Andreev Reflection in Strong Magnetic Fields

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We have studied the interplay of Andreev reflection and cyclotron motion of quasiparticles at a superconductor–normal-metal interface with a strong magnetic field applied parallel to the interface. Bound states are formed due to the confinement introduced by both the external magnetic field and the superconducting gap. These bound states are a coherent superposition of electron and hole edge excitations similar to those realized in finite quantum-Hall samples. We find the energy spectrum for these Andreev edge states and calculate transport properties.

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Rapid progress in fabrication techniques has made it possible to investigate phase-coherent transport in a variety of mesoscopic conducting devices [1]. In recent years, the study of hybrid systems consisting of superconductors in contact with normal metals has continued to attract considerable interest, mainly because of the novel effects observed in superconductor-semiconductor microjunctions [2]. Many of the unusual experimental findings arise due to the phenomenon of *Andreev reflection*, i.e., the (partial) retroreflection of an electron incident on a superconductor (S)–normal-metal (N) interface as a hole [3,4]. Phase coherence between the electron and hole states is maintained during the reflection process. Hence, coupled-electron-hole (Andreev) bound states [3] having energies within the superconducting gap are formed in mesoscopic devices with multiple interfaces, e.g., S-N-S systems [5], or S-N-I-N-S structures [6]. (I denotes an insulating barrier.) Recently, measurements of transport across the interface between a superconductor and a two-dimensional electron gas (2DEG) were performed with a strong magnetic field H applied in the direction perpendicular to the plane of the 2DEG [7]. While the magnetic field did not exceed the critical field of the superconductor, it was still large enough so that the Landau-level quantization of the electronic motion in the 2DEG was important [8]. With these experiments, a link has been established between mesoscopic superconductivity and quantum-Hall physics [9] which needs theoretical exploration.

In this Letter, we study a novel kind of Andreev bound state that is formed *at a single S-N interface* in a strong magnetic field [10]. This bound state is a coherent superposition of an electron and a hole propagating along the interface in a new type of current-carrying edge state that is induced by the superconducting pair potential. Andreev reflection (AR) gives rise to the contribution

$$G_{\mathrm{AR}} = \frac{e^2}{\pi\hbar} \sum_{n=1}^{n^*} B_n \tag{1}$$

to the small-bias conductance, which we obtained by generalizing the familiar Büttiker description [11] of transport in quantum-Hall samples. In Eq. (1), the summation is over Andreev-bound-state levels that intersect with the chemical potential, and B_n is the hole probability for a particular bound-state level. It turns out that n^* is twice the number of orbital Landau levels occupied in the bulk of the 2DEG, and $B_n \leq 1/2$ depends weakly on magnetic field H for an ideal interface but oscillates strongly with H for a nonideal interface. G_{AR} can be measured directly as the two-terminal conductance in a S-2DEG-S system [7]. Our treatment in terms of Andreev edge states provides a clear physical description of transport in such devices and explains oscillatory features in the conductance that were observed experimentally [7] and also obtained in previous numerical studies [12].

Let us start by recalling the classical and quantum-mechanical descriptions of electron dynamics in an external magnetic field. When considered to be classical charged particles, bulk-metal electrons execute periodic cyclotron motion with a frequency $\omega_c = eH/(mc)$. A surface that is parallel to the magnetic field interrupts the cyclotron orbits of nearby electrons and forces them to move in skipping orbits along the surface [13]. Within the more adequate quantum-mechanical treatment, the kinetic energy for electronic motion in the plane perpendicular to the magnetic field is quantized in Landau levels [14] which are at constant eigenvalues $\hbar\omega_c(n + 1/2)$ for electron states localized in the bulk but are bent upward in energy for states localized close to the surface [15]. Applying the classical picture of cyclotron and skipping orbits to a S-N interface, one finds that Andreev reflection leads to electrons and holes alternating in skipping orbits along the interface [16]. [See Fig. 1(a).] In what follows, we provide a full quantum-mechanical description of these alternating skipping orbits in terms of current-carrying Andreev bound states. [See Fig. 1(b).]

