Phenomenology of a Realistic Accelerating Universe Using Only Planck-Scale Physics

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Modern data are showing increasing evidence that the Universe is accelerating. So far, all attempts to account for the acceleration have required some fundamental dimensionless quantities to be extremely small. We show how a class of scalar field models (which may emerge naturally from superstring theory) can account for acceleration which starts in the present epoch with all the potential parameters $O(1)$ in Planck units.

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Current evidence that the Universe is accelerating [1], if confirmed, requires dramatic changes in the field of theoretical cosmology. Until recently, there was strong prejudice against the idea that the Universe could be accelerating. There simply is no compelling theoretical framework that could accommodate an accelerating universe. Since the case for an accelerating universe continues to build (see, for example, [2]), attempts have been made to improve the theoretical situation, with some modest success. Still, major open questions remain.

All attempts to account for acceleration [3–21] introduce a new type of matter (the "dark energy" or "quintessence") with an equation of state $p_Q = w_Q \rho_Q$ relating pressure and energy density. Values of $w_Q < -0.6$ today are preferred by the data [22] and in many models w_Q can vary over time. (In this framework, $w_Q = -1; \dot{w}_Q = 0$ gives a cosmological constant.)

One challenge faced by quintessence models is similar to the old "flatness problem" which is addressed by cosmic inflation. Consider $\Omega_{\text{tot}} \equiv \rho_{\text{tot}}/\rho_c$, where ρ_c is the critical density (achieved by a perfectly flat universe). The dynamics of the standard big bang drive Ω_{tot} away from unity and require extreme fine-tuning of initial conditions for Ω_{tot} to be as close to unity as it is today (inflation can set up the required initial conditions). In models with quintessence one must consider $\Omega_Q \equiv \rho_Q/\rho_c$ which is observed to obey

$$\Omega_Q \approx \Omega_{\text{other}} \equiv (\rho_{\text{tot}} - \rho_Q)/\rho_c \tag{1}$$

today. The "fine-tuning" problem in quintessence models comes from the tendency for Ω_Q to evolve away from Ω_{other}. Equation (1) is achieved in these models either (i) by fine-tuning initial conditions or (ii) by introducing a small scale into the fundamental Lagrangian which causes ρ_Q to only start the acceleration today. This second category includes cosmological constant models and also a very interesting category of "tracker" quintessence models [10,11,18] which achieve the right behavior *independently* of the initial conditions for the quintessence field. One then has to explain the small scale in the Lagrangian, and this may indeed be possible [23].

Here we discuss a class of quintessence models which behave differently. Like the tracker models, these models predict acceleration independently of the initial conditions for the quintessence field. These models also have ρ_Q(today) fixed by parameters in the fundamental Lagrangian. The difference is that all the parameters in our quintessence potential are $O(1)$ in Planck units. As with all known quintessence models, our model does not solve the cosmological constant problem: We do not have an explanation for the location of the zero point of our potential. This fact limits the extent to which any quintessence model can claim to "naturally" explain an accelerating universe. Recently Steinhardt [24] has suggested that M-theory arguments specify the zero point of potentials in $3 + 1$ dimensions. Our zero point coincides with the case favored by Steinhardt's argument.

We start by considering a homogeneous quintessence field ϕ moving in a potential of the form

$$V(\phi) = e^{-\lambda\phi}. \tag{2}$$

We work in units where $M_P \equiv (8\pi G)^{(-1/2)} = \hbar = c = 1$. The role of spatial variations in such a field has been studied in [5,8,12,21,31]. Inhomogeneities can be neglected for our purpose, which is to study the large-scale evolution of the Universe. We assume inflation or some other mechanism has produced what is effectively a flat Friedmann-Robertson-Walker universe and work entirely within that framework. Cosmological fields with this type of exponential potential have been studied for some time and are well understood [25–30]. (A nice review can be found in [31].)

