Noncausal Time Response in Frustrated Total Internal Reflection?

John J. Carey, Justyna Zawadzka, Dino A. Jaroszynski, and Klaas Wynne*
Femtosecond Research Centre, Department of Physics and Applied Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom
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Tunneling of photons in frustrated total internal reflection has been studied in the time domain with single-cycle femtosecond pulses. It is seen that both the phase and energy of the pulse travel faster than the speed of light in vacuum. Theoretical analysis of the experiments shows that the time-response function for electromagnetic waves propagating in the air gap is noncausal. However, it is found that superluminal signal propagation is not possible in this case because of the inevitable diffractive spreading of the signal beam.

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Waves propagating, for example, along interfaces, through waveguides below the cutoff frequency or through periodic structures at a frequency in the band gap, are evanescent. The wave vector of an evanescent field has one or more imaginary elements resulting in an exponential decay of the field amplitude with distance in one or more spatial directions. The quantum-mechanical equivalent of evanescent-wave propagation is tunneling in which the wave function decays inside a classically forbidden barrier [1,2]. Evanescent-wave phenomena are becoming ever more important as they are a crucial component of many new techniques and devices. For example, evanescent electromagnetic waves are central to many surface-sensitive spectroscopies and various forms of near-field microscopy. Evanescent electron waves occur in submicrometer semiconductor devices [3]. Tunneling has been studied extensively since its discovery early this century. However, the vast majority of these studies were performed in the frequency domain, thereby obscuring some bizarre time-domain phenomena. The most curious phenomenon in tunneling is superluminal propagation in which it may appear that signals travel faster than the speed of light in vacuum. In this paper, new data will be presented demonstrating superluminal propagation entirely measured and analyzed in the time domain.

Superluminal propagation is a result of the evanescent character of a wave. Since the evanescent component of a wave does not oscillate with distance, it does not accumulate any phase and therefore propagates through the evanescent region with zero delay. A typical example is propagation through a waveguide whose transverse dimension is less than half a wavelength. Studies of evanescent waves in waveguides, using long microwave pulses [4] and frequency-domain methods [5], have shown that the phase and group velocity [6] can be superluminal. In addition, it has been claimed [7] that in this case *information* is transferred superluminally in contradiction with special relativity theory. The usual argument against superluminal information transfer [2,8,9] is that communication systems obey the principle of causality; that is, the transmitted wave is related to the input wave by

$$\psi_{\mathrm{out}}(t) = \int_{-\infty}^{\infty} d\tau \; \psi_{\mathrm{in}}(t - \tau) r(\tau - x/c), \qquad (1)$$

where the response function $r(t)$ is zero for negative argument. It can be shown [10] that if the principle of causality holds, any point of nonanalyticity (i.e., a discontinuity in the value of ψ_{in} or its derivatives) will travel at most at the speed of light. This property has been used to argue that these points must therefore define "information." The counter argument [7] is that points of nonanalyticity have finite power at infinite frequency, which is unrealizable with practical communication devices.

The solution to this paradox [11] derives from a practical application of the principle of causality. Any system that obeys Eq. (1) must have a cutoff frequency above which the waves travel with velocity c. As all communication systems have to be switched on and off, the spectrum of the signal is inevitably broadened. Therefore, any signal advanced by superluminal propagation has nonevanescent components traveling subluminally. Since evanescent waves are strongly attenuated, whereas nonevanescent waves are not, it can be shown that superluminal communication is impossible due to a low signal-to-noise ratio exactly in those situations where the temporal advance made is larger than the inverse bandwidth of the signal. All experimental "demonstrations" of superluminal communication [7] have therefore only shown temporal advances less than the width of a signal pulse. The above argument against the possibility of *useful* superluminal communication depends on the assumption that the principle of causality applies. For example, causality does not appear to apply to the simple case of a wave breaking on a beach. When a wave front approaches a beach with velocity v and angle of incidence θ, the point where the wave breaks travels with velocity $v/\sin\theta$, which can exceed the speed of light for small θ (a shadow effect). This scheme cannot be used for superluminal

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communication but perhaps there are other, less trivial, schemes where it can.

