Higgs-Mediated $B^0 \to \mu^+\mu^-$ in Minimal Supersymmetry

K. S. Babu[1] and Christopher Kolda[2,3]
[1]*Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078*
[2]*Theory Group, MS 50A-5101, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720*
[3]*Department of Physics, University of California, Berkeley, California 94720*
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We demonstrate a new source for flavor-changing neutral currents within the minimal supersymmetric standard model. At moderate to large $\tan\beta$, it is no longer possible to diagonalize the masses of the quarks in the same basis as their Yukawa couplings. This generates flavor-violating couplings of the form $\overline{b}_R d_L \phi$ and $\overline{b}_R s_L \phi$ where ϕ is any of the three neutral, physical Higgs bosons. These new couplings lead to rare processes in the B system such as $B^0 \to \mu^+\mu^-$ and $B^0 - \overline{B}^0$ mixing. We show that the latter are anomalously suppressed, while the former is in the experimentally interesting range, with an observable signal possible at Run II of the Tevatron if $m_A \lesssim 400-700$ GeV.

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Extensions of the standard model containing more than one Higgs SU(2) doublet generically allow flavor-violating couplings of the neutral Higgs bosons. Such couplings, if unsuppressed, will lead to large flavor-changing neutral currents, in direct opposition to experiment [1]. Models such as the minimal supersymmetric standard model (MSSM) avoid these dangerous couplings by segregating the quark and Higgs fields so that one Higgs (H_u) can couple only to u-type quarks while the other (H_d) couples only to d-type. Within unbroken supersymmetry this division is completely natural; in fact, it is required by the holomorphy of the superpotential.

However, after supersymmetry is broken, there is nothing left to protect this division. In fact, it has been known for some time that couplings of the form $QU^cH_d^*$ and $QD^cH_u^*$ are generated at one loop [2]. Hall, Rattazzi, and Sarid (HRS) [3] showed that at moderate to large $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$ the contributions to d-quark masses coming from the nonholomorphic operator $QD^cH_u^*$ can be equal in size to those coming from the usual holomorphic operator QD^cH_d despite the loop suppression suffered by the former. This is because the operator itself gets an additional enhancement of $\tan\beta$. That is, the product $\tan\beta/16\pi^2$ need not be very small as $\tan\beta$ approaches its upper bound of 60 to 70.

The HRS result was followed shortly by Ref. [4] which analyzed the entire d-quark mass matrix in the presence of these corrections and found appreciable contributions to the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles. It has also recently been realized that the HRS corrections can significantly alter the (flavor-conserving) couplings of the Higgs bosons [5,6]. In this Letter we take our analysis from Ref. [6] one step further and show that flavor-changing couplings of the neutral Higgs bosons are also generated. We will show that these couplings can be appreciable and can be so even without invoking squark mixing and/or nonminimal Kähler potentials [7], and remain large even in the limit of heavy squarks and gauginos. These new couplings generate a variety of flavor-changing processes, including $\overline{B}^0 - B^0$ mixing and decays such as $B^0 \to \mu^+\mu^-$ which we will study in this Letter. A more complete discussion of these and other effects will be found in a forthcoming paper [8].

We begin by writing the effective Lagrangian for the interactions of the two Higgs doublets with the quarks in an arbitrary basis:

$$-\mathcal{L}_{\text{eff}} = \overline{D}_R \mathbf{Y_D} Q_L H_d + \overline{D}_R \mathbf{Y_D}[\epsilon_g + \epsilon_u \mathbf{Y_U^\dagger Y_U}] Q_L H_u^* + \text{H.c.} \tag{1}$$

Here $\mathbf{Y_D}$ and $\mathbf{Y_U}$ are the 3×3 Yukawa matrices of the microscopic theory, while the $\epsilon_{g,u}$ are the finite, loop-generated nonholomorphic Yukawa coupling coefficients derived by HRS. The leading contributions to ϵ_g and ϵ_u are generated by the two diagrams in Fig. 1.

