Finite Flux Solutions of the Quantum Boltzmann Equation and Semiconductor Lasers

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We propose and illustrate in the context of the semiconductor laser that, in nonequilibrium fermionic systems with sources and sinks, the family of finite flux stationary solutions of the quantum Boltzmann equation is central and more important then the zero flux Fermi-Dirac spectrum. We present the quantum analog of the finite flux Kolmogorov spectra which are central to understanding nonequilibrium classical systems such as high Reynolds number hydrodynamics and the wave turbulence encountered in water waves, plasmas, and optics. In particular, we show how semiconductor laser efficiency can be improved by maximizing the flux of carriers (electrons and holes) towards the lasing frequencies.

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The main goal (and principal novel feature) of this Letter is to point out the importance of a richer class of stationary spectra of fermionic systems. Unlike the Fermi-Dirac (FD) spectrum, which is a special case, these spectra allow for a finite transport of carriers between source energies at which the system is pumped and sink energies at which the system loses particles and energy to other states. Such finite flux distributions are the quantum analog of the finite flux Kolmogorov spectra of classical wave turbulence [1,2]. These exact solutions of the kinetic equations describe how, for example, energy, momentum, and particle number inserted by the wind into ocean surface waves is transported by four wave resonances (for wavelengths $\lambda > 2$ cm, gravity waves dominate) and three wave resonances (for $\lambda < 2$ cm surface tension is important) throughout the spectrum. Indeed such spectra have been observed experimentally [3]. The inverse cascade of predominantly particle number is responsible for "old" waves, namely, the appearance of very long and fast traveling waves which outrun the storm and which are not directly driven by the wind but can only be indirectly generated by nonlinear processes. We suggest the same scenario is also vital to understanding the behavior of the quantum fermionic systems. We illustrate the idea in the context of the semiconductor laser (ubiquitous and advantageous in a wide range of applications) and demonstrate how, by using finite flux solutions, the laser output can be enhanced. This means that the optical properties of a semiconductor material can be effectively altered without actually changing the material itself.

For the most part the semiconductor laser operates in a manner similar to the "textbook" two-level laser. Optical feedback is organized by a cavity and the coherent light output is generated by in-phase transitions of an electron from a higher to lower energy state. In semiconductors, the lower state is a valence band, from which electrons are excited by pumping into a conduction band, leaving behind positively charged holes with opposite momenta and spins. But they also differ from two-level lasers. First, since light emission is a recombination process, there must be both an electron and a hole at the same absolute momentum and spin values. Second, there is a continuum of transition energies parametrized by the electron momentum $\mathbf{k}$ and the laser output is a weighted sum of contributions from polarizations corresponding to each momentum value. (Spin is included as part of the momentum vector.) In this regard, the semiconductor laser resembles an inhomogeneously broadened two-level laser. Third, and most important of all, electrons and holes interact with each other via screened Coulomb forces [4]. This leads to a mechanism for the redistribution of carriers between different momentum states, a process well described by coupled quantum Boltzmann or kinetic equations (QKE). The electron and hole probability densities $n_k^- = \langle a_k^\dagger a_k \rangle$, $n_k^+ = \langle b_k^\dagger b_k \rangle$ evolve according to [4,5]:

