Quantum Dots with Even Number of Electrons: Kondo Effect in a Finite Magnetic Field

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We show that the Kondo effect can be induced by an external magnetic field in quantum dots with an even number of electrons. If the Zeeman energy B is close to the single-particle level spacing Δ in the dot, the scattering of the conduction electrons from the dot is dominated by an anisotropic exchange interaction. A Kondo resonance then occurs despite the fact that B exceeds by far the Kondo temperature T_K. As a result, at low temperatures $T \ll T_K$ the differential conductance approaches a unitary limit $G_K \sim e^2/\pi\hbar$. A possible experimental realization of this effect is discussed.

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Zero-bias anomaly of tunneling conductance, discovered in the early 1960s, has been explained in terms of scattering by magnetic impurities located in the insulating layer of the tunneling junction [1], in close analogy with the Kondo explanation of the resistivity minimum in metals [2]. Recently, this problem gained renewed attention following theoretical predictions that very similar effects should be detectable in tunneling of electrons through small semiconductor quantum dots [3,4]. It was indeed observed in quantum dots formed in GaAs/AlGaAs heterostructures by the gate-depletion technique [5–8].

In a quantum dot, a finite number $\mathcal{N}$ of electrons is confined in a small region of space. The electrostatic potential of the dot can be tuned with the help of a capacitively coupled gate electrode. By varying the gate voltage, one can switch between Coulomb blockade valleys, where an addition or removal of a single electron to the dot is associated with large charging energy E_c. In this regime, fluctuations of charge are suppressed, and $\mathcal{N}$ is a well defined integer, either even or odd. Transport, however, is still possible by means of virtual transitions via excited states of the dot (this mechanism is known as cotunneling). If $\mathcal{N} =$ odd, the dot has nonzero total spin, and the cotunneling can be viewed as a magnetic exchange. The spin-flip processes make a dominant contribution $G_K \sim e^2/\pi\hbar$ to the differential conductance at low temperature $T \ll T_K$, when the scattering cross section approaches the unitary limit [3]. At finite values of the source-drain voltage $eV \gg T_K$ the nonequilibrium-induced decoherence cuts off the sequence of the spin-flip events that lead to the formation of the Kondo resonance [4] in a similar way as thermal fluctuations do. Therefore, the width of the peak of differential conductance at zero bias is of the order of T_K. If a magnetic field is applied to the system, the zero-bias peak splits into two peaks at $eV = \pm B$, where $B = g\mu_B B_\parallel$ is the Zeeman energy [9]. These peaks are observable even at $eV, B \gg T_K$ [4,6,7]. However, even at $T = 0$, the value of the differential conductance at the peaks never reaches the unitary limit [4,10,11].

For $\mathcal{N} =$ even this consideration is inapplicable, since in the ground state of the *spin-degenerate* quantum dot all single-particle energy levels are occupied by pairs of electrons with opposite spins, and the total spin of the dot is zero. Therefore, the Kondo physics is not expected to emerge in this case. Yet, as we demonstrate below, quantum dots with $\mathcal{N} =$ even subject to an external magnetic field may exhibit a generic Kondo effect, which shows up at a certain value of the Zeeman energy $B \gg T_K$.

In order to elucidate this peculiar scenario, let us recall that in quantum dots charge and spin excitations are controlled by two energy scales (E_c and Δ, respectively), which typically differ by an order of magnitude [12]. This distinction enables one to change the spin state of the dot leaving its charge state intact. If $\mathcal{N} =$ even, the ground state of the dot has spin $S = 0$. The lowest excited state with nonzero spin $S = 1$ has energy Δ (see Fig. 1). If an external magnetic field is now applied, these two states are affected quite differently. In particular, for $B = \Delta$, they become degenerate (see Fig. 2). Since they differ by flipping the spin of a single electron in the dot, this system is a natural candidate for realizing the Kondo effect. Moreover,

