Phenomenology of the Randall-Sundrum Gauge Hierarchy Model

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We explore the phenomenology of the localized gravity model of Randall and Sundrum where a 5-dimensional nonfactorizable geometry generates the gauge hierarchy by an exponential function called a warp factor. The Kaluza-Klein (KK) tower of gravitons in this scenario has different properties from those in factorizable models. We derive the KK graviton interactions with the standard model fields and obtain constraints from their direct production at hadron colliders as well as from virtual KK exchanges. We study the KK spectrum in e^+e^- annihilation and show how to determine the model parameters if the first KK state is observed.

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The large hierarchy between the electroweak and apparent gravity scales is a primary mystery of particle physics. It is possible that the observed 4-dimensional (4D) value of the Planck scale, $M_{\rm Pl}$, is not truly fundamental. A scenario of this type due to Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1] proposes the existence of n additional compact dimensions of volume V_n and relates the fundamental $(4 + n)$D Planck scale, M, to $M_{\rm Pl}$ via $M_{\rm Pl}^2 = V_n M^{2+n}$. Setting $M \sim$ TeV to remove the above hierarchy necessitates large extra dimensions compactified at the scale $\mu_c = 1/r_c \sim$ eV–MeV for $n = 2$–7. This introduces another hierarchy between μ_c and M, which must be stabilized. Nonetheless, this scenario has received much attention as it affords concrete phenomenological [2] and astrophysical [3] tests. Since it is experimentally determined that the standard model (SM) fields do not feel the effects of additional dimensions of this size, they are confined to a wall, or 3-brane, while gravity propagates in the full higher-dimensional space, or bulk. Kaluza-Klein (KK) towers of gravitons, which interact with the wall fields, result from compactification of the bulk. The coupling of each KK excitation is $M_{\rm Pl}$ suppressed; however, the mode spacing is determined by μ_c and is thus very small compared to typical collider energies. This allows the summation over an enormous number of KK states which can be exchanged or emitted in a physical process, thereby reducing the summed suppression from $1/M_{\rm Pl}$ to $1/M$, or $\sim$TeV^{-1}.

An alternative scenario has recently been proposed by Randall and Sundrum (RS) [4], where the hierarchy is generated by an exponential function of the compactification radius, called a warp factor, in a 5D nonfactorizable geometry, based on a slice of AdS_5 spacetime. Two 3-branes with opposite tensions reside at S_1/Z_2 orbifold fixed points, $\phi = 0, \pi$, where ϕ is the angular coordinate parametrizing the extra dimension. The 4D Poincaré invariant solution to Einstein's equations for this configuration is

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + r_c^2\,d\phi^2, \tag{1}$$

where the Greek indices run over 4D spacetime, $\sigma(\phi) = kr_c|\phi|$ with r_c being the compactification radius of the extra dimension, and $0 \leq |\phi| \leq \pi$. Here $k \sim M_{\rm Pl}$ and relates the 5D Planck scale M to the cosmological constant. Similar configurations have also been found to arise in M/string theory [5], and an extension of this scenario where $r_c \to \infty$ is discussed in Ref. [6]. Examination of the 4D effective action in the RS scenario yields [4]

$$\overline{M}_{\rm Pl}^2 = \frac{M^3}{k}\,(1 - e^{-2kr_c\pi}) \tag{2}$$

for the reduced 4D Planck scale. Assuming that we live on the 3-brane located at $|\phi| = \pi$, it is found that a field on this brane with the fundamental mass parameter m_0 will appear to have the physical mass $m = e^{-kr_c\pi}m_0$. TeV scales are thus generated from fundamental scales of order $M_{\rm Pl}$ via a geometrical exponential warp factor. The hierarchy is reproduced if $kr_c \simeq 12$ and no additional hierarchies are generated. It has been demonstrated [7] that the size of μ_c in this scenario can be stabilized without the fine-tuning of parameters.