Here we will describe a study of frustrated total internal reflection (FTIR) in which evanescent electromagnetic waves travel in the air gap between two prisms [12,13]. It has been shown using an indirect technique [14] that light travels superluminally in this gap. It was suggested that a short *pulse* would, in fact, propagate subluminally in this case. Our experiments have been performed entirely in the time domain with laser pulses containing only a single cycle of the electromagnetic field. This allows analysis of the results without the ambiguities inherent in indirect techniques. It is found that the pulses are advanced more than ever seen before: more than a pulse width. Theoretical analysis shows that in FTIR part of the incoming pulse travels backwards in time. This apparently violates the principle of causality in a way that is different from the shadow effect. However, it will be shown that FTIR cannot be used for superluminal signal exchange.

The generation and detection of single-cycle terahertz pulses is described more fully elsewhere [11,15]. The laser system used is a Ti:sapphire oscillator and regenerative amplifier producing 150-fs pulses at a 250-kHz repetition frequency resulting in 600-mW average power at 800 nm. The beam from this laser is split into two as it enters the THz setup. One is the gate beam, which is used for detection by electro-optic sampling with a $\langle 110 \rangle$ cut 1-mm-thick ZnTe crystal [15]. The other 78% is incident on a photoconductive dipole antenna consisting of a 5×7-mm piece of low-temperature-grown GaAs (LT-GaAs) joined to two copper strips with silver paint giving an exposed area of 3×5 mm. The antenna is placed after the focus of an $f = 5$ cm lens so that the beam covers the exposed area. With 2-kV bias applied, a 0.8-ps THz pulse with a 1-mm center wavelength is produced. A parabolic mirror ($f = 12$ cm) is used to collimate the beam emitted by the antenna, while a second identical mirror placed 20 cm away focuses the beam onto the ZnTe crystal while allowing the gate beam to pass through. The polarization of the gate beam is rotated by the Pockels effect induced by the THz pulse and is detected with a quarter-wave plate and two balanced photodiodes. Scanning the delay of the gate beam at 6 Hz gives a real-time electric-field trace. The traces are averaged for 5–10 min to give a signal-to-noise ratio of ~100 even when the transmission through the setup is low.

Two right-angle Teflon prisms of 40-mm side length are placed in the beam between the two parabolic mirrors. One prism is fixed to the table while the other is translated as shown in Fig. 1 with a high precision (10 nm) motorized translation stage. Field traces with the gap between the prisms set to zero ($L = 0$) are directly compared with traces at nonzero gap. Translating the second prism away from $L = 0$ results in the creation of an air gap between the prisms and the removal of a column of air outside the prism pair. If the speed of light in air were independent of the presence of the prisms, the two experiments (at $L = 0$ and $\neq 0$) would give the same result apart from optical path length changes due to refraction. Any changes in the optical path length are indicated in these experiments as a relative delay. The refractive index of Teflon was measured using a 7-mm slab cut from the same piece used to fabricate the prism pair. Taking into account systematic errors leads to an index of 1.430 ± 0.006 in the wavelength range studied with negligible absorption. The critical angle for total internal reflection is therefore $44.4° \pm 0.2°$.

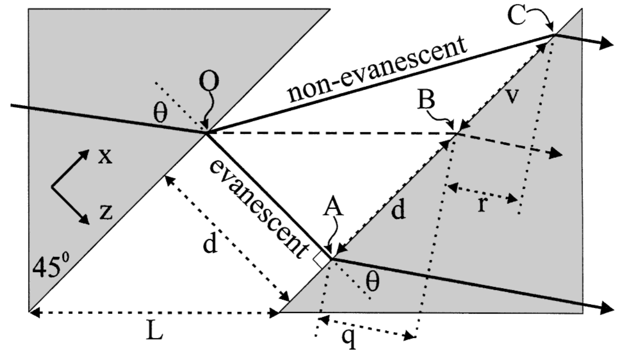

FIG. 1. Light paths through prisms in frustrated total internal reflection (FTIR).

