

Vortices Freeze like Window Glass: The Vortex Molasses Scenario

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We overview several recent experimental and numerical observations, which are at odds with the vortex glass theory of the freezing of disordered vortex matter. To reinvestigate the issue, we performed numerical simulations of the overdamped London-Langevin model, and use finite size scaling to analyze the data. Upon approaching the transition the initial vortex-glass-type criticality is arrested at some crossover temperature. Below this temperature the time scales continue growing very quickly, consistent with the Vogel-Fulcher form, while the spatial correlation length ξ stops exhibiting any observable divergence. We call this mode of freezing the vortex molasses scenario.

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The influence of disorder on vortex matter is one of the most paradigmatic problems. The vortex lattice, formed in clean systems, is inherently unstable towards a less ordered state even for infinitesimally small disorder [1]. At small magnetic fields, or equivalently, at weak disorder, a dislocation free phase emerges, which thus retains a topological order. Small angle neutron scattering [2], and Bitter decoration experiments [3] seem to support this picture. The theoretical foundation for such a Bragg glass was provided by scaling arguments [4], and variational calculations [5]. Its cornerstone is the logarithmic behavior of vortex correlations at large distances. Numerical simulations also reported a strongly suppressed dislocation density [6], and confirmed the logarithmic behavior of correlations [7] below a critical field strength. It is noteworthy, however, that the largest scale imaging studies [8] did not find evidence for logarithmic correlations; thus the details of the dislocation free regime are still subject to discussion.

We also understand the influence of increasing fields, or disorder. The key phenomenon here is the appearance of dislocation loops, accompanied by the entanglement of vortices [9,10]. Experimental support for this idea is the sharp enhancement of the critical current from magnetization measurements [11–14], the rapid destruction of the Bragg peaks in neutron scattering [2], and the pronounced dips in the electric field–current density, or E - J curves [15]. Numerical studies found evidence for dislocation loops destroying a quasiordered state in frustrated XY models [16], in Lawrence-Doniach representations [6], and in realistic London-Langevin approaches [7].

The nature of the high field phase is still very much in debate. The thermally assisted flux flow (TAFF) picture predicts that vortices move in bundles, and overcome barriers via thermal excitations. This destroys superconductivity because the linear resistivity assumes a finite value, governed by an activated temperature dependence $R(T) \sim R_0 \exp(-U/T)$ [17]. An influential alternative was put forward in the form of the vortex glass (VG) theory

[18,19]. The proposed vortex glass phase is distinguished by an unbounded distribution of barrier heights. This results in the vanishing of the linear resistivity, thus restoring superconductivity, and inherently nonlinear E - J characteristics. Numerical support for this picture emerged from the study of the isotropic gauge glass model, ignoring the effects of screening [20]. Experimental confirmation soon followed, on heavily twinned YBCO films [21]. A key evidence was provided by observing the scaling of a crossover current J_x [22].

Recent experimental and numerical work, however, has raised new questions about the vortex glass picture.

(i) The values of the correlation length, creep, and dynamical exponents, ν , μ , and z , respectively, seem to depend on temperature, current, and sample quality [23] in a very nonuniversal way. ν was found between 1.3–2, z between 3.1–6.5, and μ between 0.2–0.5.

(ii) The above values of ν and z are much higher than their mean field values, indicating that the lower critical dimension might be close to 3. It is already accepted that there is no finite temperature VG phase in 2D [20].

(iii) Recent experiments in completely untwinned YBCO samples found that the E - J curves remained completely linear down to the lowest measurable values of the current. Correspondingly, no scaling behavior of the E - J curves was found [24,25]. This suggests that in previous works the twin boundaries might have played the role of extended defects, and in fact the observed scaling behavior was that of the Bose glass.

(iv) When the twin boundaries were removed in YBCO samples, the crucial crossover current J_x was found to saturate, instead of exhibiting a scaling behavior [26].