We now provide details of our calculation. A planar interface is considered, located at $x = 0$ between a semi-infinite region ($x < 0$) occupied by a type-I superconductor and a semi-infinite normal region ($x > 0$). A uniform magnetic field is applied in the z direction, which is screened from the superconducting region due to the

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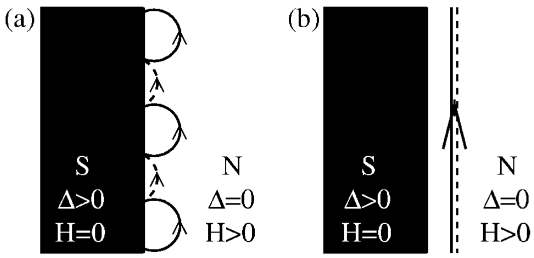

FIG. 1. Andreev bound state of an electron (solid lines) and a hole (dashed lines) at an S-N interface in a magnetic field. (a) Classical picture of electron and hole alternating in skipping orbits. (b) The quantum-mechanical analysis finds that both the electron and the hole occupy Landau-level states that are extended parallel to the interface.

Meissner effect. Neglecting inhomogeneities in the magnetic field due to the existence of a finite penetration layer [17], we assume an abrupt change of the magnetic-field strength at the interface: $H(x) = H\Theta(x)$, where $\Theta(x)$ is Heaviside's step function. The energy spectrum of Andreev bound states at the S-N interface is found by solving the Bogoliubov–de Gennes (BdG) equation [18],

$$\begin{pmatrix} H_{0,+} + U_{\rm ext} & \Delta \\ \Delta^* & -H_{0,-} - U_{\rm ext} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}, \quad (2)$$

with spatially nonuniform single-electron/hole Hamiltonians $H_{0,\pm}$ and pair potential [19] $\Delta(x) = \Delta_0 \Theta(-x)$. We introduced the potential $U_{\rm ext}(x) = U_0 \delta(x)$ to model scattering at the interface, and allow the effective mass and the Fermi energy to be different in the superconducting and normal regions. Choosing the vector potential $\vec{A}(x) = xH\Theta(x)\hat{y}$, we have

$$H_{0,\pm} = \begin{cases} \frac{p_x^2 + p_z^2}{2m_N} + \frac{m_N \omega_c^2}{2}(x \mp X_{p_y})^2 - \epsilon_{\rm F}^{(N)}, & x > 0, \\ \frac{p_x^2 + p_y^2 + p_z^2}{2m_S} - \epsilon_{\rm F}^{(S)}, & x < 0. \end{cases} \quad (3)$$

The operator $X_{p_y} = p_y \ell^2 \operatorname{sgn}(eH)/\hbar$ is the guiding-center coordinate in the x direction for cyclotron motion of electrons in the normal region, and $\ell = \sqrt{\hbar c/|eH|}$ denotes the magnetic length. Uniformity in the y and z directions suggests the separation ansatz

$$u(x,y,z) = f_X(x) e^{iyX/\ell^2} e^{ikz}/\sqrt{L_y L_z}, \quad (4a)$$

$$v(x,y,z) = g_X(x) e^{iyX/\ell^2} e^{ikz}/\sqrt{L_y L_z}. \quad (4b)$$

The lengths L_y, L_z are the sample sizes in the y and z directions, respectively. Solutions of Eq. (2) for the S-N junction are found by matching appropriate wave functions that are solutions in the normal and superconducting regions, respectively [4]. The motion in z direction can trivially be accounted for by renormalized Fermi energies $\tilde{\epsilon}_{\rm F}^{(N,S)} = \epsilon_{\rm F}^{(N,S)} - \hbar^2 k^2/(2m_{N,S})$. Nontrivial matching conditions arise only in the x direction. For fixed X and $|E| < \Delta_0$, we have to match at $x = 0$ the wave function

$$\begin{pmatrix} f_X \\ g_X \end{pmatrix}_{x>0} = \begin{pmatrix} a\chi_{\varepsilon_+}(x - X) \\ b\chi_{\varepsilon_-}(x + X) \end{pmatrix}, \quad (5a)$$

corresponding to a coherent superposition of an electron and a hole in the normal region, to that of evanescent excitations in the superconductor,

$$\begin{pmatrix} f_X \\ g_X \end{pmatrix}_{x<0} = d_+ \begin{pmatrix} \gamma \\ 1 \end{pmatrix} e^{ix\lambda_-} + d_- \begin{pmatrix} \gamma^* \\ 1 \end{pmatrix} e^{-ix\lambda_+}. \quad (5b)$$