The graviton KK spectrum is quite different in this scenario than in the case with factorizable geometry, resulting in a distinctive phenomenology. As we will see below, the masses and couplings of each individual KK excitation are determined by the TeV scale, implying that they can be separately produced on resonance with observable rates at colliders. We will examine the cases of KK graviton production in Drell-Yan and dijet events at hadron colliders as well as the KK spectrum line shape at high-energy linear e^+e^- colliders. If a resonance is observed, we demonstrate how the parameters of this model can be uniquely determined. In the case where no direct production is observed, we compute the bounds on the parameter space in the contact interaction limit.

We now calculate the mass spectrum and couplings of the graviton KK modes in the effective 4D theory on the 3-brane at $\phi = \pi$. The starting point is the 5D Einstein's equation for the RS configuration, which is given

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in Ref. [4]. We parametrize the tensor fluctuations $h_{\alpha\beta}$ by taking a linear expansion of the flat metric about its Minkowski value, $\hat{G}_{\alpha\beta} = e^{-2\sigma}(\eta_{\alpha\beta} + \kappa^* h_{\alpha\beta})$, where κ^* is an expansion parameter. To obtain the mass spectrum, we consider the 4D components of Einstein's equation with the replacement $G_{\alpha\beta} \to \hat{G}_{\alpha\beta}$, keeping terms up to $\mathcal{O}(\kappa^*)$, working in the gauge with $\partial^\alpha h_{\alpha\beta} = h^\alpha_\alpha = 0$. Upon compactification the graviton field is expanded into

$$h_{\alpha\beta}(x,\phi) = \sum_{n=0}^{\infty} h^{(n)}_{\alpha\beta}(x)\,\frac{\chi^{(n)}(\phi)}{\sqrt{r_c}}\,, \tag{3}$$

where the $h^{(n)}_{\alpha\beta}(x)$ correspond to the KK modes of the graviton on the background of Minkowski space on the 3-brane. In a gauge where $\eta^{\alpha\beta}\partial_\alpha h^{(n)}_{\beta\gamma} = \eta^{\alpha\beta} h^{(n)}_{\alpha\beta} = 0$, the equation of motion for $h^{(n)}_{\alpha\beta}$ with masses $m_n \geq 0$

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu - m_n^2)h^{(n)}_{\alpha\beta}(x) = 0\,. \tag{4}$$

Using the KK expansion (3) for $h_{\alpha\beta}$ in $\hat{G}_{\alpha\beta}$, Einstein's equation in conjunction with the above yields

$$\frac{-1}{r_c^2}\,\frac{d}{d\phi}\left(e^{-4\sigma}\,\frac{d\chi^{(n)}}{d\phi}\right) = m_n^2 e^{-2\sigma}\chi^{(n)}. \tag{5}$$

The orthonormality condition for $\chi^{(n)}$ is found to be $\int_{-\pi}^{\pi} d\phi\, e^{-2\sigma}\chi^{(m)}\chi^{(n)} = \delta_{mn}$. We have used $(d\sigma/d\phi)^2 = (kr_c)^2$ and $d^2\sigma/d\phi^2 = 2kr_c[\delta(\phi) - \delta(\phi - \pi)]$, as required by the orbifold symmetry for $\phi \in [-\pi, \pi]$ [4]. The solutions for $\chi^{(n)}$ are [8]

$$\chi^{(n)}(\phi) = \frac{e^{2\sigma(\phi)}}{N_n}\,[J_2(z_n) + \alpha_n Y_2(z_n)]\,, \tag{6}$$

where J_2 and Y_2 are Bessel functions of order 2, $z_n(\phi) = m_n e^{\sigma(\phi)}/k$, N_n represents the wave function normalization, and α_n are constant coefficients.

Defining $x_n \equiv z_n(\pi)$, and working in the limit that $m_n/k \ll 1$ and $e^{kr_c\pi} \gg 1$, the requirement that the first derivative of $\chi^{(n)}$ be continuous at the orbifold fixed points yields $\alpha_n \sim x_n^2 e^{-2kr_c\pi}$ and $J_1(x_n) = 0$, so that the x_n are simply roots of the Bessel function of order 1. The first few values of x_n are given by $x_1 \simeq 3.83$, $x_2 \simeq 7.02$, $x_3 \simeq 10.17$, and $x_4 \simeq 13.32$. Note that the masses of the graviton KK excitations, given by $m_n = kx_n e^{-kr_c\pi}$, are dependent on the roots of J_1 and are not equally spaced, contrasted to most KK models with one extra dimension. For $x_n \ll e^{kr_c\pi}$, we see that $\alpha_n \ll 1$, and hence $Y_2(z_n)$ can be neglected compared to $J_2(z_n)$ in Eq. (6). We thus obtain for the normalization

$$N_n \simeq \frac{e^{kr_c\pi}}{\sqrt{kr_c}}\,J_2(x_n); \qquad n > 0\,, \tag{7}$$

and the normalization of the zero mode is simply $N_0 = 1/\sqrt{kr_c}$.

