Comment on "Delocalization in the 1D Anderson Model with Long-Range Correlated Disorder"

In a recent Letter de Moura and Lyra [1] considered quantum particles in a one-dimensional Anderson model with on-site energies ϵ_n exhibiting long-range correlations. They made the interesting discovery that for energy sequences with persistent increments (that are size dependent) the inverse localization length vanishes within a finite range of the band and conclude the presence of an Anderson-like metal-insulator transition for these self-affine energy landscapes.

In this Comment we show that this conclusion is not valid. The self-affine landscapes considered in [1] are given by the trace of a fractional Brownian particle with Hurst exponent H. Since the fluctuations of the potential increase with increasing length scale ℓ according to $\langle(\epsilon_{n+\ell} - \epsilon_n)^2\rangle \sim \ell^{2H}$, the fluctuations increase with increasing system size L. de Moura and Lyra imposed a normalization condition $\sigma \equiv \frac{1}{L}\sum_{n=1}^{L} \epsilon_n^2 - (\frac{1}{L}\sum_{n=1}^{L} \epsilon_n)^2 = 1$, which keeps the fluctuations fixed for all L and is equivalent to dividing the local potentials by L^H. If the sequences are generated by the Fourier transform method (as is done in [1]), the rescaling is contained in the normalization factors.

Figure 1(a) demonstrates the effect of the rescaling. When the system size is increased, the *local* fluctuations of the potentials decrease, and this creates the apparent "extended" states for $H > 1/2$. *Without* rescaling, the disorder width increases with increasing system size resulting in *strongly* localized states for all Hurst exponents $H > 0$. Figure 1(b) shows, for normal (not normalized) self-affine potential landscapes with $H = 3/4$, the localization length $\lambda(E)$ calculated by the transfer-matrix method for several system sizes L. For small L and E near the band center, $\lambda(E)$ increases with L, but slower than linearly. This indicates weakly localized states. For large L and at the band edges, however, this behavior is reversed: $\lambda(E)$ decreases drastically with increasing system size, indicating strongly localized states. A similar behavior is observed for all $H > 1/2$, where all eigenstates become strongly localized in the limit of infinite system size. For the rescaled potentials the apparent transition to extended states depends on the value chosen for the constant σ, and the critical line can be derived analytically [2].

It is interesting to note that a system-size dependence of the localization length similar to the one reported in [1] can be observed also for uncorrelated random potentials ϵ_n, if, similar to above, the ϵ_n are divided by L^H (except those close to the band edges). While all states are localized with $\lambda(w) \sim w^{-2}$ for *fixed* disorder width w, a system-size dependent disorder width $w \sim L^{-H}$ leads to $\lambda/L \sim L^{2H-1}$, which decreases with increasing system size L for $H < 1/2$, while it increases for $H > 1/2$ (apparent extended states). Similarly to the self-affine landscapes, this does not present an Anderson-like metal-insulator transition.

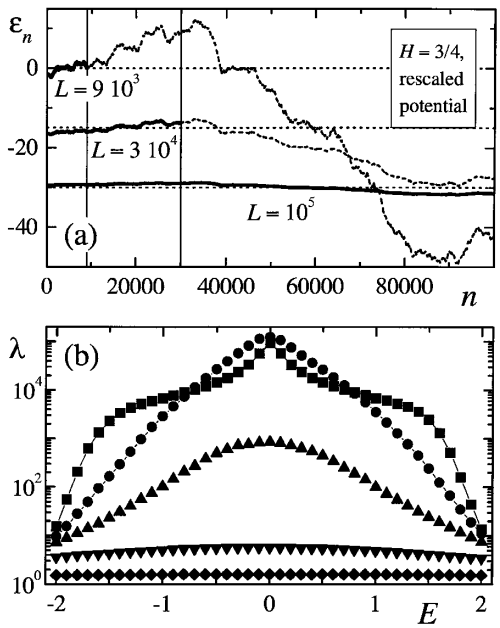

FIG. 1. (a) Illustration of a rescaled self-affine potential landscape with $H = 3/4$ for system sizes $L = 9 \times 10^3$, 3×10^4, and 10^5 (thick lines, shifted by multiples of 15). It is obvious that for increasing system sizes the potential landscape becomes smoother due to the rescaling with fixed variance $\sigma = 1$ in the considered interval of size L. (b) Plot of the localization length λ versus energy E for nonrescaled self-affine potential landscapes with $H = 3/4$ and for five system sizes $L = 2^{11}$ (boxes), $L = 2^{13}$ (circles), $L = 2^{15}$ (triangles up), $L = 2^{17}$ (triangles down), and $L = 2^{19}$ (diamonds). λ was averaged logarithmically over 1000 configurations for each symbol. For large L, the localization length decreases drastically with increasing system size.

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