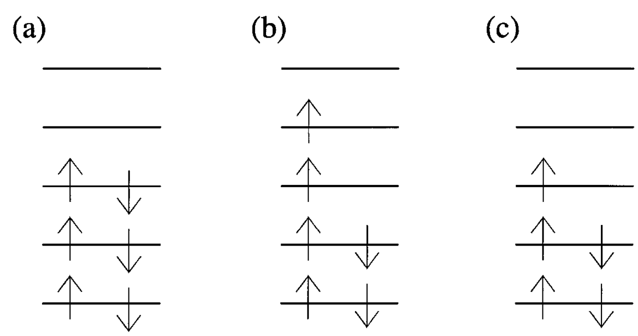

FIG. 1. (a) The spinless ground state of the dot with $\mathcal{N} =$ even electrons. (b) Excited state which has $S^z = 1$. States (a) and (b) differ by adding a spin-down or spin-up electron accordingly to the state $|\Omega\rangle$ of $\mathcal{N} - 1$ electrons in the dot, shown at (c). The states (a) and (b) are denoted as $|\downarrow\rangle$ and $|\uparrow\rangle$ in (5).

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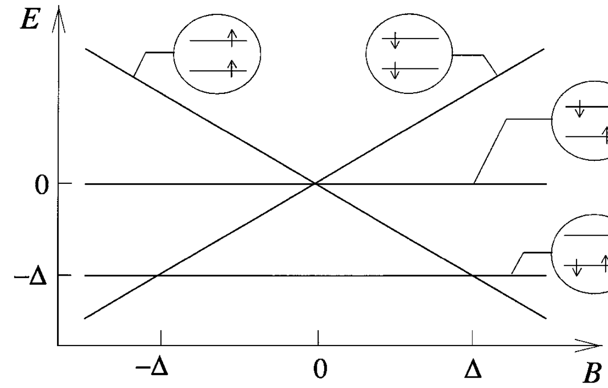

FIG. 2. Low-energy states of a spin-degenerate quantum dot in magnetic field.

this Kondo effect is unique in a sense that the presence of a large magnetic field $B = \Delta \gg T_K$ is a necessary condition for its very occurrence.

To check this concept we consider a quantum dot such as the one studied in [5–8]. For simplicity of presentation we assume, for the moment, that it is attached to a *single* metallic electrode. An appropriate Hamiltonian for modeling it is then

$$\mathcal{H} = H_0 + H_d + H_T\,. \tag{1}$$

Here the Hamiltonian of the lead electrons is

$$H_0 = \sum_{k\sigma} \epsilon_k \psi^\dagger_{k\sigma}\psi_{k\sigma}\,, \tag{2}$$

where ϵ_k is the energy measured from the Fermi level ϵ_F and the fermion operator $\psi_{k\sigma}$ annihilates electron of momentum k and spin projection σ. The dot Hamiltonian H_d is diagonal in the space containing just the two single-particle states whose energy levels are closest to ϵ_F. (Note that the Anderson model description of the $\mathcal{N}$ = odd case [3,4,10] is based on a similar approximation, which is valid if the transparency of the tunneling junctions is small [13].) Thus,

$$H_d = \sum_{p\sigma} \frac{1}{2}\,(p\Delta - \sigma B) d^\dagger_{p\sigma} d_{p\sigma} + E_c(N-2)^2, \tag{3}$$

where $N = \sum_{p\sigma} d^\dagger_{p\sigma} d_{p\sigma}$, $p = \pm 1$ refers to single-particle energy levels in the dot, and $\sigma = \pm 1$ stands for up and down spin. In writing the interaction term in (3) we assumed that the system is tuned to the middle of the $\mathcal{N}$ = even valley of the Coulomb blockade. Note also that an in-plane magnetic field has no influence on the two-dimensional electron gas in the lead, provided that $B \ll \epsilon_F$ (which is the case for $B \sim \Delta$) [9]. The coupling between the dot and the electron gas is described by the tunneling Hamiltonian

