Nematic Phase of the Two-Dimensional Electron Gas in a Magnetic Field

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The two-dimensional electron gas (2DEG) in moderate magnetic fields in ultraclean AlAs-GaAs heterojunctions exhibits transport anomalies suggestive of a compressible anisotropic metallic state. Using scaling arguments and Monte Carlo simulations, we develop an order parameter theory of an electron nematic phase. The observed temperature dependence of the resistivity anisotropy behaves like the orientational order parameter if the transition to the nematic state occurs at a finite temperature $T_c \sim 65$ mK, and is slightly rounded by a small background microscopic anisotropy. We propose a light scattering experiment to measure the critical susceptibility.

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Recently [1] two of us introduced the concept of liquid crystal phases of the two-dimensional electron gas (2DEG) in large magnetic fields, as an extension of earlier work on high temperature superconductors [2,3]. Electronic liquid crystal phases are quantum mechanical analogs of classical liquid crystals, and are predicted [2,3] to be a generic feature of strongly correlated fermionic systems. In the case of the 2DEG, the competing effects of repulsive (Coulomb) interactions and the quenching of the kinetic energy of electrons in Landau levels lead naturally to the existence of such phases. Pursuing this analogy, the phases of a 2DEG in order of increasing symmetry breaking were characterized as (a) isotropic liquids, (b) nematic liquids, (c) smectic liquid crystals, and (d) insulating crystals. The fluid character of states (a) and (b) is obvious, as they are translationally invariant. The smectic breaks translational symmetry in only one direction, and so is also a fluid. Insulating crystals break the translational symmetry down to a discrete subgroup, such that there are an integer number of electrons per unit cell.

Among the isotropic liquids are the various quantum Hall fluids, while insulating crystals are simply the Wigner crystal and its generalizations. A smectic (or stripe) phase has been found in Hartree-Fock calculations, presumably accurate in high Landau levels [4–7], which provides a qualitative picture of a smectic state. Moreover, a recent exact diagonalization study of a system with 12 electrons in the $N \geq 2$ Landau levels found results consistent with a smectic (stripe) ground state [8] (up to yet poorly understood finite size effects). Wigner crystals and "bubble phases" (crystalline states with several particles per unit cell) have also been found as Hartree-Fock variational states. In addition, a predicted charge-density-wave instability of the smectic phase at Hartree-Fock level [1,3,9,10] leads to an insulating "stripe-crystal" phase with a parallelogram-shaped unit cell. However, a microscopic theory of the nematic phase does not presently exist.

The low energy physics of a quantum smectic can be understood in terms of a theory of quantum fluctuations of the smectic [1,3,9,10], including the phase transition to the stripe-crystal phase. The long-distance behavior of a quantum nematic phase is completely determined by symmetry and by the associated Goldstone modes. Both smectic and nematic states break rotational symmetry, and as such transport properties of both states are anisotropic. In Ref. [1] we argued that, while the smectic and nematic states are both natural candidates to explain the anisotropy observed in recent experiments [11,12] on 2DEG in high mobility AlAs-GaAs heterostructures, at least at finite temperature, there are strong reasons to favor the nematic. The smectic, if pinned at the boundaries, has an infinite conductivity, at least in one direction, whereas the measured (anisotropic) conductivity has a finite $T \rightarrow 0$ limit. In addition, the data shows a pronounced temperature dependence of the resistivity, consistent with the existence of a finite temperature phase transition; since the energy of a dislocation is still finite even for a Coulomb interaction the smectic always melts at *any* nonzero temperature [13]. In the present paper we explore the universal properties of the 2D electron nematic.