Consider the first diagram in Fig. 1. If all $\tilde{Q}_i$ masses are assumed degenerate at some scale M_{unif} then, at lowest order, $i = k$ and the diagram contributes only to ϵ_g:

$$\epsilon_g \simeq \frac{2\alpha_3}{3\pi}\,\mu^* M_3 f(M_3^2, m_{\tilde{Q}_L}^2, m_{\tilde{d}_R}^2), \tag{2}$$

where $f(x,y,z)$ is defined in Ref. [3], and $f(x,x,x) = 1/(2x)$. Meanwhile, the second diagram of Fig. 1 contributes to ϵ_u:

$$\epsilon_u \simeq \frac{1}{16\pi^2}\,\mu^* A_U f(\mu^2, m_{\tilde{Q}_L}^2, m_{\tilde{u}_R}^2). \tag{3}$$

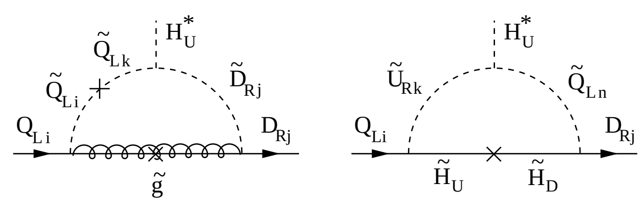

FIG. 1. Leading contributions to ϵ_g and ϵ_u. Indices i, j, k, and n label flavors.

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(We assume that the trilinear A terms can be written as some flavor-independent mass times $\mathbf{Y_U}$.) For typical inputs, one usually finds $|\epsilon_g|$ is about 4 times larger than $|\epsilon_u|$.

However, there is another sizable contribution to ϵ_u, this one coming from the *first* diagram in Fig. 1. It is well known that $\tilde{Q}_i$ degeneracy is broken by radiative effects induced by Yukawa couplings. While this would appear to be a higher-order effect, for $M_{\rm unif} \gg M_{\rm SUSY}$ it is amplified by a large logarithm and thus can be $\mathcal{O}(1)$. At the supersymmetric (SUSY) scale, we can write the $\tilde{Q}$ mass matrix in the form [9]

$$\mathbf{m}^2_{\tilde{Q}} \simeq \overline{m}^2(\mathbf{1} + c\mathbf{Y_U^\dagger Y_U} + c\mathbf{Y_D^\dagger Y_D}), \tag{4}$$

where

$$c \simeq -\frac{1}{8\pi^2}\frac{3m_0^2 + A_0^2}{\overline{m}^2}\log\left(\frac{M_{\rm unif}}{M_{\rm SUSY}}\right). \tag{5}$$

m_0 and A_0 are the common scalar mass and trilinear soft term at $M_{\rm unif}$, and $\overline{m}^2$ is a flavor-independent mass term. The effect of this nonuniversality is to generate a contribution to ϵ_u proportional to α_3 and thus potentially large (the $\mathbf{Y_D^\dagger Y_D}$ piece is irrelevant to flavor-changing questions). Specifically,

$$\Delta\epsilon_u \simeq \begin{cases} -c\epsilon_g/3 & (m^2_{\tilde{Q}} \simeq M_3^2) \\ -c\epsilon_g/2 & (m^2_{\tilde{Q}} \gg M_3^2) \end{cases}. \tag{6}$$

If $M_{\rm unif}$ is identified as the grand unified theory (GUT) scale, then c is typically in the range $-1 \lesssim c \lesssim -\frac{1}{4}$. Thus, this second contribution can either dramatically increase ϵ_u or potentially cancel much of it off, depending on their relative (model-dependent) signs. Perhaps more importantly, this contribution can still lead to large ϵ_u even if the A terms at the weak scale are small compared to the squark masses.

Now we return to Eq. (1). We can simplify it considerably by working in a basis in which $\mathbf{Y_U} = \mathbf{U}$ and $\mathbf{Y_D} = \mathbf{DV}^{0\dagger}$ where $\mathbf{V}^0$ is the CKM matrix at lowest order (the meaning of this will be clear shortly) and $\mathbf{U}$ and $\mathbf{D}$ are both diagonal. Then

$$-\mathcal{L}_{\rm eff} = \overline{D}_R\mathbf{DV}^{0\dagger}Q_LH_d + \overline{D}_R\mathbf{DV}^{0\dagger}[\epsilon_g + \epsilon_u\mathbf{U}^\dagger\mathbf{U}]Q_LH_u^* + \text{H.c.} \tag{7}$$

It is clear that in the absence of the ϵ_u term, all pieces of the effective Lagrangian can be diagonalized in the same basis, preventing the appearance of flavor-changing neutral currents (FCNCs).