$$\frac{\partial}{\partial t} n_k^s = 4\pi \int |T_{kk_1k_2k_3}|^2 \delta(\mathbf{k} + \mathbf{k_1} - \mathbf{k_2} - \mathbf{k_3}) \{ [n_{\mathbf{k_2}}^s n_{\mathbf{k_3}}^s (1 - n_{\mathbf{k_1}}^s)(1 - n_{\mathbf{k}}^s) - n_{\mathbf{k}}^s n_{\mathbf{k_1}}^s (1 - n_{\mathbf{k_2}}^s)(1 - n_{\mathbf{k_3}}^s)]$$
$$\times \delta(\omega_k^s + \omega_{k_1}^s - \omega_{k_2}^s - \omega_{k_3}^s) + [n_{\mathbf{k_2}}^s n_{\mathbf{k_3}}^{-s} (1 - n_{\mathbf{k_1}}^{-s})(1 - n_{\mathbf{k}}^s) - n_{\mathbf{k}}^s n_{\mathbf{k_1}}^{-s} (1 - n_{\mathbf{k_2}}^s)(1 - n_{\mathbf{k_3}}^{-s})]$$
$$\times \delta(\omega_k^s + \omega_{k_1}^{-s} - \omega_{k_2}^s - \omega_{k_3}^{-s}) \} d\mathbf{k_1} d\mathbf{k_2} d\mathbf{k_3}, s = +, - . \quad (1)$$

In (1), T_{kk_1,k_2k_3} is the coupling coefficient due to Coulomb forces and is proportional to $1/(k^2 + \kappa^2)$, with κ being the inverse screening length. The kinetic energies are $\hbar\omega_k^- = \frac{\hbar^2 k^2}{2m^-}$, $\hbar\omega_k^+ = \frac{\hbar^2 k^2}{2m^+}$ with $k = |\mathbf{k}|$, and m^- and m^+ are the electron and hole masses, respectively. For electric field pulses exceeding 1 ps the carrier redistribution is the *fastest* process ($\simeq$100 fs). Therefore, carriers relax adiabatically to attracting manifolds $n_{0\mathbf{k}}^s$ which are stable stationary solutions of the QKE consistent

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with the presence of sources (pumping) and sinks (laser output at lower momenta and dissipation via nonradiative recombination). It is commonly assumed [4,5] that this equilibrium state corresponds to the thermodynamic equilibrium of fermion gases, which is described by a FD distribution. However, because of the presence of sources and sinks, a semiconductor laser is far from equilibrium. In the case where the sources and sinks are located at different parts of the momentum spectrum, carriers and energy flow between them. But the FD solutions do not capture the relevant physics because they carry no flux. Whereas it has been appreciated by several authors (e.g., Jahnke and Koch [6]) that a finite flux of carriers is essential to compensate for the action of a sink (e.g., hole burning in semiconductor lasers), it has not been generally recognized that there are exact stationary solutions of the QKE which describe the constant flux of carriers between isolated sources and sinks. We will now demonstrate, in the semiconductor laser context, that such solutions are not only relevant but are realized and can be used to enhance the laser output.

Assuming isotropy, the most general steady state solutions of the QKE (1) belong to a *six* parameter family. The *six* parameters are the total numbers of electrons and holes

$$N^s = \int n_k^s \, d\mathbf{k} = \int N_\omega^s \, d\omega, s = +, -,$$
$$\hbar\omega = (\hbar^2 k^2)/(2m),$$
$$1/m = 1/m^- + 1/m^+,$$
$$N_\omega^s = \Omega_0 \left(\frac{dk}{d\omega}\right) k^{d-1} n_k^s[\mathbf{k}(\omega)]$$

(where Ω_0 is the surface area of a d-dimensional unit sphere), the total energy

$$E = \int E_\omega \, d\omega,$$
$$E_\omega = (\epsilon_{\text{gap}} + \hbar\omega m/m^-)N^- - \hbar\omega m/m^+ N^+,$$

and the three fluxes Q^-, Q^+, and P of the conserved densities N_ω^-, N_ω^+, and E_ω. It is convenient to write (1) in conservation law form

$$\frac{\partial N_\omega^s}{\partial t} = \frac{\partial Q^s}{\partial \omega}, \qquad \frac{\partial E_\omega}{\partial t} = -\frac{\partial P}{\partial \omega}, \tag{2}$$

where Q^s (P) is taken positive when the flux of N_ω^s (E_ω) is towards lower (higher) energies. Stationary solutions occur when Q^s and P are constants. The FD states $(n_k^s)^{-1} = \exp[\frac{\hbar}{T}(\frac{\omega m}{m^s} - \mu_s)] + 1$, belong to a special zero flux three parameter submanifold of solutions for which $Q^s = P = 0$. Here T can be interpreted as the common electron and hole temperature, μ_- and μ_+ as chemical potentials. FD states with zero flux are most relevant in situations where the damping and pumping are broad band and locally in balance in ω space.

