Density Dependence of the Transition Temperature in a Homogeneous Bose-Einstein Condensate

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Transition temperature data obtained as a function of particle density in the ^{4}He-Vycor system are compared with recent theoretical calculations for 3D Bose-condensed systems. In the low density dilute Bose gas regime we find, in agreement with theory, a positive shift in the transition temperature of the form $\Delta T/T_0 = \gamma(na^3)^{1/3}$. At higher densities a maximum is found in the ratio of T_c/T_0 for a value of the interaction parameter, na^3, that is in agreement with path-integral Monte Carlo calculations.

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The role of interparticle interactions in the determination of the properties of low density Bose-Einstein condensed (BEC) systems has been a topic of interest for many years. In spite of a long history of theoretical investigation [1] dating back to the 1950s, elementary questions, such as the possible shift in the transition temperature, T_c, with density and interaction strength, have remained unsettled until the recent past. In the case of the repulsive interactions in the dilute Bose gas, there has now emerged a consensus [2–6] that T_c will be an increasing function of the interaction parameter, na^3, where a is the hard sphere diameter and n the particle density. This may seem a surprising result, since it is well known that in the case of liquid ^{4}He the superfluid transition occurs at a temperature well below the transition temperature for an ideal Bose gas, T_0, with the same particle mass and number density. Moreover, a number of the earlier calculations [7] had found that the transition temperature would be reduced as a consequence of interparticle interaction.

Motivated by the recent theoretical work in this area, we have examined the dependence of the transition temperature on the superfluid particle number for the ^{4}He-Vycor system. In our early work with this system we demonstrated, for the first time in 1983 [8], an experimental realization of the weakly interacting or "dilute" Bose gas. For a further account of this work see Ref. [9]. The lowest density achieved in these experiments was on the order of 2×10^{18} per cm^3. This is sufficiently low to provide a region of overlap, in terms of the interaction parameter, with the values of na^3 currently accessible to the BEC systems realized with ^{87}Rb [10] or ^{23}Na [11] atoms confined within magnetic or optical traps. In the case of Bose-condensed atomic hydrogen [12] the small s-wave scattering length and limits on the particle density set by recombination restrict this system to values of na^3 several orders of magnitude below the values that can be achieved with Bose-condensed Na, Rb, or He.

For questions such as the effect on T_c of increasing the interaction parameter, the ^{4}He-Vycor system offers advantages over the BEC systems of trapped atomic gases because in the Vycor case the interaction parameter can be varied continuously from the low density, weakly interacting limit to the strongly interacting regime, which is currently inaccessible with the alkali vapor systems. Further, working with the ^{4}He-Vycor system allows much larger sample sizes, on the order of a centimeter cubed, with an essentially unlimited time for observation of the BEC superfluid state.

In this Letter we give only a brief summary of our experimental methods. The cryogenic techniques and thermometry methods are those standard in the study of superfluid ^{3}He, and the reader is referred elsewhere [13] for details. For the present experiments, we have used a torsional oscillator technique to obtain a signal proportional to the superfluid particle density. The period of the oscillator is determined by the torsion constant and the total moment of inertia, which includes the Vycor and that fraction of helium atoms locked to the surface of the Vycor. Above the superfluid transition temperature the entire helium mass is locked to the substrate, and the oscillator period is essentially constant independent of temperature. However, when the temperature is lowered below the transition, the particles constituting the superfluid become increasingly uncoupled from the substrate and a shift in the oscillator period proportional to the superfluid mass is observed. The number density is estimated from an extrapolation of the superfluid signal to zero temperature and the calibrated mass sensitivity of our torsional oscillator. A more detailed discussion of these procedures can be found in earlier papers [8,9].

The interior channels of the porous Vycor glass used for these measurements range in diameter from 4 to 8 nm and form a highly interconnected 3D network. The superfluid helium atoms are constrained by van der Waals forces to move over the complex 3D-connected surface provided by the pores. It is important to appreciate that at low temperatures the thermal wavelength of the mobile superfluid particles can be larger than the pore size and that the Feynman exchange cycles characterizing the BEC or superfluid state link many pores at low particle density [14]. Therefore we model the superfluid phase as a homogeneous Bose gas constrained within the volume of the Vycor sample. We anticipate, however, that the influence of the substrate and pore geometry may be reflected in a small modification of the effective mass of the mobile superfluid atoms. We then expect to observe, in the low density limit, thermodynamic properties similar to those of the "free" or ideal Bose gas with an effective mass, m^*.