To see how this works with ϵ_u included, it is sufficient to keep only the Yukawa couplings of the third generation so that $(\mathbf{U})_{ij} = y_t\delta_{i3}\delta_{j3}$ and $(\mathbf{D})_{ij}y_b\delta_{i3}\delta_{j3}$. The flavor-conserving pieces of $\mathcal{L}_{\rm eff}$ then have a form proportional to

$$\mathbf{DV}^{0\dagger} = y_b\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V^0_{ub} & V^0_{cb} & V^0_{tb} \end{pmatrix}, \tag{8}$$

while the flavor-changing piece has the form

$$\mathbf{DV}^{0\dagger}\mathbf{U}^\dagger\mathbf{U} = y_t^2y_b\,\text{diag}(0,0,V^0_{tb}). \tag{9}$$

We can define a physical eigenbasis by rotating the d component of Q_L by a new matrix $\mathbf{V}$ defined by diagonalizing the mass matrix: $(\mathbf{V}^\dagger\mathcal{Y}^\dagger\mathcal{Y}\mathbf{V})_{ij} = \text{diag}(\overline{y}_d^2, \overline{y}_s^2, \overline{y}_b^2)$ where the $\overline{y}_i$ are defined to be the "physical" Yukawa couplings, e.g., $m_b = \overline{y}_b v_d$, and

$$\mathcal{Y} = \mathbf{DV}^{0\dagger}[1 + \tan\beta(\epsilon_g + \epsilon_u\mathbf{U}^\dagger\mathbf{U})], \tag{10}$$

the $\tan\beta$ coming from the v_u which multiplies the loop-induced terms. $\mathbf{V}$ can now be interpreted as the physical CKM matrix.

In the physical basis, the (3, 3) element of the mass matrix gives us the corrected b-quark mass:

$$\overline{y}_b \simeq y_b[1 + (\epsilon_g + \epsilon_u y_t^2)\tan\beta]. \tag{11}$$

To get to this equation we used the fact that one finds no large (i.e., $\tan\beta$ enhanced) corrections to V_{tb} [4], so that we can replace $V^0_{tb} \simeq V_{tb} \simeq 1$.

The corrected CKM elements are the elements of $\mathbf{V}$. In particular,

$$V_{ub} \simeq V^0_{ub}\left[\frac{1 + \epsilon_g\tan\beta}{1 + (\epsilon_g + \epsilon_u y_t^2)\tan\beta}\right]. \tag{12}$$

The same form also holds for the corrected V_{cb}, V_{td}, and V_{ts}. Notice that V_{ub} reduces to V^0_{ub} in the limit that $\epsilon_u = 0$.

For $\epsilon_u \neq 0$, however, the rotation that diagonalized the mass matrix does not diagonalize the Yukawa couplings of the Higgs fields. Redefining D_L and D_R as the mass eigenstates, the effective Lagrangian for their couplings to the neutral Higgs fields is

$$-\mathcal{L}_{d,\rm eff} = \overline{D}_R\mathbf{DV}^{0\dagger}\mathbf{V}D_LH_d^0 + \overline{D}_R\mathbf{DV}^{0\dagger}[\epsilon_g + \epsilon_u\mathbf{U}^\dagger\mathbf{U}]\mathbf{V}D_LH_u^{0*} + \text{H.c.} \tag{13}$$

Keeping only the flavor changing pieces, this simplifies after some algebra to

$$\mathcal{L}_{\rm FCNC} = \frac{\overline{y}_bV^*_{tb}}{\sin\beta}\chi_{\rm FC}[V_{td}\overline{b}_Rd_L + V_{ts}\overline{b}_Rs_L] \times (\cos\beta H_u^{0*} - \sin\beta H_d^0) + \text{H.c.} \tag{14}$$

with the quark fields in the physical/mass eigenbasis, and defining

$$\chi_{\rm FC} = \frac{-\epsilon_u y_t^2\tan\beta}{(1 + \epsilon_g\tan\beta)[1 + (\epsilon_g + \epsilon_u y_t^2)\tan\beta]} \tag{15}$$

to parametrize the amount of flavor changing induced.

The flavor-changing couplings between the Higgs mass states and the fermion mass states are

$$\left.\begin{aligned} h^0\overline{b}_Rd_L&: i\cos(\beta - \alpha) \\ H^0\overline{b}_Rd_L&: i\sin(\beta - \alpha) \\ A^0\overline{b}_Rd_L&: 1 \end{aligned}\right\} \times \frac{\overline{y}_bV_{td}V^*_{tb}}{\sqrt{2}\sin\beta}\chi_{\rm FC}. \tag{16}$$

A similar expression holds for the Higgs couplings to $\overline{b}_Rs_L$ with V_{td} replaced by V_{ts}. One nontrivial check of this result is to take the Higgs decoupling limit in which $m_{A^0} \to \infty$, driving $\alpha \to \beta - \frac{\pi}{2}$. There the $h^0\overline{b}_Rd_L$ coupling goes to zero as it should in any single Higgs doublet model.