But broad band pumping, and the FD distributions generated by it, may be inefficient. Observe from Fig. 1(a) that at room temperatures and typical operating conditions, the FD distributions are much broader than the gain band for lasing and therefore much energy is used in exciting transitions at momenta which, because there are no fluxes, do not directly contribute to lasing. Therefore, we are led to investigate what happens if, instead of pumping broadly, we pump the semiconductor in a relatively narrow spectral region around an energy value $\hbar\omega_0$ that is greater than the lasing energy $\hbar\omega_L$ [see Fig. 1(b)]. Such local pumping is possible through optical pumping [7] or through resonant tunneling of electrons through multiple quantum well structures [8]. In this case, a significant portion of carriers and their associated energies will flow back from the source ($\hbar\omega_0$) to sink (lasing at $\hbar\omega_L$) energies and thereby involve electrons and holes at all momenta and energies in lasing. At the same time, and because of the conservation of carriers and energy, some carriers will flow to higher energy ($\hbar\omega_R$) values and their energies will be absorbed at various energy levels $\hbar\omega_R > \hbar\omega_0$ due to many processes, e.g., (i) absorption of the charge carriers with high kinetic energies that leave the optically active region and thus contribute to the electrical pumping current without contributing to the light amplification; (ii) nonradiative recombination of electron-hole pairs mediated by

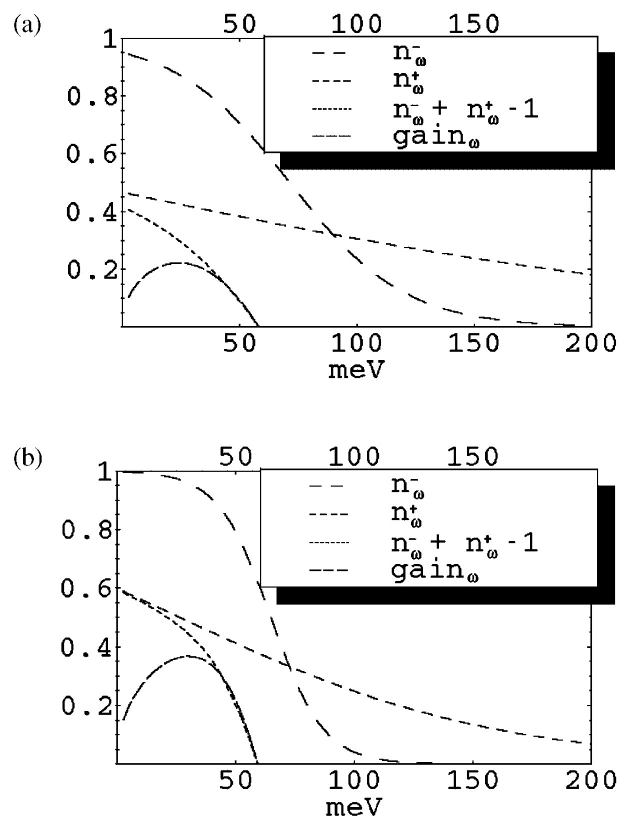

FIG. 1. (a) Stationary Fermi-Dirac electron and hole distributions n_ω^-, n_ω^+, the inversion $n_\omega^- + n_\omega^+ - 1$, and gain $\sqrt{\omega}\,(n_\omega^- + n_\omega^+ - 1)$ for the broadly pumped semiconductor laser. (b) Stationary (finite flux) distributions for the narrow band pumping case. Lasing occurs only over energies where the inversion is positive. Note the effect of the finite flux is to compress the original FD distribution.

impurities, dislocations, and interface roughness; (iii) Auger processes.

