \sqrt{1/3}\,m_0, && \text{all other } \tilde{q},\tilde{\ell} : m_0\,.\\ &\tilde{t}_L,\tilde{b}_L : \sqrt{2/3}\,m_0, && H^\pm, A, H^0 : m_0\,.\end{aligned} \quad (11)$$

Exact values of $m_{\tilde{u}_L}$ are presented in Fig. 1. All squarks, sleptons, and heavy Higgs scalars may therefore have masses $\gtrsim 1-2$ TeV, and may be beyond the reach of the Large Hadron Collider (LHC) and proposed linear colliders. The discovery of such heavy scalars then requires some even more energetic facility, such as the envisioned muon or very large hadron colliders.

As may be seen from Fig. 1, however, fine-tuning constraints do not allow multi-TeV $M_{1/2}$. A similar conclusion applies to μ, as may be seen in Fig. 3. We therefore expect all gauginos and Higgsinos to be within the kinematic reach of the LHC. Note that some regions of low μ are unnatural. In these regions, large cancellations in the function f of Eq. (2) occur, and the simple definition $c \propto \mu^2$ is inadequate.

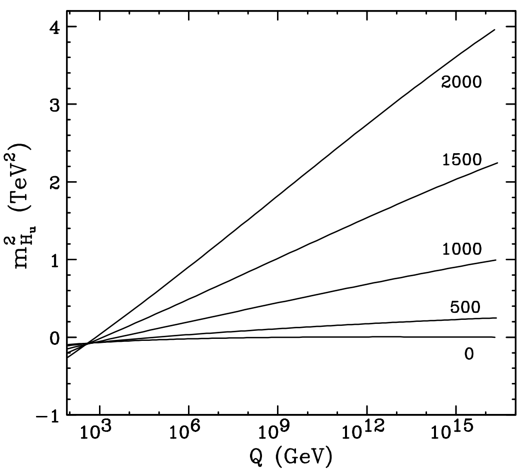

FIG. 2. The RG evolution of $m_{H_u}^2$ for fixed $M_{1/2} = 200$ GeV, $A_0 = 0$, $\tan\beta = 10$, $m_t = 175$ GeV, and several values of m_0 (shown, in GeV). The RG behavior of $m_{H_u}^2$ exhibits a focus point near the weak scale, where $m_{H_u}^2$ takes its weak scale value $\sim -(300\text{ GeV})^2$, irrespective of m_0.

In addition to the gauginos and Higgsinos, the lightest Higgs boson is, of course, still required to be light. Contours of lightest Higgs mass m_h are also presented in Fig. 3. Very heavy top and bottom squarks increase m_h through radiative corrections: for low $M_{1/2}$, m_h increases by roughly 6 GeV as m_0 increases from 500 GeV to 2 TeV. However, in the multi-TeV m_0 scenario, naturalness requires $A_0 \sim M_{\rm weak}$ (see below), and so left-right squark mixing is suppressed. The upper bound on m_h in Fig. 3 is thus approximately 120 GeV, well below limits achieved for TeV squarks with maximal left-right mixing, and within the $(3-5)\sigma$ discovery range of Higgs searches at the Tevatron with luminosity 10–30 fb^{-1} [18].

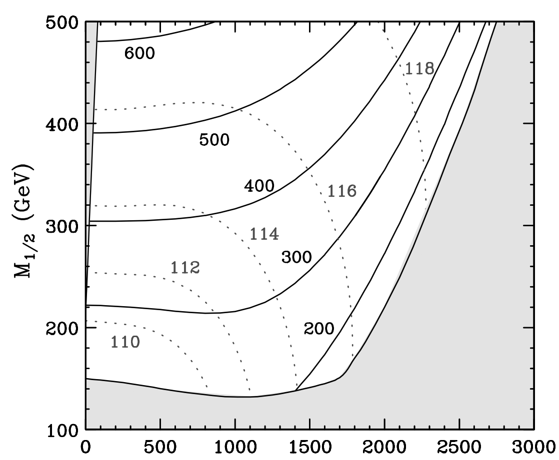

FIG. 3. Contours of μ (solid) and m_h (dotted) in GeV for input parameters as in Fig. 1.

The focus point analysis presented above (for small Y_b) relied heavily on the universality of the H_u, U_3, and Q_3 soft masses. It is not hard to show, however, that GUT scale boundary conditions of the form $(m_{H_u}^2, m_{U_3}^2, m_{Q_3}^2) = (1, 1 - x, 1 + x)$, for any x, also exhibit the focus point behavior. With respect to the other supersymmetry breaking parameters, the focus point is fairly robust. The mechanism is independent of all other scalar masses. Also, in the analysis above, *any* natural particular solution would do. Arbitrary and nonuniversal gaugino masses and trilinear couplings of order M_{weak} are therefore allowed. (Similarly, deviations in $m_{H_u}^2$, $m_{U_3}^2$, and $m_{Q_3}^2$ of order M_{weak}^2 do not destabilize the focus point.) Note, however, that multi-TeV gaugino masses and A parameters are not allowed. The required hierarchy between the scalar masses and the gaugino mass, A, and μ parameters may result from an approximate $\mathrm{U}(1)_{R+PQ}$ symmetry or from the absence of singlet F terms [19]. $B\mu$ may also be suppressed by such a symmetry, and so leads to an experimentally viable scenario with naturally large $\tan\beta \approx m_{H_d}^2/(B\mu)$, which is typically difficult to realize [20].

Although the focus point mechanism depends on a relation between m_t and $\ln(M_{\text{GUT}}/M_{\text{weak}})$, it is not extraordinarily sensitive to these values. The focus point is still near the weak scale if m_t is varied within its experimental uncertainty of 5 GeV, and, in fact, natural regions with multi-TeV m_0 are also possible if the high scale is raised to $\sim 10^{18}$ GeV [17].

We stress, however, that, if Y_t is included among the free and fundamental parameters, multi-TeV m_0 would be considered unnatural. For example, for $\tan\beta = 10$ and $A_0 = 0$, $c_{Y_t} < 25$ (50) corresponds to $m_0 \lesssim 500$ GeV (800 GeV) [17]. We have presented above our rationale for not including Y_t among the a_i, although a definitive resolution of this issue most likely requires an understanding of the fundamental theory of flavor.

In conclusion, for moderate and large $\tan\beta$, multi-TeV scalars are natural in minimal supergravity. (For small $\tan\beta \lesssim 5$, multi-TeV scalars are unnatural, both because the focus point differs significantly from the weak scale and because m_Z becomes sensitive to m_{H_d}.) In view of this result, the discovery of squarks, sleptons, and heavy Higgs scalars may be extremely challenging even at the LHC. In addition, it is not surprising that these scalars have so far escaped detection, as present bounds are far from excluding most of the natural parameter space. Finally, it is tempting to speculate that what appears to be an accidental conspiracy between m_t and the ratio of high to weak scales may find some fundamental explanation. If gauginos and Higgsinos are discovered, but all supersymmetric scalars escape detection at the LHC, the preservation of the naturalness motivation for supersymmetry, as currently understood, will require either an explanation of large cancellations between supersymmetry breaking soft masses at the weak scale, or the above scenario with a top mass fixed to be near 175 GeV. The latter possibility is, in our view, far more compelling and is supported by experimental data.

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