Given the solutions for $\chi^{(n)}$, we can now derive the interactions of $h^{(n)}_{\alpha\beta}$ with the matter fields on the 3-brane. Starting with the 5D action and imposing the constraint that we live on the brane at $\phi = \pi$, we find the usual form of the interaction Lagrangian in the 4D effective theory,

$$\mathcal{L} = -\frac{1}{M^{3/2}}\,T^{\alpha\beta}(x)h_{\alpha\beta}(x, \phi = \pi)\,, \tag{8}$$

where $T_{\alpha\beta}(x)$ is the Minkowski space energy-momentum tensor of the matter fields and we have used $\kappa^* = 2/M^{3/2}$. Expanding the graviton field into the KK states of Eq. (3) and using the above normalization in Eq. (7) for $\chi^{(n)}(\phi)$ we find via Eq. (2)

$$\mathcal{L} = -\frac{1}{\overline{M}_{\rm Pl}}\,T^{\alpha\beta}(x)h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi}\,T^{\alpha\beta}(x)\sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x)\,. \tag{9}$$

Thus the zero mode separates from the sum and couples with the usual strength, whereas all the massive KK states are suppressed by only Λ_π^{-1}, where $\Lambda_\pi = e^{-kr_c\pi}\overline{M}_{\rm Pl} \sim$ TeV.

We have assumed $k < M$ with $M \sim \overline{M}_{\rm Pl}$, so that the 5D curvature is small compared to M and the solution for the bulk metric can be trusted [4]. This implies that the ratio $k/\overline{M}_{\rm Pl}$ cannot be too large. As we will see, the value of this ratio is central to the phenomenology of this model. In order to get a feel for the natural size of $k/\overline{M}_{\rm Pl}$, we perform an estimate using string theoretic arguments. The string scale M_s can be related [9] to $\overline{M}_{\rm Pl}$ in 4D heterotic string theories by $M_s \sim g_{\rm YM}\overline{M}_{\rm Pl}$, where $g_{\rm YM}$ is the 4D Yang-Mills gauge coupling constant, and the tension τ_3 of a D 3-brane is given by $\tau_3 = M_s^4/g(2\pi)^3$, where g is the string coupling constant. For $g_{\rm YM} \sim 0.7$ and $g \sim 1$, we find $\tau_3 \sim 10^{-3}\overline{M}_{\rm Pl}^4$. In the RS scenario, the magnitude of the 3-brane tension is given by $V = 24\overline{M}_{\rm Pl}^2 k^2$. Requiring that $V = \tau_3$ suggests $k/\overline{M}_{\rm Pl} \sim 10^{-2}$. We take the range $0.01 \leq k/\overline{M}_{\rm Pl} \leq 1$ in our analysis; however, the above discussion favors the lower end of this range.