The parameters γ and $\lambda_\pm$ are defined in the usual way [5]. The functions $\chi_{\varepsilon_\pm}(\xi)$ solve the familiar one-dimensional harmonic-oscillator Schrödinger equation,

$$\frac{\ell^2}{2} \frac{d^2 \chi_{\varepsilon_\pm}}{d\xi^2} - \left[\frac{\xi^2}{2\ell^2} - \frac{\varepsilon_\pm}{\hbar\omega_c} \right] \chi_{\varepsilon_\pm} = 0, \quad (6)$$

with eigenvalues $\varepsilon_\pm = \epsilon_{\rm F}^{(N)} \pm E - \hbar^2 k^2/(2m_N)$ and are assumed to be normalized to unity in the normal region. Hence, they are proportional to the fundamental solutions of Eq. (6) that are well behaved for $x \to \infty$; these are the *parabolic cylinder functions* [20] $U(-\varepsilon_\pm/\hbar\omega_c, \sqrt{2}\,\xi/\ell)$. The matching conditions yield a homogeneous system of four linear equations for the coefficients $a, b, d_\pm$ whose secular equation determines the allowed values of E.

It is straightforward to calculate the probability and charge currents for any particular Andreev-bound-state solution of the BdG equation (2) that is labeled by guiding-center coordinate X and energy E. It turns out that currents flow parallel to the interface. The total (integrated) quasiparticle *probability* current is given by

$$I_X^{(P)} = \frac{1}{\hbar} \frac{\ell^2}{L_y} \frac{\partial E}{\partial X}. \quad (7)$$

The total *charge* current can be written as the sum of three contributions, $I_X^{(Q)} = I_X^{(Q,n)} - I_X^{(Q,a)} + I_X^{(Q,s)}$, where

$$I_X^{(Q,n)} = \frac{e}{\hbar} \frac{\ell^2}{L_y} \frac{\partial E}{\partial X}, \quad (8a)$$

$$I_X^{(Q,a)} = \frac{e}{\hbar} \frac{\ell^2}{L_y} \frac{\partial E}{\partial X} 2 \int_x |g_X(x)|^2, \quad (8b)$$

$$I_X^{(Q,s)} = \frac{e}{\hbar} \frac{\ell^2}{L_y} 2\Delta \int_x \Theta(-x) \left[g_X^* \frac{df_X}{dX} - f_X^* \frac{dg_X}{dX} \right]. \quad (8c)$$

Note that $I_X^{(Q,n)}$ is the current that would flow in an ordinary quantum-Hall edge state [15], i.e., due to normal reflection at the interface. The existence of Andreev reflection is manifested in the contribution $-I_X^{(Q,a)}$ to the Hall current; it is proportional to the hole probability $B(X) = \int_x |g_X(x)|^2$. The part $I_X^{(Q,s)}$ of the quasiparticle charge current is converted into a supercurrent.

Numerical implementation of the matching procedure is straightforward. More detailed insight is gained, however, when considering the limit $|X| \ll \sqrt{\varepsilon_\pm/(2m_N\omega_c^2)}$ for which analytical progress can be made. Using an asymptotic form for the parabolic cylinder functions [20], the secular equation can be written as

$$\cos(\varphi_+) + \Gamma(\varphi_-) = \frac{2s}{s^2 + w^2 + 1} \frac{E \sin(\varphi_+)}{\sqrt{\Delta_0^2 - E^2}}. \quad (9)$$

Here we used the Andreev approximation [3] ($E, \Delta_0 \ll \tilde{\epsilon}_F^{(N)}, \tilde{\epsilon}_F^{(S)}$), and the abbreviations

$$\varphi_+ = \pi \frac{E}{\hbar\omega_c} + \frac{2X}{\hbar}\sqrt{2m_N \tilde{\epsilon}_F^{(N)}}, \quad (10a)$$

$$\varphi_- = \pi \frac{\nu}{2} + \frac{EX}{\hbar}\sqrt{\frac{2m_N}{\tilde{\epsilon}_F^{(N)}}}, \quad (10b)$$

$$\Gamma(\alpha) = \frac{[s^2 + w^2 - 1]\sin(\alpha) + 2w\cos(\alpha)}{s^2 + w^2 + 1}. \quad (10c)$$

The variable $\nu = 2\tilde{\epsilon}_F^{(N)}/(\hbar\omega_c)$ coincides with the *filling factor* of quantum-Hall physics [9] when the N region is a 2DEG. The parameter $s = [\tilde{\epsilon}_F^{(S)} m_N/(\tilde{\epsilon}_F^{(N)} m_S)]^{1/2}$ measures the Fermi-velocity mismatch for the junction, and $w = [2m_N U_0^2/(\hbar^2 \tilde{\epsilon}_F^{(N)})]^{1/2}$ quantifies interface scattering. We discuss briefly results for two limiting cases [21].