The experiments of Refs. [11] and [12] have revealed the existence of regimes of magnetic fields in which the 2DEG exhibits characteristics of a compressible fluid with an unexpectedly large and temperature-dependent anisotropy in its transport properties. This occurs when the Landau level index N lies in the range $2 \leq N \leq 6$. The large anisotropy is seen only in high mobility samples. In the same samples, similar behavior has also been seen in the first $N = 1$ Landau level when the magnetic field is tilted [14]. A large number of fractional quantum Hall (FQH) states are observed, and the subtle FQH state at $\nu = 5/2$ is sharp. The experimental facts are as follows: (i) Measurements on square samples show that, as the temperature is lowered below 100 mK, the longitudinal *resistance* R_{xx} grows very rapidly while R_{yy} becomes smaller; as $T \rightarrow 0$, their ratio approaches a constant [15] in the range $100 < R_{xx}/R_{yy} < 3500$, where x and y are orthogonal lattice directions. (ii) In Hall bars [11,16], R_{yy}

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is essentially temperature independent while R_{xx} increases by a factor of 5–10 as the temperature is lowered below 100 mK. (iii) The compressible (dissipative) regime occupies a finite range of magnetic fields ΔB, centered around the middle of the partially filled Landau level, and, unlike the conventional transition between plateaus, ΔB does not shrink as the temperature is lowered. (iv) In the same range of magnetic fields the Hall resistance varies continuously with the magnetic field. (v) At the peak, the *conductivity* σ_{yy} is typically of the order of e^2/h (see below and Refs. [1] and [17]). (vi) An in-plane magnetic field tunes the anisotropy through zero, and reverses the roles of the x and y directions. (vii) In the region of the resistivity peak, the I-V curves are highly nonlinear but show no threshold (depinning) behavior [11,16]. (ix) There are reentrant integer quantum Hall plateaus symmetrically located for magnetic fields *outside* the compressible regime. (x) The anisotropy has not been reported in lower mobility samples which show instead the (expected) phase transition between quantum Hall states.

A natural interpretation of the experiments is that, in regimes in which interactions dominate over the effects of disorder, instead of the expected transition between plateaus in the middle of the partially filled Landau level, the 2DEG forms a compressible anisotropic fluid. Because a continuous (rotational) symmetry cannot be spontaneously broken in $D = 2$, for such an anisotropy to be observable [1] the sample must have a small background microscopic anisotropy whose effect is greatly amplified at low temperatures by the collective properties of the state. Specifically, we will show that the experimentally observed temperature dependence of the anisotropy can be understood as evidence for a finite temperature Kosterlitz-Thouless (KT) transition from a two-dimensional nematic to an isotropic fluid [13,18], which is rounded by a symmetry breaking field representing the effects of the background anisotropy. We present an analysis (Fig. 1) of the experimental data of Lilly *et al.* [11], and a fit with the results of a Monte Carlo simulation of a model of a classical nematic in a symmetry breaking field (Fig. 2). The results strongly support our earlier claim [1] that the anisotropic transport occurs where the 2DEG is in a nematic phase (at least at finite temperature). They also give some indirect support to the further conjecture, made in Ref. [1], that there is a direct transition as a function of B from the nematic state to an insulating stripe-crystal phase, which was identified with the innermost of the reentrant quantum Hall liquids. (See also Ref. [19].)

In 2D, the nematic has only quasi-long-range order; its finite temperature transition to a disordered liquid can be described by the two-dimensional classical XY model with a director order parameter [13]. Such a description should fail at (very) low temperatures where quantum fluctuations (and/or quenched disorder) become important. Since the order parameter of the nematic state is a director field,

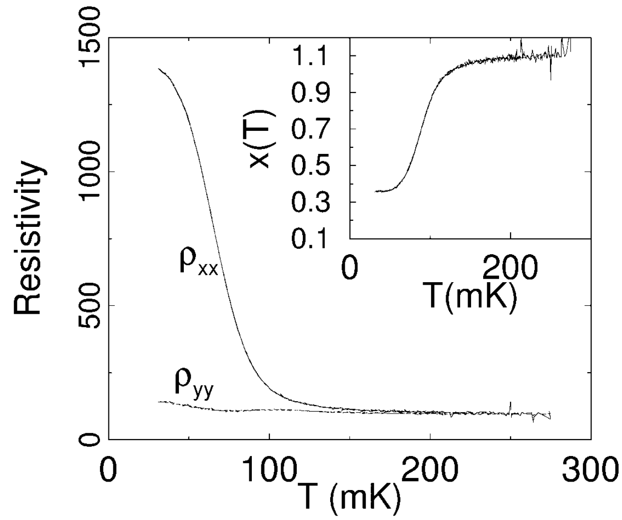

FIG. 1. Resistivities ρ_{xx} and ρ_{yy} determined from the resistance data of Lilly *et al.* [11,16] at $\nu = 9/2$, after deconvoluting the effects of the geometry; ρ_{yy} is essentially constant for the entire range of temperatures, as in Hall bars. Inset: The function $x(T)$.