Constraints on the parameters of this model can be obtained by direct collider searches for the first graviton excitation, $G^{(1)}$, at the Tevatron or LHC. The cleanest signal for graviton resonance production will be either an excess in Drell-Yan events, $q\bar{q}, gg \to G^{(1)} \to \ell^+\ell^-$, or in the dijet channel, $q\bar{q}, gg \to G^{(1)} \to q\bar{q}, gg$. Note that gluon-gluon initiated processes now contribute to Drell-Yan production. This differs from the ADD scheme where individual resonances associated with graviton exchange are not observable. Using the Lagrangian (9), the production cross section, decay widths, and branching fractions relevant for graviton production can be obtained. We assume that the first excitation decays only into SM states, so that for a fixed value of its mass, m_1, the value of $k/\overline{M}_{\rm Pl}$ completely determines all of the above quantities. In fact, its total width, Γ_1, is found to be proportional to $(k/\overline{M}_{\rm Pl})^2$. Keeping in mind that theoretic arguments favor a smaller value for this parameter, and to get a handle on the possible constraints that arise from these channels, we employ the narrow width approximation. This is strictly valid only for $k/\overline{M}_{\rm Pl} \lesssim 0.3$ but well approximates the true search reach obtained via a more complete analysis [10]. The lack of any signal for a new resonance in either the Drell-Yan or dijet channel in the existing Tevatron data [11] then provides a constraint on $k/\overline{M}_{\rm Pl}$ for any given value of m_1 as shown in Fig. 1(a). Similarly, we estimate the future 95% C.L. parameter exclusion regions at both Run II at the Tevatron and at the LHC; these results are displayed in Figs. 1(a) and 1(b). The dijet constraints for Run II were estimated by a simple luminosity (and $\sqrt{s}$) rescaling of the published Run I results. Note that the Drell-Yan and dijet channels play complementary roles at the Tevatron in obtaining these limits. We expect a dijet search at the LHC to yield poor results due to the large QCD background at this higher $\sqrt{s}$.

The discovery of the first graviton excitation as a resonance at a collider will allow the determination of all of the fundamental model parameters through measurements of its mass and width. To demonstrate this, we make use of the two relations $\Lambda_\pi = m_1\overline{M}_{\rm Pl}/(kx_1)$ and $\Gamma_1 = \rho m_1 x_1^2(k/\overline{M}_{\rm Pl})^2$, where x_1 is the first nonzero root of the J_1 Bessel function and ρ is a constant which depends on the number of open decay channels; it is fixed if the graviton decays only to SM fields. Using these relations we find that $r_c = \log[kx_1/m_1]/(k\pi)$ with $k = \overline{M}_{\rm Pl}[\Gamma_1/m_1\rho x_1^2]^{1/2}$. In addition, the spin-2 nature of the graviton can be determined via angular distributions of its decay products, and hence graviton production is clearly differentiable from other possible new resonaces such as a Z' boson.

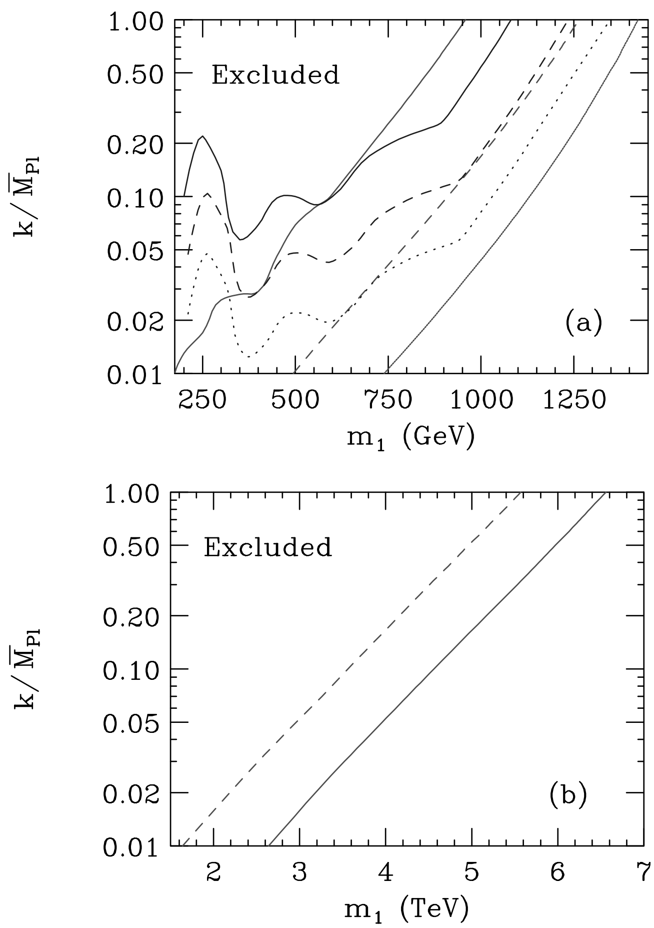

FIG. 1. Exclusion regions for resonance production of the first KK graviton excitation in (a) Drell-Yan (diagonal lines) and dijet (the bumpy curves) channels at the Tevatron and (b) Drell-Yan production at the LHC. (a) The solid curves represent the results for Run I, while the dashed and dotted curves correspond to Run II with 2 and 30 fb^{-1} of integrated luminosity, respectively. (b) The dashed and solid curves correspond to 10 and 100 fb^{-1}.