In the discussion of our experimental results, we shall first consider the data for the dilute Bose gas regime. The quantities required for a comparison with theory are the particle number density and the corresponding transition temperature. Following the recent calculations [2–6], we expect that for low densities T_c will be given by

$$T_c = T_0[1 + \gamma(na^3)^{1/3}], \tag{1}$$

where $T_0 = [2\pi\hbar^2/m^* k_B \zeta(3/2)^{2/3}]n^{2/3}$ is the transition temperature for an ideal Bose gas with particle mass m^* and density n. The coefficient γ is positive as required for T_c to rise above T_0 as the interaction parameter, na^3, increases. Although there is agreement on the form of Eq. (1), the theoretical estimates for γ range over more than an order of magnitude, from 0.34 (Grüter, Ceperley, and Laloë (GCL) [3]), 0.7 (Holzmann *et al.* [4]), 2.3 $\pm$ 0.25 (Holzmann and Krauth [6]), 2.9 (Baym *et al.* [5]), to a value of 4.66 (Stoof [2]).

For a convenient comparison to experimental results we cast Eq. (1) in a linear form with the variable, $n^{1/3}a$, as

$$\frac{T_c}{n^{2/3}} = \frac{2\pi\hbar^2}{m^* k_B \zeta(3/2)^{2/3}}[1 + \gamma(n^{1/3}a)]. \tag{2}$$

In Fig. 1 we plot $T_c/n^{2/3}$ against the parameter $n^{1/3}a$, taking a value of 0.22 nm [15] for the helium hard core diameter. As expected from theory, a linear fit gives a good representation for our data in the low density regime. The zero density intercept of the fit serves to determine an effective mass ratio of $m^*/m = 1.34 \pm 0.2$, where m is the ^{4}He mass, and the slope yields a value of 5.1 $\pm$ 0.9 for γ, which within error is in agreement with the value given by Stoof [2].

It is of interest to compare the range of the interaction parameter explored in the various BEC experiments. The highest particle densities achieved to date are 4×10^{14} atoms/cm^3 in the case of ^{87}Rb and 3×10^{15} cm^3 for ^{23}Na; for both the alkali atoms, however, the s-wave scattering lengths are much larger than for the respective value for the ^{4}He atom. The larger scattering lengths for the alkalis more than compensate for the lower particle densities and result in a range of overlap in terms of the interaction parameter between all three systems. This is illustrated in Fig. 1, where we indicate the estimated range of $n^{1/3}a$ for the ^{1}H, ^{23}Na, and ^{87}Rb experiments. The scattering lengths used in these estimates are as follows: 0.0648 nm for ^{1}H [16], 2.75 nm for ^{23}Na [17], and 5.77 nm for ^{87}Rb [18]. Since the ^{4}He-Vycor system clearly exhibits superfluid properties, one may also expect superfluidity to exist in the Bose-condensed alkali systems at comparable values of the interaction parameter and sufficiently low temperature.

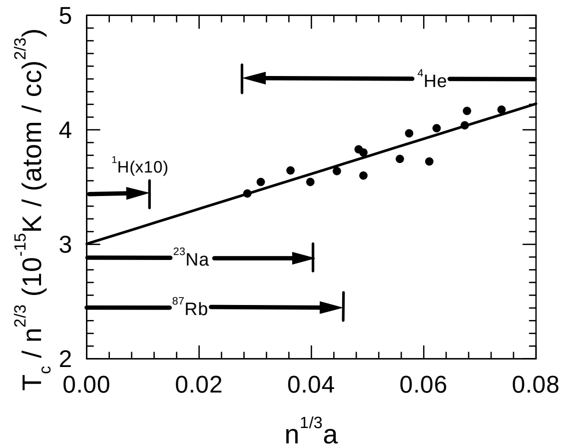

FIG. 1. The quantity $T_c/n^{2/3}$ is plotted as a function of the parameter $n^{1/3}a$. The straight line is our fit to these data. The arrows indicate the range of interaction parameter covered in the ^{1}H, ^{4}He, ^{87}Rb, and ^{23}Na experiments. Note that in the case of ^{1}H the scale has been enlarged by a factor of 10.

In Fig. 2 we show a more conventional plot of the low density data. Here we have plotted T_c as a function of the particle density, n. The curve through the data is the theoretical expression given in Eq. (1) with our best fit values for γ and m^* as determined previously. Two curves based on the γ estimates of Stoof and Baym *et al.* are also shown. The lowest curve is for the noninteracting Bose gas [i.e., $\gamma = 0$ in Eq. (1)]. The relatively close agreement with the estimate of Stoof is clear in this plot.