Let us review some key features: A quintessence field with this potential will approach a "scaling" solution, independent of initial conditions. During scaling Ω_Q takes on a fixed value which depends only on λ (and changes during the radiation-matter transition). In general, if the density of the dominant matter component scales as $\rho \propto a^{-n}$, then after an initial transient the quintessence field obeys $\Omega_\phi = \frac{n}{\lambda^2}$, effectively "locking on" to the dominant matter component. Figure 1 (upper panel) shows $\Omega_Q(a)$ for scaling solutions in exponential potential models, where a is the scale factor of the expanding universe [a(today) $= 1$].

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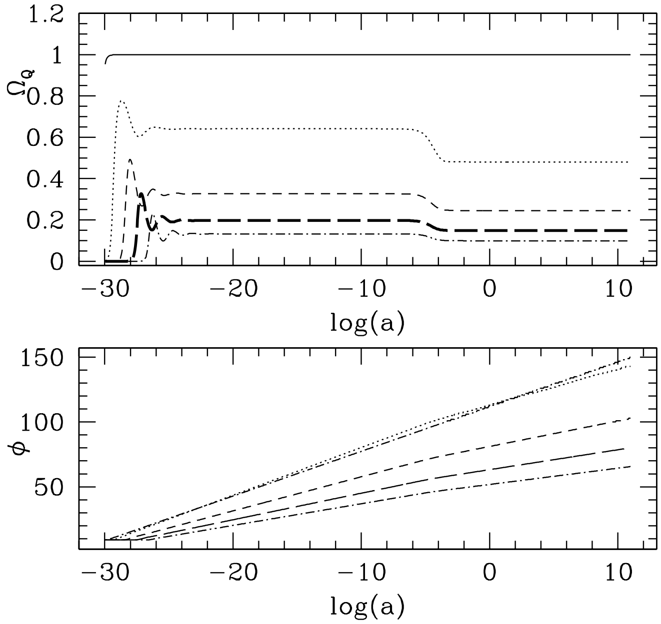

FIG. 1. The upper panel shows $\Omega_Q(a)$ for different constant values of λ. Each solution shows scaling behavior after the initial transient. The radiation-matter transition is evident at $a \approx 10^{-5}$. The heavy curve saturates a generous interpretation of the nucleosynthesis bound but still does not generate acceleration today. The lower panel shows the evolution of $\phi(a)$ for the same solutions. Note how ϕ varies very little while a and ρ_Q vary by many orders of magnitude. Today $a = 1$, $a \approx 10^{-10}$ at nucleosynthesis, and $a \approx 10^{-30}$ at the Planck epoch. The curves correspond to $\lambda = 1.5, 2.5, 3.5, 4.5$, and 5.5 (going from top to bottom in the upper panel).

At the Planck epoch $a \approx 10^{-30}$. In Fig. 1 (upper panel) one can see the initial transients which all approach the unique scaling solution determined only by λ. In [32] it is shown that exponential potentials are the *only* potentials that give this scaling behavior.

Scaling models are special because the condition $\Omega_Q \approx \Omega_{\text{other}}$ is achieved naturally through the scaling behavior. The problem with these exponential models is that no choice of λ can give a model that accelerates today *and* is consistent with other data. The tightest constraint comes from requiring that Ω_Q not be too large during nucleosynthesis [33] (at $a \approx 10^{-10}$). The heavy curve in Fig. 1 (upper panel) just saturates a generous $\Omega_Q < 0.2$ bound at nucleosynthesis and produces a subdominant Ω_Q today. The combined effects of subdominance and scaling cause $w_Q = 0$ in the matter era, so this solution is irrelevant to a universe which is accelerating today.

The lower panel of Fig. 1 shows how the value of ϕ changes by only about an order of magnitude over the entire history of the Universe, while the scale factor (and ρ_Q) change by many orders of magnitude. This effect, which is due to the exponential form of the potential, plays a key role in what follows. The point is that modest variations to the simple exponential form can produce much more interesting solutions. Because ϕ takes on values throughout history that are $O(1)$ in Planck units, the parameters in the modified $V(\phi)$ can also be $O(1)$ and produce solutions relevant to current observations.