Several series of experiments were performed in which L was varied between 0 and 20 mm, the incidence angle was varied between 35° and 55°, and with p and s polarization of the THz beam. Experimentally the difference between the two polarization directions is slight except for higher transmission for p-polarized light. Figures 2 and 3 show data for angles of incidence where the waves in the gap are either evanescent or nonevanescent. The data were

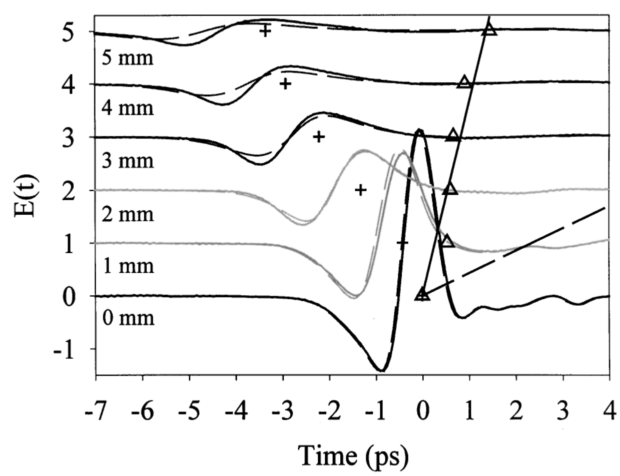

FIG. 2. Experimental FTIR data, unmodified except for a vertical offset for clarity. Shown are the experimental (solid line) and theoretical (dashed line) transmitted pulse shapes of a p-polarized THz pulse for prism spacing $L = 0-5$ mm. The angle of incidence is $\theta = 44.8°$ (evanescent propagation). The crosses are the centroid delays (average arrival time of the energy) calculated from the experimental transmitted pulse shapes and the triangles are the perpendicular-crossing times calculated from the experimental centroid delays by correcting for optical path length changes in the second prism. The dashed line indicates the delays expected for a perpendicular crossing at the speed of light.

initially analyzed by Fourier transformation and calculation of transmission, phase, phase delay, and group delay. In the evanescent cases, it is found that the transmission varies from unity at low frequencies to zero at high frequencies ($\sim$60 cm^{-1}) and that the transmission spectrum narrows with increasing gap. In both the evanescent and nonevanescent cases the advance as measured by the group or the centroid delay [11] is on the order of a few picoseconds and larger than the pulse width.

In our theoretical description of FTIR, it is assumed that the two prisms are right-angle prisms as shown in Fig. 1. The input beam has an angle of incidence on the first glass-air interface of θ, crosses the gap, and propagates into the second prism with an exit angle of θ. It is convenient to separate the field into components propagating parallel or perpendicular to the gap. Each monochromatic component of the electric field can then be written as [2]

$$\tilde{E}(x,z,\omega) = \tilde{E}(z,\omega)\exp\{i[(n\omega x/c)\sin\theta - \omega t]\}, \quad (2)$$

where x is the coordinate parallel to the interface and z perpendicular, and n is the refractive index of the prisms. The z component of the field is governed by a scalar wave equation [2], which can be used to calculate transmission and reflection coefficients. The x component is a propagating wave with phase and group velocity $c/(n\sin\theta)$. When multiple reflections across the gap are included, the transmission of the z component is

$$\tilde{T}(\omega) = \alpha e^{-\omega\gamma}/(1 + \beta e^{-2\omega\gamma}), \quad (3)$$

where $\gamma \equiv sd/c$, $s \equiv \sqrt{n^2\sin^2\theta - 1}$, and d is the spacing between the prisms perpendicular to the gap. For p-polarized light,

$$\alpha = i4ns\cos\theta/(\cos\theta + nis)^2,$$
$$\beta = -(\cos\theta - nis)^2/(\cos\theta + nis)^2, \quad (4)$$

and similar expressions for s-polarized light are obtained. If θ is larger than the critical angle $\theta_c = \arcsin(1/n)$, the transmitted field is evanescent and it can be shown that $|\beta| = 1$. If $\theta < \theta_c$, $0 > \beta > -1$. Equation (3) is sufficient to describe FTIR in the frequency and time domain. However, one can gain insight in the tunneling process by considering the time-response function.