$\vec{m}(\vec{r})$, it is periodic under rotations by π, and has the form $m_x(\vec{r}) + i m_y(\vec{r}) = \exp(2i\theta_{\vec{r}})$. The classical Hamiltonian of this system is thus

$$H = -J \sum_{\vec{r},\mu=x,y} \cos(2\Delta_\mu \theta_{\vec{r}}) + h \sum_{\vec{r}} \cos(2\theta_{\vec{r}}), \qquad (1)$$

where, for simplicity we have used a square lattice of unit spacing whose sites are labeled by the lattice vectors $\vec{r}$. In Eq. (1), we have used the notation $\Delta_\mu \theta_{\vec{r}} = \theta_{\vec{r}+\vec{e}_\mu} - \theta_{\vec{r}}$, where e_μ is a unit vector along the direction $\mu = x, y$, and J is the stiffness, the energy required to rotate two nearby regions by a small angle. The quantity h breaks rotational symmetry explicitly. It represents the effects of a background symmetry breaking field, such as the anisotropy and/or the effects of a parallel magnetic field.

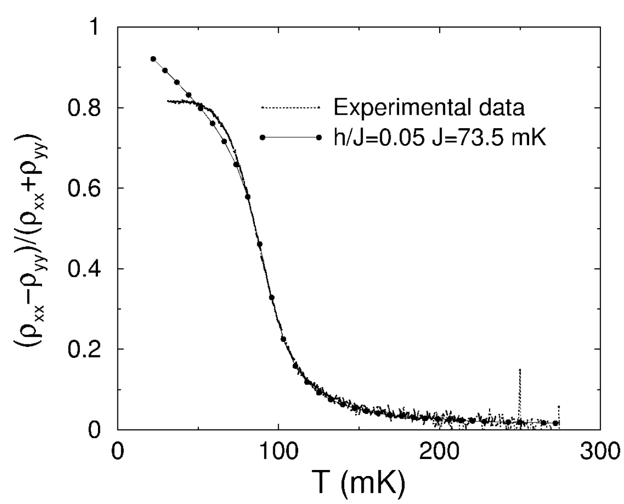

FIG. 2. Fit of the Monte Carlo data for a 100×100 lattice, to the data of Lilly *et al.* [11,16]. The best fit is found for $J = 73$ mK, $h = 0.05J = 3.5$ mK, and $T_c = 65$ mK.

Because two is the lower critical dimension for continuous symmetry breaking, for $0 < T < T_c$ the system is controlled by a line of critical points [18]. In this range of T, where classical Goldstone excitations of the nematic ("spin waves") dominate, the correlation function $G(\vec{r}) = \langle \exp 2i[\theta(\vec{r}) - \theta(\vec{0})]\rangle$ of the order parameter has power law behavior, $G(\vec{r}) \sim 1/|\vec{r}|^{\eta(T)}$, with $\eta = 2T/\pi\kappa(T)$ and a divergent susceptibility. Here $\eta \to 1/4$ as $T \to T_c$ and $\kappa(T)$ is the helicity modulus, which approaches $\kappa(0) = 4J$ as $T \to 0$ and $\kappa(T_c) = 4T_c/\pi$. In the presence of a symmetry breaking field h, the order parameter behaves like $m = \langle \exp(2i\theta)\rangle \sim |h|^{1/\delta}$, where $\delta = 4/\eta - 1$ and $\delta(T_c) = 15$. For $T > T_c$, the correlation length is finite, and diverges at T_c like $\xi \sim \exp(A/\sqrt{T - T_c})$, where A is a (nonuniversal) constant. At finite h the correlation length is always finite, and the singularities of the KT transition get rounded. In this regime, even a very small symmetry breaking field induces a very large expectation value of the order parameter. For $T > T_c$, $m \sim \chi(T)h$, where $\chi(T) \sim \xi^{7/4}$ is the susceptibility.