To exhibit how the KK tower of gravitons may appear at a collider, Fig. 2 displays the cross section for $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$, assuming $m_1 = 600$ GeV and taking various values of $k/\overline{M}_{\rm Pl}$. We see that for small values of $k/\overline{M}_{\rm Pl}$ the gravitons appear as ever widening peaks and are almost regularly spaced, with the widths and the spacing both being dependent on successive roots of J_1. However, as $k/\overline{M}_{\rm Pl}$ grows, the peaks become too wide to be identified as true resonances and the classic KK signature of successive peaks becomes lost. Instead, it would appear experimentally that there is an overall large enhancement of the cross section, similar to what might be expected from a contact interaction. At some point the cross section may grow so large as to violate the partial wave unitarity bound [2] of $\sigma_U = 20\pi/s$ for initial and final fermion states with helicity of 1. However, even for $k/\overline{M}_{\rm Pl} \sim 1$ we find that unitarity will not be violated until $\sqrt{s}$ is at least several TeV.

If the gravitons are too massive to be directly produced at colliders, their contributions to fermion pair production may still be felt via virtual exchange. The 4-fermion matrix element is easily computed from the Lagrangian (9) and is seen to reproduce that derived [2] for the ADD scenario with the replacement

$$\frac{\lambda}{M_s^4} \to \frac{i^2}{8\Lambda_\pi^2}\sum_{n=1}^{\infty}\frac{1}{s - m_n^2}\,. \tag{10}$$

Here, unlike in the factorizable case, there are no divergences associated with performing the sum, and hence uncertainties related to the introduction of a cutoff do

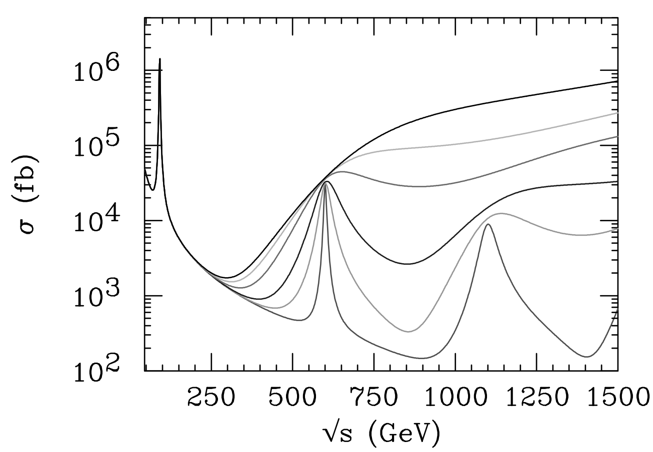

FIG. 2. The cross section for $e^+e^- \to \mu^+\mu^-$ including the exchange of a KK tower of gravitons, taking the mass of the first mode to be 600 GeV, as a function of $\sqrt{s}$. From top to bottom the curves correspond to $k/\overline{M}_{\rm Pl} = 1.0, 0.7, 0.5, 0.3, 0.2, 0.1$.