Many theorists believe that fields with potentials of the form

$$V(\phi) = V_p(\phi) e^{-\lambda\phi} \tag{3}$$

are predicted in the low energy limit of M theory, where $V_p(\phi)$ is a polynomial. As a simple example we consider

$$V_p(\phi) = (\phi - B)^\alpha + A\,. \tag{4}$$

For a variety of values for α, A and B solutions like the one shown in Fig. 2 can be produced. In this solution Ω_Q is well below the nucleosynthesis bound, and the Universe is accelerating today. We show $\Omega_Q(a)$ (dashed line), $\Omega_{\text{matter}}(a)$ (solid line), and $\Omega_{\text{radiation}}(a)$ (dotted line). The lower panel in Fig. 2 plots $w_Q(a)$ and shows that the necessary negative values are achieved at the present epoch.

Figure 3 illustrates how the solutions depend on the parameters in $V_p(\phi)$. We plot quintessence energy density ρ_Q as a function of the scale factor a. After showing some initial transient behavior each solution scales with the other matter for an extended period before ρ_Q comes to dominate. The radiation-matter transition, which occurs at around $a = 10^{-5}$, can be seen in Fig. 3 as a change in the slope in the scaling domain. The constant parameter

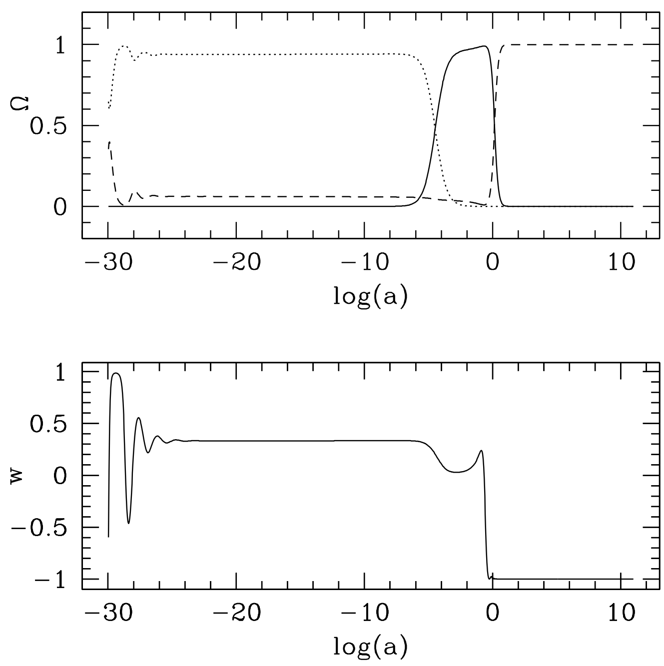

FIG. 2. Upper panel: A solution obtained by including a V_p factor in the potential. We show $\Omega_Q(a)$ (dashed line), $\Omega_{\text{matter}}(a)$ (solid line) and $\Omega_{\text{radiation}}(a)$ (dotted line). The lower panel shows $w_Q(a)$. The solution shows the normal radiation and matter dominated epochs and then an accelerating quintessence dominated epoch at the end. We have used V_p given by Eq. (4) with $B = 34.8$, $\alpha = 2$, $A = 0.01$, and $\lambda = 8$. Today $a = 1$, $a \approx 10^{-10}$ at nucleosynthesis, and $a \approx 10^{-30}$ at the Planck epoch.