$$H_T = \sum_{p\sigma} t_p \psi^\dagger_\sigma d_{p\sigma} + \text{H.c.}, \qquad \psi_\sigma = \frac{1}{\sqrt{L}} \sum_k \psi_{k\sigma}\,, \tag{4}$$

where L is a normalization constant, and we have allowed an explicit dependence of the tunneling amplitudes on p. The two states of the dot which become degenerate at $B = \Delta$ are

$$|\Uparrow\rangle = d^\dagger_{+1\uparrow}|\Omega\rangle, \qquad |\Downarrow\rangle = d^\dagger_{-1\downarrow}|\Omega\rangle\,, \tag{5}$$

with $|\Omega\rangle = d^\dagger_{-1\uparrow}|0\rangle$, in which $|0\rangle$ is the ground state of the dot with $\mathcal{N} - 2$ electrons. It is useful to define spin operators built on the states (5):

$$S^z = \frac{1}{2}\,(|\Uparrow\rangle\langle\Uparrow| - |\Downarrow\rangle\langle\Downarrow|),$$
$$S^+ = S^x + iS^y = |\Uparrow\rangle\langle\Downarrow|\,.$$

These operators act on different *spin* states of the dot. Since $E_c \gg \Delta$, virtual charge excitations to states with $N \neq 2$ can be integrated out by means of a Schrieffer-Wolf transformation. The resulting effective Hamiltonian now reads

$$H = H_0 + H_p + H_{\text{ex}}\,. \tag{6}$$

It contains a potential scattering part

$$H_p = U_c\rho + U_s\sigma^z, \tag{7}$$

and an anisotropic exchange interaction

$$H_{\text{ex}} = J^z_c \rho S^z + J^z_s \sigma^z S^z + \frac{1}{2} J^\perp(\sigma^+ S^- + \sigma^- S^+)\,. \tag{8}$$

The operators appearing in (7) and (8) act on the conduction electrons at the site of the dot. They are defined as

$$\rho = \tfrac{1}{2}(\rho_\uparrow + \rho_\downarrow), \qquad \sigma^z = \tfrac{1}{2}(\rho_\uparrow - \rho_\downarrow),$$
$$\rho_\sigma = \psi^\dagger_\sigma\psi_\sigma\,, \qquad \sigma^+ = \psi^\dagger_\uparrow\psi_\downarrow, \qquad \sigma^- = \psi^\dagger_\downarrow\psi_\uparrow\,. \tag{9}$$

The various coefficients in (8) are $J^z_s = 2U_s = 2(t^2_{+1} + t^2_{-1})/E_c$, $J^\perp = 4t_{+1}t_{-1}/E_c$, and $J^z_c = -2U_c = 2(t^2_{+1} - t^2_{-1})/E_c$. It should be noticed that the value of U_c depends strongly on the choice of intermediate states entering the calculation. Including single-particle energy levels other than those two closest to ϵ_F results in some $\sim E_c/\Delta$ additional contributions of the same order to U_c. In addition to the operators listed in (7) and (8) there also appears a term multiplying S^z. It represents a correction to the level spacing Δ, which does not lift the degeneracy of the states (5) if B is properly tuned (we assume that tunneling alone does not change the symmetry of the dot ground state).

Inspecting the Hamiltonian (6)–(8) we notice that, unlike the situation encountered in standard (bulk) Kondo effect, the charge and spin degrees of freedom are not separated. Rather, they are coupled through the term $J^z_c \rho S^z$. In addition, the potential scattering component H_p contains a term $U_s\sigma^z$ in the spin channel. The main effect of H_p is to introduce small spin-dependent corrections to the density of states [14]. Specifically, denoting by ν_σ the density of states of electrons with spin σ, and by ν_0 the (unperturbed) density of states at the Fermi level, it is sufficient to replace the weakly energy-dependent functions ν_σ by their values at ϵ_F,

$$\nu_\sigma = \frac{\nu_0}{1 + (\pi \nu_0 U_\sigma)^2}, \qquad U_\sigma = \frac{1}{2}(U_c + \sigma U_s). \tag{10}$$