(a) Ideal interface.—In the absence of scattering at the S-N interface ($w = 0$) and for perfectly matching Fermi velocities ($s = 1$), $\Gamma(\alpha)$ vanishes identically. The energy spectrum is found from solutions of the transcendental equation $\cot(\varphi_+) = E/\sqrt{\Delta_0^2 - E^2}$. It consists of several bands, and states within each band are labeled by their guiding-center quantum number X. It turns out that $a^2 = b^2$ *at any energy,* and the band dispersion is

$$\frac{\partial E}{\partial X} = -\frac{2\sqrt{2m_N\tilde{\epsilon}_F^{(N)}}}{\hbar} \frac{\sqrt{\Delta_0^2 - E^2}}{1 + \pi\sqrt{\Delta_0^2 - E^2}/(\hbar\omega_c)}. \quad (11)$$

(b) Nonideal (S-I-N) interface at low energies.—For $E \ll \min\{\Delta_0, \hbar\sqrt{\tilde{\epsilon}_F^{(N)}/(2m_N X^2)}\}$, the dependence of φ_- on E and X can be neglected. We find

$$E = \Delta_0 \frac{(2n + 1)\pi \pm \arccos(\Gamma_0) - 2X\sqrt{2m_N\tilde{\epsilon}_F^{(N)}}/\hbar}{q + \pi\Delta_0/(\hbar\omega_c)}, \quad (12)$$

where $\Gamma_0 = \Gamma(\pi\nu/2)$ and $q = 2s/(s^2 + w^2 + 1)$. For $s = 1$ and $w = 0$, the spectrum for an ideal interface at small energies emerges. In the opposite limit of a very bad interface ($s^2 + w^2 \to \infty$), we recover the spectrum of the Landau-level edge states close to a hard wall [15]. In general, Γ_0 oscillates as a function of filling factor ν. Hence, unlike in the ideal case, the bound-state energies of Eq. (12) vary oscillatory with ν. The same is true for the hole probability $B \approx b^2 \leq 1/2$, for which we find

$$B = \frac{1}{2} \frac{q^2/(1 - \Gamma_0^2)}{1 + \sqrt{1 - q^2/(1 - \Gamma_0^2)}}. \quad (13)$$

The minima in the oscillatory dependence of B on ν occur whenever $\tan(\pi\nu/2) = 2w/(1 - s^2 - w^2)$.

Results obtained in the approximate analytical treatment sketched above are expected to be valid only as long as X is not too large. It turns out, however, that they actually provide a good description at $E \approx 0$ for Andreev levels intersecting with the Fermi energy, which are important for small-bias transport. In particular, we obtained a nonvanishing dispersion $\partial E/\partial X$ close to the interface which leads to a finite Hall current $I_X^{(Q,n)} - I_X^{(Q,a)}$. [See Eqs. (8).] Obviously, no coupling via the pair potential is possible for electrons and holes with guiding center far away from the interface, and dispersionless Landau levels are solutions of