The time-response function is given by the Fourier transform of Eq. (3). When $\theta > \theta_c$, Eq. (3) has simple poles at $\omega = (m + \frac{1}{2})2\pi i + \ln\beta$, where m is an integer. Contour integration results in the real-valued time-response function

$$T_E(t) = \frac{i\alpha}{2\gamma}\,\frac{e^{(i\pi + \ln\beta)[-it/(2\gamma) - 1/2]}}{1 - e^{2\pi i[-it/(2\gamma) - 1/2]}}. \quad (5)$$

When $\theta < \theta_c$, the integral can be calculated using the geometric series, resulting in the time-response function

$$T_{\mathrm{NE}}(t) = \alpha\sum_{m=0}^{\infty}|\beta|^m\delta(t + i2\gamma m + i\gamma). \quad (6)$$

Numerically, Eq. (5) goes over into (6) at $\theta = \theta_c$. These results may be compared with the free-space case, where the time-response function is $T_{\mathrm{FS}}(t) = \delta(t - d/c)$. Plots of the time-response function are shown in Fig. 4.

In the experiments, we compare the electric field transmitted at a certain gap $L = d\sqrt{2}$ with that when $L = 0$. When the gap is zero, light exiting the first prism at point O (see Fig. 1) enters the second prism at B and travels through a certain amount of material. The transmission function $T(t)$ describes propagation in the z direction only (from O to A). To calculate the response at point B, the wave has to be propagated an additional distance q through the prism. As the gap is increased, the amount of air that the pulse has to travel through outside the prism pair is reduced by an amount b. The "differential" time response is therefore $T_{\mathrm{difference}} = T[t - (nq - b)/c]$, where $q = d\sin\theta$ and $b = L/\cos(\frac{1}{4}\pi - \theta)$. The data are analyzed in the time domain by convoluting the measured pulse at

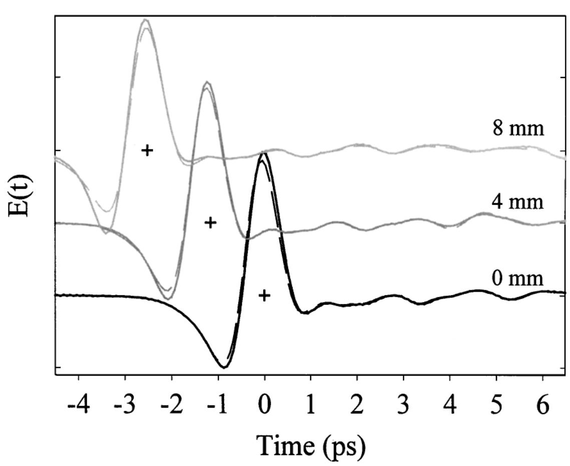

FIG. 3. Same as Fig. 2 except for a prism spacing of $L = 0$, 4, and 8 mm and an angle of incidence of $\theta = 41.1°$ (nonevanescent). The crosses are the centroid delays calculated from the experimental transmitted pulse shapes and can be used to calculate that the pulses cross the gap at a speed of $(0.99 \pm 0.01)c$.