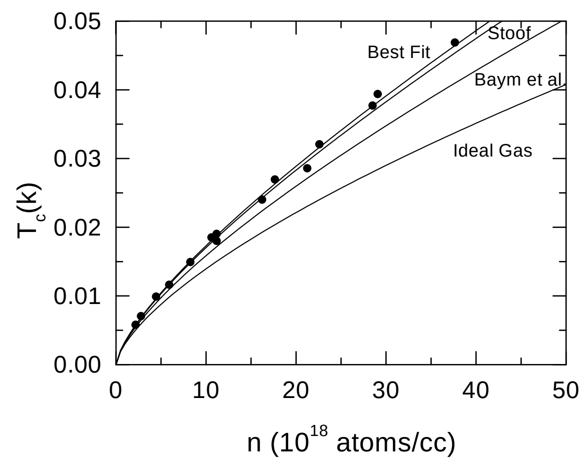

FIG. 2. The observed transition temperature T_c is plotted as a function of the particle density n. The top curve through the data is the function given by Eq. (1) with the parameters for the effective mass m^* and γ determined as from the linear fit shown in Fig. 1. The two other curves are for $\gamma = 4.66$ as given by Stoof and $\gamma = 0$, the noninteracting case.

In contrast to the other theoretical treatments of the BEC problem for the interacting Bose gas [2,4–6], the calculations of GCL are not restricted to the low density limit. The path-integral Monte Carlo technique employed by GCL allows a prediction for the ratio of T_c/T_0 over the entire range of na^3, from the dilute gas limit to densities approaching the freezing point of liquid helium. In the top panel of Fig. 3 we show the results of the GCL calculation taken from Ref. [3]. At low values of the interaction parameter, GCL also find that the ratio T_c/T_0 increases in proportion to $n^{1/3}a$, which is in qualitative agreement with the earlier calculations of Stoof [2]. An interesting feature of the GCL calculation is the maximum in T_c/T_0 found near a value of the interaction parameter $na^3 = 0.01$.

In the lower panel of Fig. 3, we show the data obtained in a number of different ^{4}He-Vycor experiments at Cornell, including a recent measurement designed specifically to map out the region of the peak found by GCL. Although the higher density data show more scatter than the data of Figs. 1 and 2, the trends are clear, and there is a pleasing agreement in the qualitative form of the experimental data as compared to the GCL calculation. In particular, there is an excellent match in the position of the peak value for the ratio T_c/T_0 as a function of the interaction parameter. A remaining problem, however, is the as yet unexplained disagreement in the magnitude of the effect estimated by GCL for low densities as compared to our experimental data or the calculations of Stoof [2] and Baym *et al.* [5].

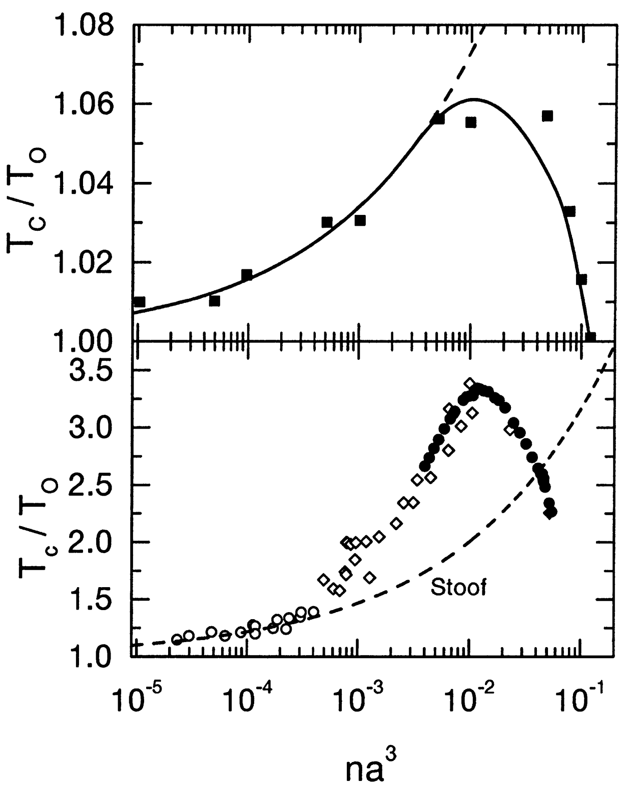

FIG. 3. The top panel shows the values of the ratio T_c/T_0 calculated by GCL as a function of the interaction parameter na^3. The curve through the lower density points is given by $T_c/T_0 = 1 + 0.34(na^3)^{1/3}$. The curve through the higher density data including the region of the peak is merely a guide for the eye. In the lower panel we plot the values of T_c/T_0, obtained from a number of different Vycor experiments (distinguished by different symbols), as a function of na^3. The dashed curve is the theoretical estimate of Stoof [2].

In conclusion, we emphasize that studies with low density Bose-condensed helium systems can provide a useful and illuminating contrast to the trapped gas systems for the study of properties of the weakly interacting BEC fluids. Particularly important aspects of the ^{4}He-Vycor system are the ability to study long term phenomena, such as the question of robustness or stability of flow states in a Bose-condensed superfluid, and to extend observations of BEC to the realm of larger interaction parameters than are accessible to the trapped gas systems.

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