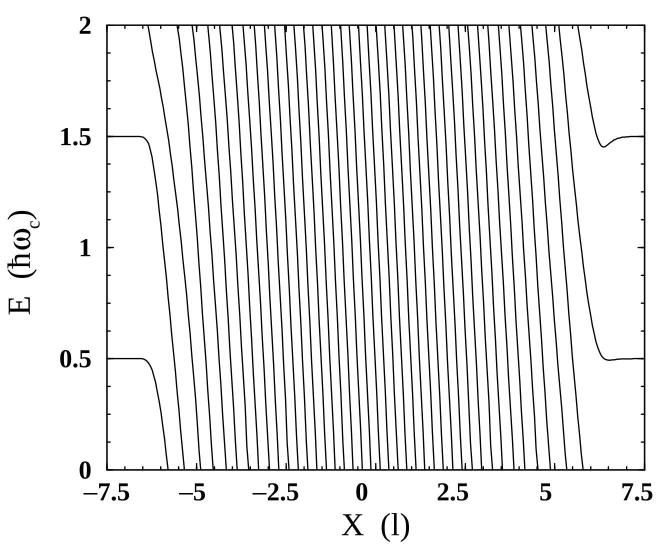

FIG. 2. Andreev-bound-state levels for the ideal S-N interface in a magnetic field, calculated numerically for $\nu = 40$ and $\Delta_0/(\hbar\omega_c) = 2$ by exactly matching solutions of the BdG equation for the normal and superconducting regions.

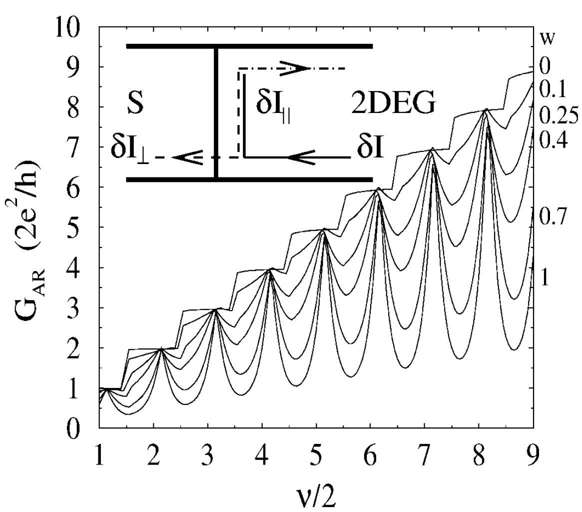

FIG. 3. Conductance G_{AR}, calculated according to Eq. (1) with hole probabilities B_n obtained from the exact numerical matching procedure. We set $\Delta_0 = 0.02\tilde{\epsilon}_F^{(N)}$ and $s = 1$. The value of w for each curve is indicated. The inset shows how the current δI carried by the lower quantum-Hall edge channel (solid line) is distributed at the left interface. (See text.)

the BdG equation. However, as the guiding-center coordinate X gets close to the interface, these Landau levels are bent upward and become Andreev-bound-state levels for $|E| < \Delta_0$. This is seen in the exact numerical calculation of the spectrum (Fig. 2), which also provides a crucial piece of information that is elusive within the approximate analytical treatment: the number n^* of Andreev levels intersecting with the Fermi energy. We find that n^* is twice the integer part of $\nu/2$.

We now apply our findings to study transport in S-2DEG-S structures [7]. In experiment, two S-N interfaces are linked by ordinary quantum-Hall edge channels whose local chemical potentials differ by $\delta\mu$. Generalizing the Büttiker formalism [11] for edge-channel transport, the following picture emerges. (See inset of Fig. 3.) From the lower edge channel, a current $\delta I = \delta\mu(e/h)$ impinges on the left S-N interface. This current divides up into a part $\delta I_{\|}$ flowing parallel to the interface in Andreev edge states studied above, and $\delta I_{\perp}$ which flows across the interface. Chirality of edge states (both Andreev and ordinary) and conservation of quasiparticle probability current completely determines the current parallel to the interface to be $\delta I_{\|} = (1 - 2B_n)\delta\mu(e/h)$. The upper edge channel collects $\delta I_{\|}$ and returns to the right interface, where a similar discussion applies. Hence, the two-terminal conductance $e\delta I_{\perp}/\delta\mu$ in the S-2DEG-S device equals G_{AR} [given in Eq. (1)]. Using amplitudes obtained from the exact numerical matching procedure, we determined the filling-factor dependence of G_{AR} (shown in Fig. 3 for $2 \leq \nu \leq 18$ and various values of w). As anticipated from the approximate analytical result $B_n \approx 1/2$, the ideal interface exhibits almost perfect conductance steps in units of $2e^2/h$. For finite scattering at the interface, oscillations appear in the conductance whose amplitude increases as the interface quality worsens. However, for certain single values of ν, the ideal conductance is reached even at a bad interface. The location of minima and maxima in the field dependence of the conductance can be obtained from our analytical calculation and compare well with results of previous numerical studies [12] based on a representation in terms of scattering states.

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