In the remainder of this Letter, we provide concrete evidence to support our idea. We numerically solve both the QKE (1) and the single mode semiconductor laser Maxwell-Bloch equations in the free carrier limit for the electric field envelope $e(t)$, polarization envelope p_ω and electron and hole distributions n_ω^-, n_ω^+ [4,9]:

$$\frac{\partial e}{\partial t} = i \frac{\Omega}{2\epsilon_0} \frac{V}{(2\pi)^d} \int d_\omega p_\omega \frac{d\mathbf{k}}{d\omega} d\omega - \gamma_E e ,$$
$$\frac{\partial p_\omega}{\partial t} = (i\Omega - i\tilde{\omega} - \gamma_P) p_\omega - \frac{i d_\omega}{2\hbar} (n_\omega^+ + n_\omega^- - 1) e ,$$
$$\frac{\partial n_\omega^s}{\partial t} = \Lambda_\omega^s (1 - n_\omega^s) - \gamma_\omega n_\omega^s + \left(\frac{\partial n_\omega^s}{\partial t}\right)_{\text{coll}} - \frac{i}{\hbar} \text{Im}(d_\omega p_\omega e^+) . \quad (3)$$

Here V is a sample volume and we use typical semiconductor laser parameters suggested in [4]. $\Omega = 1440$ meV$/\hbar$ is the cavity frequency. The electric field damping γ_E equals 6×10^{10} s^{-1}, the polarization decay (dephasing) $\gamma_P = 10^{13}$ s^{-1}, ϵ_0 is the permittivity of free space, d_ω is the dipole matrix element $d_{\omega=0}/(1 + \epsilon_k/\epsilon_{\text{gap}})$, $d_{\omega=0} = 3 \times 10^{-10}$ m $\times$ the electron charge, and the nonradiative carrier damping γ_ω equals 10^{10} s^{-1}. In (3), Λ_ω^s is the pumping due to the injection current (taken to be between 0.001 and 0.01 ps^{-1}), $1 - n_\omega^s$ is the Pauli blocking factor (because of the Pauli exclusion principle, n_ω^s cannot be pumped over unity), and $\hbar\tilde{\omega} = \epsilon_{\text{gap}} + \epsilon_{e,k} + \epsilon_{h,k}$. We further assume that all fields are isotropic and make a transformation from k $(= |\mathbf{k}|)$ to ω via the dispersion relation $\hbar\omega(\mathbf{k}) = \hbar^2 k^2/(2m)$ and define the carrier density n_ω^s as $n^s[\mathbf{k}(\omega)]$. The collision terms in (3) are given in (1) and approximated by the differential approximation [1,9–11]. It is equivalent to the assumption that the spectral transfer of carriers is very local in ω space. The differential approximation reads

$$\frac{\partial N_\omega^s}{\partial t} = -\frac{m^s}{m} \frac{\partial^2}{\partial\omega^2} K_s - s \frac{m^s}{m} \frac{\partial J}{\partial \omega}, \qquad s = +, - .$$
$$K_s(\omega) = I_1 \omega^{\gamma+6} [(n_\omega^s)^4 (1/n_\omega^s)'' + (n_\omega^s)^2 (\ln n_\omega^s)''],$$
$$J(\omega) = I_2 \omega^{\gamma+4} [(n_\omega^-)' n_\omega^+ (1 - n_\omega^+) - (m^{-1} m^-)(n_\omega^+)' n_\omega^- (1 - n_\omega^-)],$$
$$' = d/(d\omega) , \quad (4)$$