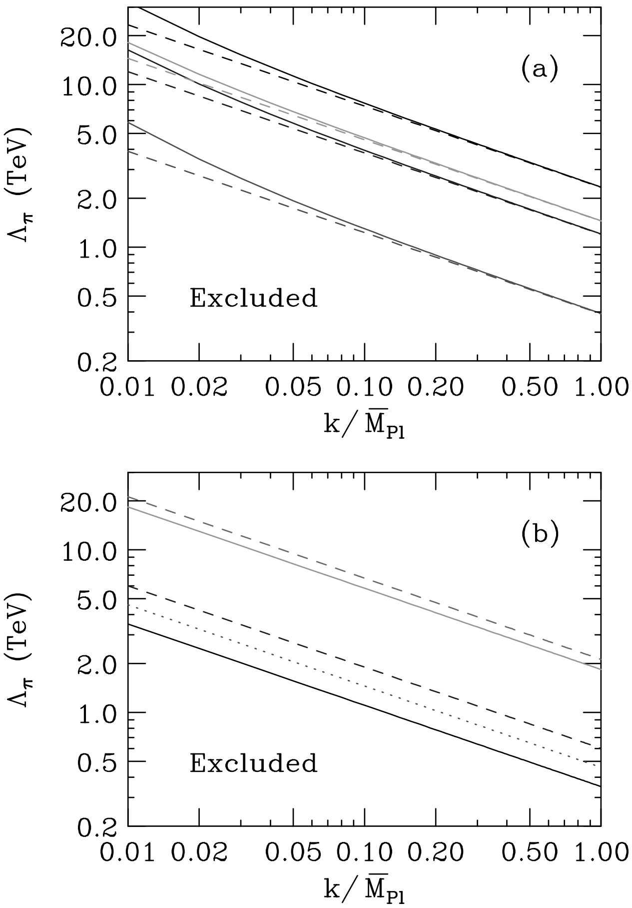

FIG. 3. Constraints in the Λ_π-$k/\overline{M}_{\rm Pl}$ plane from virtual exchange of the KK tower of gravitons. (a) The dashed curves assume $m_n \gg \sqrt{s}$, while the solid curves correspond to the case where the first two excitations are close to $\sqrt{s}$. From bottom to top the pairs of curves correspond to LEP II at 195 GeV with 2.5 $\rm fb^{-1}$ of integrated luminosity; a linear e^+e^- collider at 500 GeV with 75 $\rm fb^{-1}$; 500 GeV with 500 $\rm fb^{-1}$; and 1 TeV with 200 $\rm fb^{-1}$. (b) From bottom to top the curves correspond to the Tevatron Run I with 110 $\rm fb^{-1}$, Run II with 2 $\rm fb^{-1}$, Run II with 30 $\rm fb^{-1}$, the LHC with 10 $\rm fb^{-1}$, and 100 $\rm fb^{-1}$.

not appear. In the limit of $m_n^2 \gg s$, the sum over the KK graviton propagators becomes $[k\Lambda_\pi/\overline{M}_{\rm Pl}]^{-2}\sum_n 1/x_n^2$ which rapidly converges. The 95% C.L. search reaches in the Λ_π-$k/\overline{M}_{\rm Pl}$ plane are given in Fig. 3 for various (a) e^+e^- and (b) hadron colliders. In e^+e^- annihilation we have examined the unpolarized (and polarized for the case of high-energy linear colliders) angular distributions, summing over e, μ, τ, c, b (and t, if kinematically accessible) final states, as well as τ polarization, and included initial state radiation, heavy quark tagging efficiencies, an angular cut around the beam pipe, and 90% beam polarization where applicable. For hadron colliders we examined the lepton pair invariant mass spectrum and forward-backward asymmetry in Drell-Yan production, for both e and μ final states. We also investigated the case where the first two excitations are too close to the collider center-of-mass energy to use the approximation $m_n^2 \gg s$. The bounds in e^+e^- annihilation for this case are given by the solid curves in Fig. 3(a), showing very little difference in the resulting constraints.

As a last point, we note that, whereas graviton tower emission was an important probe of the ADD scenario, this is no longer true in the RS model since the graviton states are so massive and can be individually examined.

In this paper we have explored the phenomenology of the Randall-Sundrum localized gravity model of nonfactorizable 5D spacetime, and contrasted it with the ADD scenario. We derived the interaction of the KK tower of gravitons with the SM fields, and obtained limits on the model parameters using existing data from colliders, both through direct production searches and via virtual exchange contributions, and estimated how future colliders can extend these bounds. We also described the appearance of KK tower production at high-energy linear colliders, the possible loss of the conventional KK signature of successive peaks due to the ever growing widths of these excitations, and demonstrated how measurements of the properties of the first KK state would completely determine the model parameters.

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