How is this thermodynamic transition, which describes the breaking of rotational invariance, related to the transport properties? On general grounds, one expects that, near a phase transition, quantities which transform in the same way under the symmetry should be related, even if one is a transport coefficient and the other a thermodynamic property. In particular [1], the combination of resistivities $\zeta = (\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy})$ transforms like the order parameter $m = \langle \exp(i2\theta)\rangle$. It should be related to the order parameter through an odd *analytic* function $\zeta = f(m)$. Therefore, near T_c, if the symmetry breaking is small, the linear approximation $f(m) \propto m + O(m^3)$ should be reasonably accurate [20].

We can determine if the 2DEG is in a nematic phase by analyzing the *temperature* dependence of the resistivity in terms of the temperature dependence of the order parameter of the nematic in the presence of a symmetry breaking field. What is needed is the function $m = \Phi(T, h)$, the equation of state, which we computed by a Monte Carlo simulation of the classical XY model of Eq. (1). Notice that we relate m to a local (intensive) property such as the resistivity instead of to the resistance, which is extensive and sensitive to significant finite size effects. However, the experimental data gives the resistances R_{xx} and R_{yy} as functions of temperature, not the resistivities. Thus, in order to fit the data, we extracted the resistivities from the measured resistances, using a method discussed below, with the result shown in Fig. 1.

One result of this analysis is that the resistivities for the square sample behave exactly in the same way as the resistances of the Hall bars. Given the low T values of ρ_{xx} and ρ_{yy} in Fig. 1, and the (large) measured value of the Hall conductance, one finds that the peak value of the *conductivity* is $\sigma_{yy} = 1.12e^2/h$ and $\sigma_{xx} = 0.11e^2/h$. Notice that ρ_{xx} saturates rather sharply below 55 mK and that both ρ_{xx} and ρ_{yy} approach nonzero (and different) values as $T \to 0$. Thus, the 2DEG remains in an anisotropic compressible (metallic) state, down to the lowest accessible temperatures.

Having determined the temperature dependence of the *resistivities*, we can now see if it is consistent with a (rounded) phase transition from a high temperature isotropic fluid phase to a low temperature nematic phase. We have done this by means of a Monte Carlo Metropolis simulation of the 2D XY model of Eq. (1) on square lattices of sizes 40×40 through 120×120, for the range of symmetry breaking fields $0.01J < h < 0.5J$, and for a wide range of temperatures (see below). In Fig. 2 we show our Monte Carlo data for the order parameter as a function of temperature for $h = 0.05J$. For this range of symmetry breaking fields, we find that for $L = 100$ the finite size effects on the order parameter are very small. We have fitted the data by assuming that $\zeta = (\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy})$ is actually *equal* to the order parameter m [21]. Having done so, we fitted the data by finding the best value of J that fits the data for a given value of h, and then changed h to get the best fit.

The classical nematic does indeed explain the temperature dependence; the data is consistent with a thermodynamic Kosterlitz-Thouless transition at $T_c(h = 0) = 0.88J \sim 65$ mK [22], slightly rounded by a background anisotropy field of magnitude $h \sim 0.05J \sim 3.5$ mK, which is a very small energy scale. Notice that both the stiffness $J \sim 73$ mK and h are much smaller than the Coulomb energy, although they are comparable with the gap in the $\nu = 5/2$ (presumably paired) state, which hints a possible common origin. Below 55 mK the fit is not as good. In this temperature range the XY order parameter is big (larger than $1/2$) so there is no reason to expect $\zeta \sim m$. However, ζ strikingly saturates (unlike m which shows the characteristic linear temperature behavior of classical spin wave theory) so the discrepancy may be indicating that quantum mechanical effects (or disorder) are important at low T.

Our analysis of the experiments strongly indicates that the 2DEG in large magnetic fields in clean samples has regimes where it behaves as a nematic fluid, an anisotropic metal. Such a metallic state should have a strong signature in polarized light scattering experiments. In particular, a nematic has long range fluctuations in the orientational order, which will cause the polarization tensor correlation function (and the corresponding longitudinal and transverse susceptibilities χ_L and χ_T) to have a singularity at T_c (cut off by the anisotropy). This effect is similar to critical opalescence but for orientational order instead of density fluctuations. For nonzero and small background anisotropy h, for $T < T_c$, χ_L is $\chi_L \sim h^{-\alpha}$, with $\alpha = 1 - 1/\delta$, where $\alpha = 14/15$ at T_c. For $T > T_c$, χ_L can be written in a scaling form as $\chi(h, T) \sim \xi^{7/4}\Phi_0(h\xi^{15/8})$, where $\Phi_0(0) = 1$ and $\Phi_0(x) \sim x^{-14/15}$ as $x \to \infty$; as discussed above, $\xi(T) \sim \exp(-A/\sqrt{t})$, where $t = T/T_c - 1$.