The effect of the remaining exchange interaction H_{ex} can be studied with the help of the standard scaling procedure, which results in a familiar set of equations [15]

$$\frac{dJ^z}{d\ln D} = -(J^\perp)^2, \qquad \frac{dJ^\perp}{d\ln D} = -J^z J^\perp, \tag{11}$$

for the dimensionless coupling constants

$$J^z = (J^z_\uparrow + J^z_\downarrow)/2,$$
$$J^z_\sigma = \nu_\sigma(J^z_s + \sigma J^z_c), \qquad J^\perp = \sqrt{\nu_\uparrow \nu_\downarrow}\, J^\perp. \tag{12}$$

Since initially $J^z \approx \nu_0 J^z_s > 0$, the solution of Eqs. (11) flow to the strong coupling fixed point $J^z_\sigma, J^\perp \to \infty$.

Experimentally, the properties of this system can be probed by means of transport spectroscopy [5–8], when the dot is connected by tunneling junctions to the source and drain electrodes. To describe this situation, we add an additional index q ($q = R/L$ for the right/left electrodes) to the operators, which create or annihilate conduction electrons: $H_0 = \sum_{qk\sigma} \epsilon_k \psi^\dagger_{qk\sigma} \psi_{qk\sigma}$, $H_T = \sum_{qp\sigma}(t_{qp} \psi^\dagger_{q\sigma} d_{p\sigma} + \text{H.c.})$. We are interested in the contribution to the tunneling conductance due to the Kondo effect. The potential scattering terms do not destroy the effect, as we have seen above. Therefore, we will ignore these terms [thereby neglecting small corrections to the densities of states, similar to (10)]. It is convenient to perform a canonical transformation [3]

$$a_\sigma = \alpha_\sigma \psi_{L\sigma} + \beta_\sigma \psi_{R\sigma}, \qquad c_\sigma = \alpha_\sigma \psi_{L\sigma} - \beta_\sigma \psi_{R\sigma}, \tag{13}$$

where

$$\alpha_\uparrow = t_{L,+1}/t_\uparrow, \qquad \beta_\uparrow = t_{R,+1}/t_\uparrow,$$
$$\alpha_\downarrow = t_{L,-1}/t_\downarrow, \qquad \beta_\downarrow = t_{R,-1}/t_\downarrow,$$

and

$$t_\uparrow = \sqrt{t^2_{L,+1} + t^2_{R,+1}}, \qquad t_\downarrow = \sqrt{t^2_{L,-1} + t^2_{R,-1}}. \tag{14}$$

Unlike in [3], the coefficients in (13) are spin dependent as a result of the asymmetry of the tunneling amplitudes. It turns out that only a_σ enter the interaction terms in the effective Hamiltonian which acquire the same form, as (6) with

$$J^z_c = \frac{2(t^2_\uparrow - t^2_\downarrow)}{E_c}, \qquad J^z_s = \frac{2(t^2_\uparrow + t^2_\downarrow)}{E_c}, \qquad J^\perp = \frac{4 t_\uparrow t_\downarrow}{E_c}, \tag{15}$$

and with definitions of the operators analogous to (9) (with ψ_σ replaced by a_σ).

In the weak coupling regime $T \gg T_K$ (the characteristic energy scale of the problem—the Kondo temperature T_K—is discussed below) the Kondo contribution G_K to the differential conductance can be calculated perturbatively from (8) and (13). The resulting expression is lengthy; therefore we present it only for the symmetric case $t_{qp} = t_q$, when $t_\uparrow = t_\downarrow = t$ and $J^z_s = J^\perp = J = 4t^2/E_c$. In this case the result can be written in a compact form,

$$G_K = \frac{e^2}{\pi\hbar} g_0\left(\frac{3\pi^2/8}{\ln^2(T/T_K)}\right), \qquad g_0 = \left(\frac{2t_L t_R}{t^2_L + t^2_R}\right)^2. \tag{16}$$