(v) Recent numerical papers reported that, when a finite London screening length λ was restored into the previously studied gauge glass models, the finite temperature vortex glass transition disappeared [27–29]. Now, close to the transition the vortex correlation length is supposed to diverge, thus exceeding λ . Therefore the ultimate transition region is always in this finite λ regime.

We conclude that there is an emerging body of evidence, which is inconsistent with the vortex glass picture. Motivated by this inconsistency, in this paper we explore analogies to another widely studied glass transition, and investigate the possibility that *vortices freeze like the window glass: the vortex molasses scenario*.

We realize that there is not a uniquely accepted theory of the window glass transition. Therefore we construct the vortex molasses (VM) scenario only from those elements, which *are* common among the different theories: (i) a very rapid freezing of the dynamics, with diverging time scales, characterized by the Vogel-Fulcher law: $\tau \sim \exp[1/(T - T_G)]$; (ii) the possible divergence of the spatial correlation length ξ is rendered unobservable by this rapid freezing.

We note that, among the early alternative propositions, some emphasize the entanglement of vortices in the presence of disorder, in analogy to polymer glasses [30]. Also, extensions of the TAFF theory were constructed [31]. Finally, a vortex slush picture has been proposed, viewing the glass of vortices as a viscous liquid, driven by the remnants of the first order melting transition [32].

We start by overviewing ways to distinguish between the VG and VM scenarios. First, the predicted temperature dependence of the resistivity differs: in the VG theory the resistivity vanishes as $\rho(T) = \rho_{\text{VG}}|T - T_G|^{\nu(z-1)}$, whereas in the VM scenario one expects the resistivity to follow the Vogel-Fulcher law $\rho(T) = \rho_{\text{VM}} \exp[-1/(T - T_G)]$ [33]. However, it is hard to achieve decisive distinction between these forms, as the resistivity exponent in the VG theory is large: $\nu(z - 1) \sim 5-7$ [22].

The second method is more promising. Equating the current related free energy of a correlated volume to the thermal energy yields the above-mentioned crossover current density scale $J_x = ck_B T / (\Phi_0 \xi^2)$ [19]. Above the transition temperature the low current linear E - J of the viscous liquid is expected to cross over at J_x to the Bardeen-Stephen form at high currents. In the VG theory the correlation length ξ diverges: $\xi \sim (T - T_G)^{-\nu}$. Correspondingly, the crossover current scale J_x collapses as $J_x(T) \sim (T - T_G)^{2\nu}$. In contrast, in the VM picture the criticality of the correlation length is *unobservable*, thus J_x *does not collapse*. As mentioned, while in twinned samples J_x collapsed, in untwinned YBCO J_x *saturated* at some finite value upon approaching the transition [26].

In this paper we report numerical simulations of a realistic, London-type model for driven vortices, governed by overdamped dynamics in order to distinguish between the above two scenarios. Previous studies on the gauge glass already indicated a breakdown of the VG picture [27–29]; however, that model is rather simplified. For instance, it is isotropic, whereas in real vortex matter the external field definitely introduces a strong anisotropy. Thus it remains an open question, whether the gauge glass adequately describes the vortex matter, making our realistic simulations necessary. The Langevin equation describing the overdamped motion of vortices reads

$$\eta \frac{\partial \mathbf{R}_\mu(z, t)}{\partial t} = \boldsymbol{\zeta}_\mu(z, t) + \mathbf{F}_L - \frac{\delta H[\{\mathbf{R}_\nu(z, t)\}]}{\delta \mathbf{R}_\mu(z, t)}, \quad (1)$$

where μ labels the vortices with coordinates \mathbf{R}_μ , $\boldsymbol{\zeta}(z, t)$ is the Langevin noise, and \mathbf{F}_L is the Lorentz force. The Hamiltonian H is constructed on the basis of the London theory. Its derivative decomposes into three forces: the pairwise interactions, the single-vortex bending force, and the pinning. For more details of the method, see Ref. [7]. The trustworthiness of our code was demonstrated by the quantitatively correct reproduction of the phase diagram of disordered YBCO [7]. In our model we do not take into account vortex loops whose effect on melting is still controversial [34].