We will now consider two processes which constrain and/or provide a signal for the Higgs-mediated FCNCs: $B^0 - \overline{B}^0$ mixing and the decay $B^0 \to \mu^+\mu^-$. The case of $B^0 - \overline{B}^0$ mixing is actually quite amusing. Δm_{B_d} is very well known and usually provides one of the tightest constraints on new sources of flavor violation in the d-quark sector. And, in principle, mixing can be generated by single Higgs exchange. The leading order contribution of the three physical Higgs bosons to an effective operator $\overline{b}^i_R d^i_L \overline{b}^j_R d^j_L$ [i, j are SU(3) indices] is proportional to the product of vertex factors and propagators given by

$$\mathcal{F} \equiv \left[\frac{\cos^2(\beta - \alpha)}{m_h^2} + \frac{\sin^2(\beta - \alpha)}{m_H^2} - \frac{1}{m_A^2}\right]. \quad (17)$$

However, $\mathcal{F} = 0$ at lowest order.

It is natural to ask whether this zero survives loop corrections, and one finds that it does not. We have considered in detail the largest nonzero contribution, which arises from top-stop induced vacuum polarization on the internal Higgs line. While these propagator corrections to the Higgs are known to be large [10], we find that the leading term (which is a correction to the H_u line) is suppressed by $1/\tan^2\beta$. The next-leading term (a correction on the H_d line due to left-right stop mixing) is present but is not very large. All other radiative corrections we expect to be even smaller.

One can still derive a bound on m_A by demanding that the MSSM contribution to Δm_{B_d} is less than its observed value. Assuming degenerate MSSM spectrum and constructive interference between Eqs. (3) and (6), we find $m_A \lesssim 100$ to 125 GeV for $\tan\beta = 40$ to 60. Direct search constraints aside, it is known that models with such a light second Higgs doublet generally contribute far too much to $b \to s\gamma$ and are therefore already ruled out [11]. Thus this new source of flavor changing rules out a part of parameter space which is already known to be disfavored.

We now consider the rare decay $B^0 \to \mu^+\mu^-$. This occurs via emission off the quark current of a single virtual Higgs boson which then decays leptonically. The largest leptonic flavor-changing branching fraction would clearly be to $\tau^+\tau^-$. However, the branching fraction to μ's is suppressed only by $(m_\mu/m_\tau)^2$ times a phase space factor, which is only about 1 part in 100. Given the extreme difficulties encountered in trying to measure the τ mode experimentally, it is doubtful that it will ever provide an interesting constraint or signal in and of itself. Thus we will concentrate on the μ channel.

The partial width for the process $B^0_{(d,s)} \to \mu^+\mu^-$ is given by

$$\Gamma(B^0_{(d,s)} \to \mu^+\mu^-) = \frac{\eta^2_{\mathrm{QCD}}}{128\pi} m_B^3 f_B^2 \overline{y}_b^2 y_\mu^2 |V^*_{t(d,s)} V_{tb}|^2 \times \chi^2_{\mathrm{FC}}(a_1^2 + a_2^2), \quad (18)$$

where $a_1 = [\sin(\beta - \alpha)\cos\alpha]/m_H^2 - [\cos(\beta - \alpha) \times \sin\alpha]/m_h^2$ and $a_2 = -\sin\beta/m_A^2$. In the large m_A, large $\tan\beta$ limit, $a_1^2 + a_2^2 \simeq 2/m_A^4$. The QCD correction is identical to the usual running of a quark mass operator, which in this case gives η_{QCD} between 1.4 and 1.6 for m_A between m_Z and 500 GeV. Experimentally, $\mathcal{B}(B^0_{(d,s)} \to \mu^+\mu^-) < (6.8, 20) \times 10^{-7}$ at 90% confidence [12]. Thus $\Gamma_{(d,s)} < (2.9, 8.7) \times 10^{-19}$ GeV. Theory predicts the partial width for $B^0_s \to \mu^+\mu^-$ to be enhanced by $(V_{ts}/V_{td})^2 \simeq 25$. Thus one expects a signal in B^0_s decays before one is observed in B^0_d.