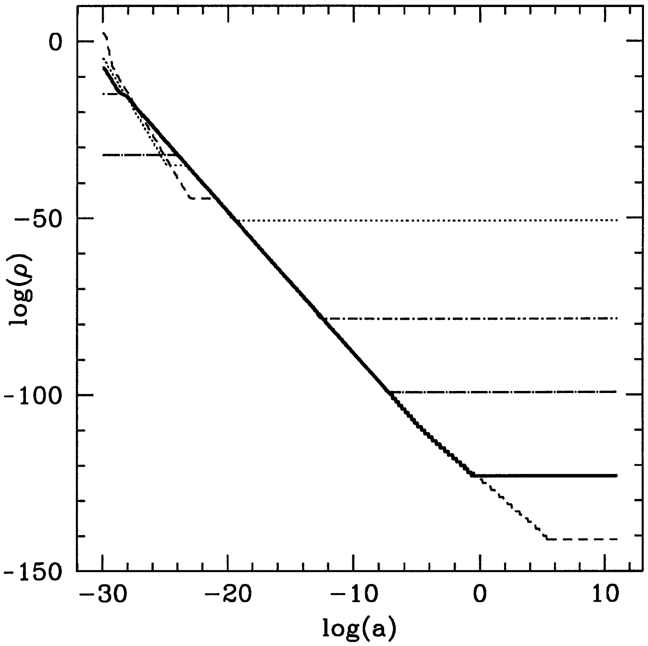

FIG. 3. Energy density (ρ_Q) vs a. The heavy line shows the solution from Fig. 2. This set of solutions shows that the point where acceleration (ρ_Q = const) starts is controlled by Planck-scale parameters in the Lagrangian. Moving from top to bottom on the right side the values of B [in Eq. (4)] are 14, 22, 28, 34.8, and 40. We also varied the initial value of ϕ between 0.3 and 10 to illustrate a range of initial transients.

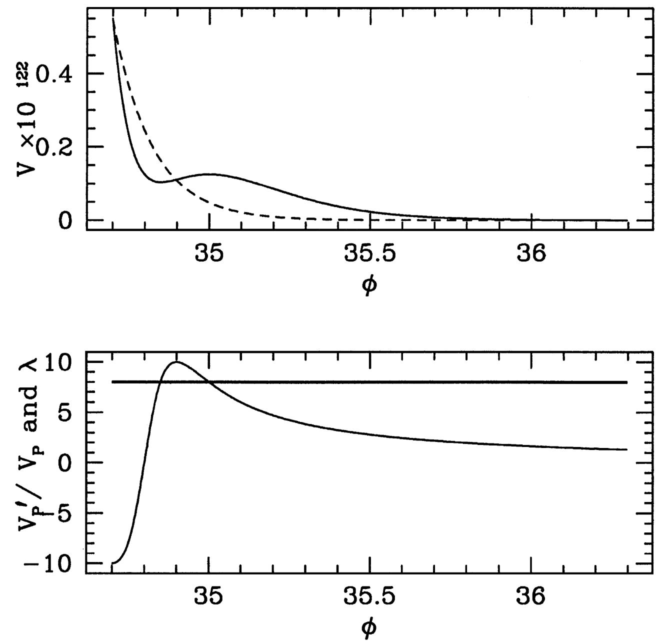

FIG. 4. The upper panel shows a closeup of the interesting region of $V(\phi)$ for the solution shown in Fig. 2. The dashed curve shows a pure $V(\phi) = \exp(-\lambda\phi)$ potential with $\lambda = 8.113$ for comparison. The onset of acceleration is caused by ϕ settling into the local minimum. The fact that the feature is introduced on such a small scale is due to the exponential factor in the potential. The lower panel shows V_p'/V_p and λ (the constant curve). Where the two curves cross $V' = 0$.

B in Eq. (4) has been selected from the range 14–40 for these models, yet the point of ϕ domination shifts clear across the entire history of the Universe. In this picture, the fact that Ω_Q is just approaching unity *today* rather than 10^{10} years ago is put in by hand, as is the case with other models of cosmic acceleration. Our models are special because this can be accomplished while keeping the parameters in the potential $O(1)$ in Planck units. Although we illustrate only the B dependence here, we have found that similar behavior holds when other parameters in $V_p(\phi)$ are varied.