There are several length scales in the model. To avoid observing some crossover instead of the asymptotic behavior, we chose the characteristic microscopic length scales small and close to each other. We use $\lambda/\xi = 4$, a large magnetic field of $H/H_{c2} = 0.2$, to make $a_0 \approx \lambda$, and finally we made the system isotropic by choosing $\epsilon = 1$. The 100–500 vortex elements produce good self-averaging, so a reasonable statistics was achieved by averaging over 10–20 disorder realizations. Simulated annealing was employed to generate the starting configurations.

In Fig. 1 we show the typical behavior of the differential resistivity $\rho(T, I) = dE/dJ$, normalized to ρ_{BS} , the Bardeen-Stephen value. At high currents $\rho(T, J) = \rho_{\text{BS}}$ as it should. After a pronounced drop with decreasing current, $\rho(T, J)$ flattens at low J , clearly indicating an Ohmic behavior: we are in the vortex liquid regime.

In our temperature sweeps $\rho(T)$ drops by 2 orders of magnitude upon approaching the freezing transition. The inset of Fig. 1 shows its temperature dependence. We also

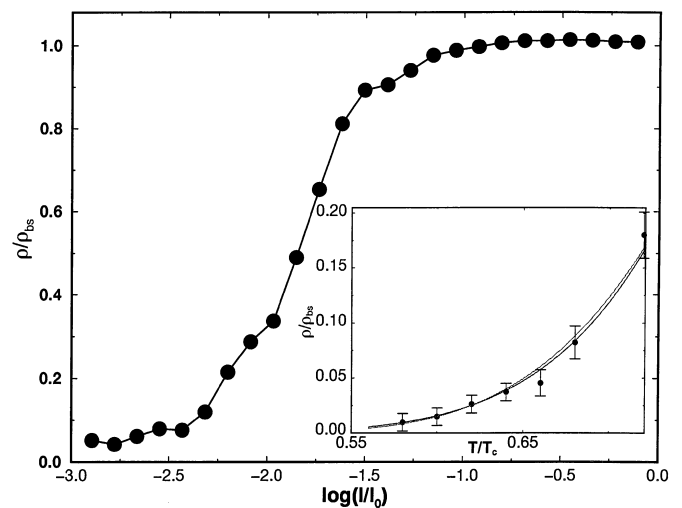


FIG. 1. The current dependence of the resistivity $\rho(T, I)/\rho_{\text{BS}}$ at $T/T_c = 0.74$. Inset: The temperature dependence of the low-current Ohmic resistance $\rho(T, I \rightarrow 0)/\rho_{\text{BS}}$, for $L = 8^3$ systems. Dark gray: fit to the Vogel-Fulcher form, $\rho(T) = \rho_0 \exp[-T_0/(T - T_G)]$; light grey: VG fit, $\rho(T) = \rho_0 [(T - T_G)/T_G]^{2\nu}$.

exhibit a Vogel-Fulcher fit, and a power law fit. As expected, both fits are comparable, and thus do not distinguish between the VM and VG theories.

However, useful information can be extracted from the $R(T)$ runs via finite size scaling. In Fig. 2 we show the results for 4^3 , 6^3 , and 8^3 systems. Upon cooling, the system exhibits increasing finite size sensitivity down to $T/T_c \sim 0.70$, which typically indicates the approaching of a phase transition. Remarkably, however, below this temperature range the finite size sensitivity decreases, as if the criticality is arrested.