Thus, at fixed but small h, as the temperature is lowered, χ_L increases very rapidly to a maximum above T_c, with a crossover to a critical behavior $\sim h^{-\alpha(T)}$, where $\alpha(T) = 2(7 + t)/(15 + t)$ for $|h| \ll T$, and $\chi_L \sim (\pi T/2h)[1/(4\pi^2 J + h)]$, for $T \ll |h|$. On the other hand, by Goldstone's theorem, $\chi_T = m/h$, where m is the order parameter (Fig. 2). χ_L and χ_T are shown in Fig. 3. Although the specific heat of the 2DEG is very hard to measure, it should have a broad bump at a temperature $T^* > T_c$ set by the core energy of the disclinations of the nematic phase, while at T_c there is a very weak essential singularity [18] (rounded by the anisotropy).

Finally, we summarize how we determined the resistivities. It was observed recently [15,23] that on square samples there is a large distortion of the current distribution. If the 2DEG has an anisotropic but homogeneous resistivity tensor [24], one can calculate the currents using conformal mappings [25]. If the principal axes are aligned with the edges, one finds

$$R_{xx}/R_{yy} = g(x)/g(1/x), \tag{2}$$

where $x = [L_y/L_x]\sqrt{\rho_{yy}/\rho_{xx}} \equiv x(T)$ measures the aspect ratio L_y/L_x and the ratio of resistivities, and

$$g(x) = \ln\left(\frac{\theta_3(i\pi x/2) + \sqrt{k}\,\theta_2(i\pi x/2)}{\theta_3(i\pi x/2) - \sqrt{k}\,\theta_2(i\pi x/2)}\right). \tag{3}$$

$\theta_2(z)$ and $\theta_3(z)$ are theta functions with modulus k

$$k = 4\sqrt{q}\,\prod_{n=1}^{\infty}\left(\frac{1 + q^{2n}}{1 + q^{2n-1}}\right)^4, \tag{4}$$

where $q = \exp(-2\pi x)$ is the period [25]. Given R_{xx}/R_{yy} at different temperatures, and using Eq. (2), we calculated the function $x(T)$ (shown in the inset of Fig. 1). At high T, $x(T)$ approaches a value somewhat larger than 1, but it is smaller than 1 at lower temperatures, and both the resistances and the resistivities show a crossing at some high temperature [26]. This effect indicates that the sample is not homogeneous at large scales. A macroscopic inhomogeneity is equivalent to an effective aspect ratio, and by choosing $L_y/L_x = 1.12$ we can make the ratio $\rho_{xx}/\rho_{yy} \to 1$ at high temperature.

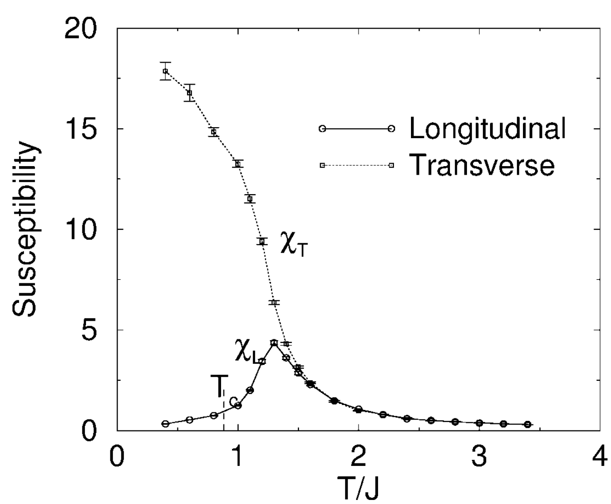

FIG. 3. Longitudinal and transverse susceptibilities of a classical nematic on a 100×100 lattice, for $h = 0.05J$.

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