Let us examine more closely what is going on: The derivative of $V(\phi)$ is given by

$$\frac{dV}{d\phi} = \left(\frac{V_p'}{V_p} - \lambda\right)V\,. \tag{5}$$

In regions where V_p is dominated by a single power-law ϕ^n behavior $V_p'/V_p \approx n/\phi$ which, unless n is large, rapidly becomes $\ll\lambda$ for values of λ large enough to evade the nucleosynthesis bound, leading to little difference from a simple exponential potential. However, there will be points where V_p can show other behavior which *can* impact V'. Using Eq. (4) gives

$$\frac{V_p'}{V_p} = \frac{\alpha(\phi - B)^{\alpha-1}}{(\phi - B)^{\alpha} + A}\,. \tag{6}$$

This varies rapidly near $\phi = B$ and for $\alpha = 2$ peaks at a value $V_p = 1/\sqrt{A}$. The upper panel in Fig. 4 shows the behavior of V near $\phi = B = 34.8$ for the solution shown in Fig. 2. The dashed curve shows a pure exponential for comparison. The lower panel shows the curves V_p'/V_p and λ (the constant curve). Where these two curves cross $V' = 0$. Because the peak value $1/\sqrt{A} > \lambda$, two zeros are produced in V' creating the bump shown in the figure. In our solution ρ_Q is coming to dominate near $\phi = B$ because the field is getting trapped in the local minimum. The behavior of the scaling solution ensures that ϕ gets stuck in the minimum rather than rolling on through (regardless of the initial conditions).

At one stage in this work we focused on potentials of the form $V(\phi) = \exp(-\lambda_{\text{eff}}\phi)$ with the idea that λ_{eff} might not be absolutely constant, but could be slowly varying with ϕ. We considered forms such as $\lambda_{\text{eff}} = \lambda[1 - (\phi/B)^{\alpha}]$ and found many interesting solutions, especially for moderately large values of B which make λ_{eff} slowly varying. For example, $\lambda = 13$, $B = 65$, and $\alpha = 1.5$ give a solution similar to Fig. 2. If this form for λ_{eff} were taken seriously for large ϕ, then these models have an absolute minimum in V which ϕ settles into (or at least approached) at the start of acceleration. But our expression may represent just an approximation to λ_{eff} over the relevant (finite) range of ϕ values. Of course we always can rewrite Eq. (3) in terms of λ_{eff} with $\lambda_{\text{eff}} = \lambda - \ln(V_p)/\phi$. In the end we focused on potentials in the form of Eq. (3) because they seem more likely to connect with ideas from M theory. Whatever form one considers for V, the concept remains the same. Simple corrections to pure $V = \exp(-\lambda\phi)$ can produce interesting solutions with all parameters $O(1)$ in Planck units.

We should acknowledge that we use $O(1)$ rather loosely here. In the face of the sort of numbers required by other quintessence models or for, say, a straight cosmological constant ($\rho_\Lambda \approx 10^{-120}$), numbers like 0.01 and 34.8 are $O(1)$. Also, the whole quintessence idea has several important open questions. Some authors argue [34] that values of $\phi > 1$ should not be considered without a full quantum gravitational treatment, although currently most cosmologists do not worry as long as the *densities* are $\ll 1$ (a condition our models easily meet). Another issue that has been emphasized by Carroll [35] is that, even with the (standard) assumption that ϕ is coupled to other matter only via gravity, there still will be other observable consequences that will constrain quintessence models and require small couplings. Because in our models $\dot{\phi} \approx 0$ today the tightest constraints in [35] are evaded, but there would still be effective dimensionless parameters $\approx 10^{-4}$ required.

Looking toward the bigger picture, a general polynomial V_p will produce other features of the sort we have noted. Some bumps in the potential can be "rolled" over classically but may produce features in the perturbation spectrum or other observable effects. We are investigating a variety of cosmological scenarios with a more general version of V_p. We are also looking at the effect of quantum decay processes which are relevant to local minima of the sort we consider here. We expect a range of possibilities depending on the nature of V_p.

In conclusion, we have exhibited a class of quintessence models which show realistic accelerating solutions. These solutions are produced with parameters in the quintessence potential which are $O(1)$ in Planck units. Without a fundamental motivation for such a potential, all arguments about "naturalness" and "fine-tuning" are not very productive. We feel, however, that this work represents interesting progress at a phenomenological level and might point out promising directions in which to search for a more fundamental picture.

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