In the strong coupling regime $T \ll T_K$ the spin-flip scattering is suppressed, and the system allows an effective Fermi-liquid description (see, for example, [16]). The zero-bias conductance then follows immediately from the Landauer formula,

$$G_K = \frac{e^2}{2\pi\hbar}\sum_\sigma \mathcal{T}_\sigma, \qquad \mathcal{T}_\sigma = (2\alpha_\sigma \beta_\sigma)^2. \tag{17}$$

In the symmetric case (17) reduces to $G_K = (e^2/\pi\hbar) g_0$. By virtue of the universality of the Kondo model, the two independent parameters, g_0 and T_K, are sufficient for the description of G_K in the whole temperature range $T \ll \Delta$. Notice that, due to the asymmetry of the coefficients in (13), the transmission probabilities $\mathcal{T}_\sigma$ retain the spin dependence even in the unitary limit, unlike in the $\mathcal{N} =$ odd Kondo effect [3]. This reveals itself in the *spin* current in response to the applied voltage, with the corresponding *spin* conductance given by $G^S_K = (e^2/2\pi\hbar)\sum_\sigma \sigma \mathcal{T}_\sigma \neq 0$. However, this effect might be difficult to measure.

If there is a finite bias $eV \gg T_K$ or if the magnetic field departs from the degeneracy points $B = \pm\Delta$, the situation resembles that encountered for the case $\mathcal{N} =$ odd [4,10]: At $B = \pm\Delta$, the conductance exhibits peaks near zero bias, whose width saturates to T_K in the Kondo regime $T \ll T_K$. When the degeneracy is lifted, each of these peaks splits into two. Therefore, the peak positions in the (B, eV) plane are located at those points which satisfy either $|B - \Delta| \approx eV$ or $|B + \Delta| \approx eV$. For a fixed $eV \neq 0$, these equations have four solutions for B.

The feasibility of experimental realizations of the proposed Kondo effect depends crucially on the value of the Kondo temperature T_K. Since for $\nu_\sigma = \nu_0$ the scaling invariant [15] $C^2 = (J^z)^2 - (J^\perp)^2 = (\frac{2\nu_0}{E_c})^2 (t^2_\uparrow - t^2_\downarrow)^2 \geq 0$, the first of the scaling equations (11) can be written as $dJ^z/d\ln D = C^2 - (J^z)^2$, which after integration results in

$$\ln\frac{E_c}{D} = \frac{1}{2C}\ln\left[\left(\frac{J^z - C}{J^z + C}\right)\left(\frac{J^z_0 + C}{J^z_0 - C}\right)\right].$$

Here we have assumed that the initial bandwidth is of the order of E_c, and J^z_0 is the bare value of J^z. The condition $J^z(D = T_k) \sim 1$ gives the logarithmic estimate of the Kondo temperature. Since $C \ll 1$, one finds

$$T_K \sim E_c \exp[-A/J^z_0], \tag{18}$$

where $A = \frac{1}{2\lambda}\ln(\frac{1+\lambda}{1-\lambda})$, $\lambda = C/J_0^z$, and $0 \leq \lambda < 1$. In the isotropic limit $\lambda \rightarrow 0$ one has $A \rightarrow 1$ and (18) reduces to the usual expression $T_K \sim E_c \exp(-1/J_0^z)$. For a given J_0^z this value is significantly higher than that corresponding to the strongly anisotropic limit. When $\lambda \rightarrow 1$ the factor A diverges as $\ln(1-\lambda)^{-1}$, and $T_K/E_c \rightarrow 0$ as $(1-\lambda)^{1/J_0^z}$. The parameters J_0^z and C which control the Kondo temperature T_K can be expressed in terms of the Kondo temperatures $T_K^{\mathcal{N}\pm 1} \sim E_c \exp(-1/\nu_0 J_{\mathcal{N}\pm 1})$ for the nearby Coulomb blockade valleys with an odd number of electrons $\mathcal{N} \pm 1$, since the corresponding exchange constants are given by $J_{\mathcal{N}-1} = 4t_\downarrow^2/E_c$ and $J_{\mathcal{N}+1} = 4t_\uparrow^2/E_c$:

$$J_0^z \approx \frac{1}{2}\left(\frac{1}{\ln E_c/T_K^{\mathcal{N}-1}} + \frac{1}{\ln E_c/T_K^{\mathcal{N}+1}}\right),$$

$$C \approx \frac{1}{2}\left|\frac{1}{\ln E_c/T_K^{\mathcal{N}-1}} - \frac{1}{\ln E_c/T_K^{\mathcal{N}+1}}\right|.$$

From these equations and from (18) it follows that

$$\min T_K^{\mathcal{N}\pm 1} \lesssim T_K \lesssim \max T_K^{\mathcal{N}\pm 1}. \tag{19}$$

Thus, T_K is estimated to be between its corresponding figures at the neighboring Coulomb blockade valleys with an odd number of electrons. It ensures the observability of the proposed effect in the systems studied in [5–8].

In conclusion, we argue in this Letter that spin-degenerate quantum dots with an even number of electrons exhibit the Kondo effect in a finite magnetic field, when the Zeeman energy is equal to the single-particle level spacing in the dot, and, therefore, is much larger than the Kondo temperature. The effect appears due to a large difference between the characteristic energy scales for spin and charge excitations of quantum dots, and cannot be realized within the conventional Kondo systems in which itinerant electrons interact with magnetic impurities.

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[1] A. F. G. Wyatt, Phys. Rev. Lett. **13**, 401 (1964); R. A. Logan and J. M. Rowell, Phys. Rev. Lett. **13**, 404 (1964); J. M. Rowell and L. Y. L. Shen, Phys. Rev. Lett. **17**, 15 (1966); J. Appelbaum, Phys. Rev. Lett. **17**, 91 (1966); P. W. Anderson, Phys. Rev. Lett. **17**, 95 (1966).

[2] J. Kondo, Prog. Theor. Phys. **32**, 37 (1964).

[3] L. I. Glazman and M. E. Raikh, JETP Lett. **47**, 452 (1988); T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).

[4] Y. Meir, N. S. Wingreen, and P. A. Lee, Phys. Rev. Lett. **70**, 2601 (1993).

[5] D. Goldhaber-Gordon *et al.*, Phys. Rev. Lett. **81**, 5225 (1998); D. Goldhaber-Gordon *et al.*, Nature (London) **391**, 156 (1998).

[6] S. M. Cronenwett, T. H. Oosterkamp, and L. P. Kouwenhoven, Science **281**, 540 (1998).

[7] J. Schmid, J. Weis, K. Eberl, and K. von Klitzing, Physica (Amsterdam) **256-258B**, 182 (1998).

[8] F. Simmel *et al.*, Phys. Rev. Lett. **83**, 804 (1999).

[9] Under the conditions of the experiments [5–8], the direction of the magnetic field should be in the plane of the two-dimensional electron gas, in order to prevent the formation of the quantum Hall states in the lead.

[10] N. S. Wingreen and Y. Meir, Phys. Rev. B **49**, 11 040 (1994).

[11] A. Kaminski, Yu. V. Nazarov, and L. I. Glazman, Phys. Rev. Lett. **83**, 384 (1999).

[12] Typical values for the systems, studied in [5–8], are $E_c \sim$ 1 meV, $\Delta \sim 0.1$ meV.

[13] L. I. Glazman, F. W. J. Hekking, and A. I. Larkin, Phys. Rev. Lett. **83**, 1830 (1999).

[14] If a sharp band cutoff is imposed, it also results in a formation of a bound state outside the band, which does not affect the low-energy properties of the system [P. A. Wolf, Phys. Rev. **124**, 1030 (1961)]; see also A. Zawadowski *et al.*, Phys. Rev. B **56**, 12 947 (1997), for a recent discussion.

[15] P. W. Anderson, J. Phys. C **3**, 2436 (1970).

[16] P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).