To view this from a different perspective, we follow Young *et al.* [27] by studying an analog of the Binder ratio, $\log[\rho(L)/\rho(L')]/\log[L/L']$. Far from a transition, where finite size sensitivity is small, this should be close to 0. Approaching a transition, the increased finite size sensitivity is signalled by the data on different system sizes splaying out. Eventually, however, they come together and cross at $T = T_G$. In Fig. 3 we show this ratio for our model. As the temperature decreases, the initial splaying shows an impending transition. The curves, however, do not cross. Instead, they turn back up: the transition is arrested. This again signals the decrease of finite size sensitivity, which is most readily interpreted as the initial increase of the correlation length ξ being arrested around $T/T_c \sim 0.70$.

The best measure of ξ is via the crossover current J_x , plotted in Fig. 4. As $\xi \sim J_x^{-1/2}$, decreasing J_x indicates increasing correlation length. However, J_x , and thus ξ , saturates with decreasing T , around $T/T_c \sim 0.68$, in quantitative accord with the finite size scaling. Note that a very similar flattening of J_x was observed in untwinned YBCO [26]. All these three tests can be interpreted as follows. Upon decreasing the temperature a vortex glass criticality starts to develop. However, this critical behavior gets arrested around $T/T_c = 0.69 \pm 0.01$, and crosses over to a vortex molasses criticality. This is characterized by a

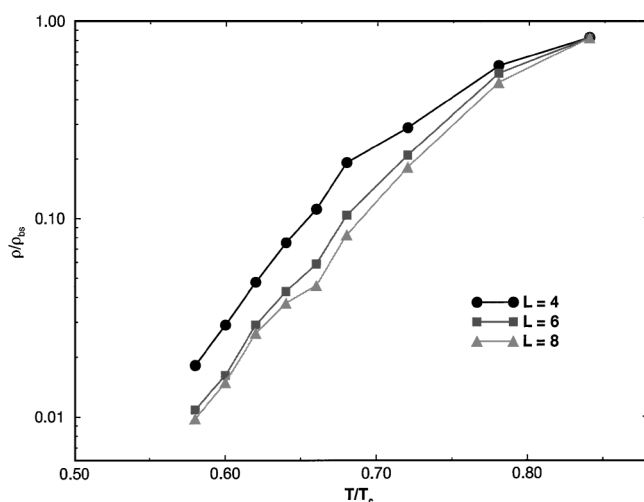


FIG. 2. The temperature dependence of $R(T)$ for systems with $L = 4, 6$, and 8 .

rapid, Vogel-Fulcher-type decrease of the resistivity, but at the same time an essentially noncritical behavior of the correlation length ξ .

The above results established that the freezing transition is unlikely to be governed by vortex glass theory, but rather it looks more like a window glass transition. However, there is not a single theory agreed upon by the window glass community. For a review of different approaches, see Ref. [33]. Some theories propose that the correlation length diverges as a power law, but the freezing of the dynamics is so rapid that it renders this divergence unobservable. Others believe that, in fact, ξ does not diverge at all; it remains noncritical even on the longest time scales. Finally, there are theories which envision that there is no true transition at any finite temperatures, but a rapid, continuous increase of the viscosity, diverging only at $T = 0$. This latter view was imported to the vortex problem by Ref. [27]. Setting up the scaling theory for finite size systems with a correlation length $\xi \sim T^{-\nu}$ gives for the nonlinear E - J relation: $E/(JR) = \tilde{E}(J/T^{1+2\nu}, L^{1/\nu}T)$, where E is a universal scaling function. Adopting the accepted definition of the crossover current density, $E/(J_x R) = 2$ yields $J_x = T^{1+2\nu} f(L^{1/\nu}T)$, where f is another universal function. Reference [27] finds J_x to be a universal function of $L^{1/\nu}T$, with $\nu = 1$. To test this proposition, we also plotted $J_x/T^{1+2\nu}$ as a function of $L^{1/\nu}T$. However, we found no universal dependence whatsoever. This clearly eliminates the possibility of a $T = 0$ fixed point governing the freezing behavior of the London model. Thus, remarkably, regarding the freezing transition the gauge glass and the realistic vortex simulations give qualitatively different results.