where I_1, I_2 are semiconductor relaxation time constants [9] and γ equals unity for semiconductors. The fluxes are given by $Q^s = -m^s m^{-1} \frac{\partial K_s}{\partial \omega} - s m^s m^{-1} J$ and $P = -\epsilon_{\text{gap}} Q^- + \hbar[\omega \frac{\partial}{\partial\omega}(K_+ + K_-) - (K_+ + K_-)]$. We point out that J is proportional to $-\frac{(m^{-1}m^-)}{\hbar} \frac{\partial}{\partial\omega} \ln \frac{n^-}{1-n^-} + \frac{(m^{-1}m^+)}{\hbar} \frac{\partial}{\partial\omega} \ln \frac{n^+}{1-n^+}$, or to the difference between the local inverse temperatures $(T^-)^{-1}(\omega) - (T^+)^{-1}(\omega)$ (for FD distributions, these are constant and equal) and thus the role of the cross term is to equalize electron and hole temperatures. We tested the validity of the model and of our numerical model by simulating the broad band pumping and reproducing the expected (but slightly modified due to hole burning) FD carrier distribution and laser turn-on and output characteristics. Note that even for the broad band pumping case, there is some flux of carriers across the spectrum. Indeed, the maximum of the pumping $\Lambda_\omega^s(1 - n_\omega^s)$ is located at the middle part of the spectrum, see Fig. 1(a). The maximum of the carrier absorption is located at small ω. Therefore one unavoidably has a flux of carriers across the spectrum.

We also observed, consistent with theoretical arguments one can make from (4) [9], that the net effect of the positive finite fluxes is to *compress* the FD spectra to smaller ω values. Such a compression, which is particularly strong for the electron distribution, can be interpreted as an *effective* temperature and chemical potential decrease. Therefore finite fluxes effectively increase inversion in the lasing part of the spectrum. In addition, the operation of a semiconductor laser may be optimized by choosing the pumping energy $\hbar\omega_0$ to (i) make $\hbar\omega_0$ big enough to minimize Pauli blocking, because electrons are best pumped where n_ω^- is small, (ii) to make $\hbar\omega_0$ small enough (closer to the lasing energy $\hbar\omega_L$) to increases the flux of carriers towards the lasing energy. The flux formulas are given in the next paragraph.

In the numerical experiments, we solve (3) for $t \geq 0$ on $\omega_1 \leq \omega \leq \omega_0$ with ω_1 just less than ω_L, the lasing frequency. The boundary conditions and pumping rates are (a) broad band case: $Q^+ = Q^- = 0$ at $\omega = \omega_1, \omega_0$, $\Lambda_\omega =$ FD, typical pump profile; (b) narrow band case: $Q^+ = Q^- = P = 0$ at $\omega = \omega_1$, $Q^+ = Q^- = Q_L$, $P = -(\epsilon_{\text{gap}} + \hbar\omega_L)Q_L$, $\omega = \omega_0$; $\Lambda_\omega^s = 0$. In both cases the background dissipation $\gamma^s n_\omega^s$ is the same. Q_L is calculated as follows. Consider Fig. 2. From the conservation of electrons, holes and total energy, the assumptions that $P_R = -\omega_R(Q_R^+ + Q_R^-)$, $P_L = -\omega_L(Q_L^+ + Q_L^-)$ (namely, that both carriers and energy are absorbed by the dissipation and laser states, respectively) and charge neutrality $Q_L^+ = Q_L^-$, we find after a little calculation that $Q_L = Q_0(\omega_R - \omega_0)/(\omega_R - \omega_L)$, $\frac{-P_L}{-P_0} = \frac{e_{\text{gap}} + \hbar\omega_L}{\epsilon_{\text{gap}} + \hbar\omega_0} \frac{\omega_R - \omega_0}{\omega_R - \omega_L} \simeq \frac{\omega_R - \omega_0}{\omega_R - \omega_L}$ since $\epsilon_{\text{gap}} \simeq 1400$, $\hbar\omega_L \simeq 40$, and $\hbar\omega_0 \simeq 200$ meV. To make the comparison with broad band pumping, we choose $Q_0 = \int \Lambda_\omega (1 - n_\omega^-) \frac{dk}{d\omega} d\omega$ such that the net particle and energy input rates are the same (energy $\simeq e_{\text{gap}} \times$ particle number). Because of Pauli blocking, the absorption of power for the pump may be slower in the broad band case than in the narrow band case.