A quick estimate can give us an impression of the importance of these new contributions. For nearly degenerate MSSM particles, one finds $|\epsilon_g| \approx 1/80$ and $|\epsilon_u| \approx (1/4)|\epsilon_g|$, not including in ϵ_u the contribution of Eq. (6). We derive a bound on m_A from the limit on $B^0_s \to \mu^+\mu^-$ and using $f_B = 180$ MeV and $|V_{ts}| = 0.04$. The bound depends sensitively on the signs of ϵ_g and ϵ_u as well as the size of the c parameter of Eq. (6), which we take in the range $-3/4 \le c \le 0$. We also demand that $y_b \le y_t$ to avoid problems with perturbation theory and consistency with unification; this places an upper bound on $\tan\beta$ as a function of ϵ_g, ϵ_u, and c. Varying over all of these, the strongest bounds are $m_A > (225, 175, 230, 215)$ GeV for $\tan\beta = (29, 65, 38, 65)$, $c = (-3/4, 0, 0, -3/4)$, and the signs of $\{\epsilon_g, \epsilon_u\}$ being $(--, +-, -+, ++)$, respectively. Minimal GUT models with $b - \tau$ unification would prefer ϵ_g to be negative [3,13], where our bounds are more stringent.

Like the case of $B^0 - \overline{B}^0$ mixing, we are finding ourselves in the range already constrained by $b \to s\gamma$ and direct searches. However, unlike the mixing case where the MSSM contribution was typically smaller than the standard model prediction, here we are still far above the standard model which predicts $\mathcal{B}(B^0_{(d,s)} \to \mu^+\mu^-) \simeq (1.5, 35) \times 10^{-10}$ [14]. And because the $B^0 - \overline{B}^0$ mixing constraint is so much weaker than naive expectation, processes such as $B^0 \to \mu^+\mu^-$ which are usually less sensitive to new sources of flavor changing can now be their primary probe. Thus further experimental data can significantly improve the bounds on m_A or find a nonzero signal induced by supersymmetry.

So what is implied for Run II at the Tevatron? Assuming no change in their efficiencies and acceptances, CDF can in principle place a bound $\mathcal{B}(B^0_s \to \mu^+\mu^-) < 1 \times 10^{-7}$ given 1 fb^{-1} of data, a factor of 20 stronger than present. Thus the region probed in m_A will increase by $20^{1/4} \simeq 2$:

$$m_A > (475, 370, 490, 450) \text{ GeV} \quad (19)$$

for the same sets of inputs as previously. After collecting 5 fb^{-1} these masses increase by another 50%, up to 725 GeV. This can be a very important signal for supersymmetry since *this source of flavor changing does not decouple as* $M_{\mathrm{SUSY}} \to \infty$ so long as m_A does not also get very heavy. That is to say, the bound on m_A is roughly independent of M_{SUSY}. Therefore supersymmetric spectra in the multihundred GeV to TeV range may be probed at the Tevatron through rare B decays even when direct production of supersymmetry (including the second Higgs doublet) cannot be observed.

Since the precise predictions for $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ are highly dependent on the individual model, these estimates should be taken only as indicative. Furthermore, there are nontrivial and very interesting correlations between $B^0 \to \mu^+\mu^-$ and other loop-generated processes such as $b \to s\gamma$ and $(g - 2)_\mu$. A signal in one of these may be a harbinger of a signal in another, though no model-independent statement can be made since each decouples in a different limit (large m_A, $m_{\tilde{Q}}$, and $m_{\tilde{L}}$, respectively). Further work exploring these correlations and others will be forthcoming [8].

It is also possible to look for new sources of flavor changing in inclusive semileptonic decays $B \to X_s\mu^+\mu^-$. The width for this process can be extracted from Eq. (17) by replacement of f_B with m_B and dividing by $192\pi^2$ for the 3-body phase space. Comparing to current bounds [15] yields constraints on m_A that are weaker by a factor of 1.8 than the bounds from the purely leptonic mode. The ability of future experiments to extract information from this mode will be discussed in [8].

In summary, we have found that neutral Higgs bosons are capable of mediating flavor-changing interactions within the MSSM. This result is generic and does not rely on assumptions about sparticle mass nonuniversality which are usually required in order to get FCNCs. These interactions are enhanced at large $\tan\beta$ and are in the range that will be experimentally probed in the near future. In particular, we have shown that SUSY may be discovered in Run II of the Tevatron in rare B decays long before direct production of sparticles is observed.

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