We now understand that structural (window) glasses and systems with quenched disorder often behave quite similarly [35]. Their glassy phase exhibits different aging phenomena [36]. Measuring the two time correlation functions [36] of vortices, and comparing to the predicted power law relaxation forms, would be a constructive test of the vortex molasses.

A word on the appropriateness of models with strong screening. Reference [27] recalls that in the analogous

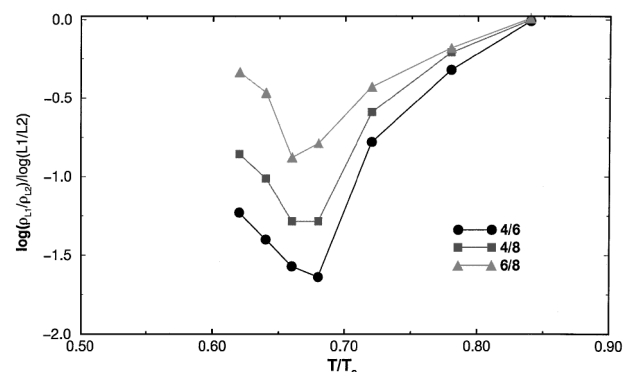


FIG. 3. The logarithmic resistance ratio as a function of the temperature.

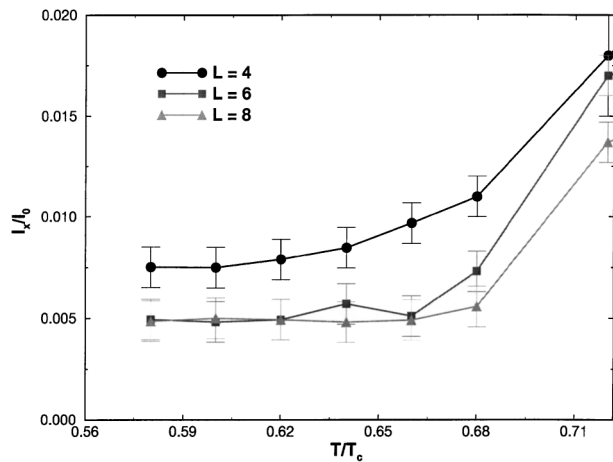


FIG. 4. The temperature dependence of the crossover current I_x , for $L = 4, 6,$ and 8 .

3D XY model *both* the screening length λ and correlation length ξ diverge, when T_c is approached from *below*. In the critical region the exponent of λ is half of ξ 's, and, hence, close enough to T_c the proper characterization of the system *should* involve strong screening. *Above* T_c the screening length on macroscopic scales is infinite, as we are in the vortex liquid. On the scale of intervortex separation, however, λ equals its bare value. In general, the presence of the other vortices generates a renormalized, scale dependent $\lambda(x)$. Whether the model is in the strong or weak screening limit will then be determined by $\lambda(\xi)$ being greater or smaller than ξ . By invoking that the critical behavior around T_c is typically symmetric, and that below T_c we *are* in the strong screening limit, we expect $\lambda(\xi) < \xi$, i.e., the screening remaining essential for understanding the physics of the model, the starting point of our simulations.

In conclusion, we collected several numerical and experimental results, which are at odds with the vortex glass theory of the freezing of the disordered vortex matter. Previous confirmations of the VG theory were reinterpreted in terms of twin boundaries and proper accounts of the screening. To reinvestigate the issue, we performed careful numerical simulations of the overdamped London-Langevin model, and used finite size scaling to analyze the data. We found that upon approaching the transition the initial vortex-glass-type criticality is arrested at some crossover temperature, where the vortex correlation length catches up with the screening length. Below this temperature the time scale continues growing very quickly, consistent with the Vogel-Fulcher form, while the spatial correlation length ξ stops exhibiting any observable divergence. We call this mode of freezing the vortex molasses scenario.

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