In Fig. 3 we present the output power as a function of the pumping strength. We observe that the output power is consistently higher for narrow band pumping than for broad band pumping, especially for weak pumping. The lasing threshold value is less for narrow band pumping.

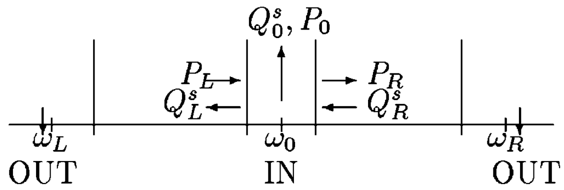

FIG. 2. This picture explains the setup for input-output fluxes. Carriers (electrons and holes) and energy are added at ω_0 with rates Q_0^s, $s = +, -$ and P_0. Energy and some carriers are dissipated at $\omega_R > \omega_0$ (an idealization) and are absorbed by the laser at ω_L. Finite flux stationary solutions are realized in the windows (ω_L, ω_0) and (ω_0, ω_R) although in practice there will be some losses in both of these regions. The electron and hole fluxes to the left (right) are Q_L^-, Q_L^+ $(-Q_R^-, -Q_R^+)$. The rate of energy flowing to the left (right) is $-P_L$ (P_R). Note that fluxes are defined so that $Q_L^s > 0$, $s = +, -$, $Q_R^s < 0$, $P_L < 0$, and $P_R > 0$.

The distribution functions are more compressed towards lower energies for narrow band pumping than they are for broad band pumping because of the higher leftward fluxes associated with the former. The results for different mass ratios $m^+ = m^-$, $m^+ = 2m^-$ are similar.

In this Letter, we have shown that the QKE has a new and richer class of steady state solutions than the FD distribution. These new solutions generalize FD to include finite fluxes of the conserved quantities. We demonstrated how these finite flux solutions of the QKE can be used in the semiconductor laser context and, in particular, how the finite fluxes of carriers and energy can be exploited to give improved laser efficiency. But semiconductor lasers are just one particular example. There is a huge number of other potential applications of these spectra in situations where the system is driven far from equilibrium by the presence of sources and sinks operating at different energies. Two other examples: the growth of (Bose) condensates in superfluids is a direct consequence of an inverse cascade of particles; likewise, the onset of intermittency in (classical) optical waves of diffraction in nonlinear media where the refractive index increases with intensity is a result of filaments triggered by instabilities of long waves which in turn are driven by an inverse cascade of particles [10]. We point out also that optoelectronics is a fast growing field and the idea of effectively changing optical properties without changing the materials themselves is certainly worth more attention.

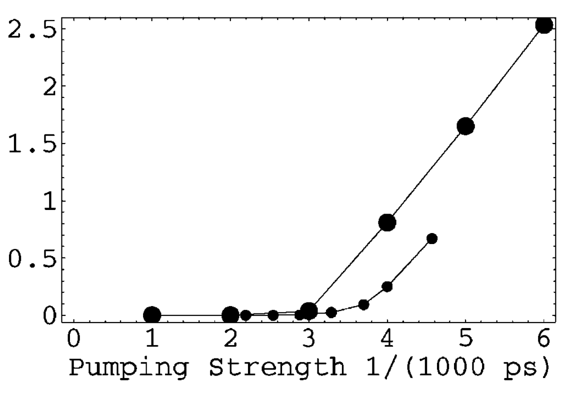

FIG. 3. The output power (in arbitrary units) as a function of pumping strength for narrow band and broad band pumping for $m^+ = 6m^-$. The narrow band output power is shown by large circles; the broad band output power is shown by small circles.

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