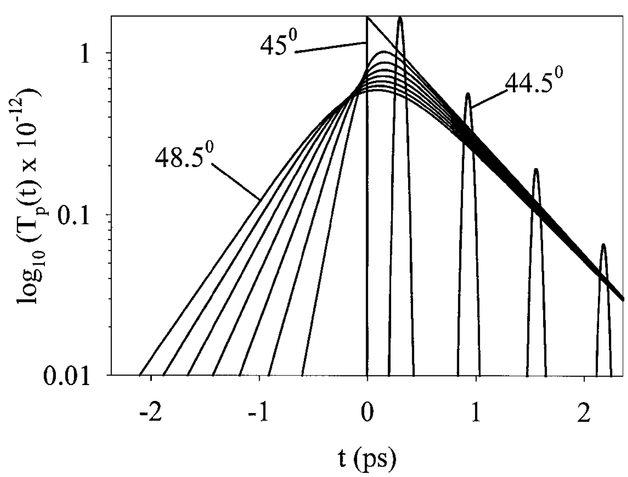

FIG. 4. Plot of the temporal response function $T(t)$ for frustrated total internal reflection. $n = \sqrt{2}$, $L = 1$ mm ($d/c = 2.4$ ps) and $\theta = 44.5, 45, \ldots, 48.5°$. The critical angle is 45°. $T(t)$ has been multiplied with 10^{-12}.

$L = 0$ with the time-response function. Least-squares fitting of the data at $L = 2$ mm is then used to find the exact angle of incidence. Figure 2 shows an example of data acquired under conditions where the transmitted radiation is evanescent between the prisms ($\theta = 44.8°$). Figure 3 shows data taken for $\theta = 41.1°$ ($\theta < \theta_c$) when the transmitted radiation is nonevanescent. In the nonevanescent case, the transmission is nearly unity and data could be obtained for gaps that are orders of magnitude larger than the average wavelength in the pulse.

The raw data presented here show temporal advances larger than a pulse width for both evanescent and nonevanescent waves. In the evanescent case, the temporal advance across the gap is, in fact, much larger than apparent because the waves have to travel through more prism material as the gap is increased (see Fig. 1). In the nonevanescent case, the measured temporal advance is due to the fact that the diffracted beam travels through less material in the second prism as the gap is increased. From the change in the experimentally determined centroid delay [11] as a function of the gap $\Delta\tau_{\text{centroid}}$, one can calculate the perpendicular-crossing time (from point O to A) from $\Delta\tau_{\text{centroid}} - (nq - b)/c$. For the nonevanescent waves, one can calculate the time required to cross from O to C from $\Delta\tau_{\text{centroid}} + (nr + b)/c$, where $r = v\sin\theta$. In the evanescent case, the perpendicular-crossing times shown in Fig. 2 are found to be very close to zero, consistent with the time-response functions shown in Fig. 4. In the nonevanescent case, the centroid of the pulses is found to cross the gap at speed $(0.99 \pm 0.01)c$.

The theory used to describe these data appears to show that the principle of causality does not apply in FTIR. However, in the case of nonevanescent wave propagation, any apparent violation of the principle of causality is due to a trivial shadow effect. In the case of evanescent-wave propagation, the time-response function has a value at negative time (extending to negative infinity) inconsistent with a shadow effect. Of course, to obtain this superluminal effect, the pulses have to travel through prism material in which the pulses are delayed with respect to free space. However, in principle, the gap could be made arbitrarily large and the superluminal advance would outstrip the delay due to the prism material. The transmission of evanescent waves is poor, but this could be compensated for by using a powerful input pulse.

Do the results presented here imply that superluminal communication is possible and causality is violated in a nontrivial way? According to Fig. 4, if the angle of incidence is chosen close to but larger than the critical angle, the signal will travel superluminally with transmission close to unity. However, to have a single angle of incidence, the signal beam has to be perfectly collimated necessitating an infinitely extended beam. This requires infinitely large prisms and hence an infinitely large delay compared to free space. In practice therefore, the prisms must have a finite extent resulting in a distribution of incidence angles. Using Gaussian beam theory [6], it can be shown that there is an uncertainty relation between the $1/e$ widths of the spatial and angular distribution in the beam: $e_\theta e_x = \lambda/\pi n$, where λ is the wavelength and n the refractive index. In our experiment, we measured $e_x = 1.5$ cm implying that the electric-field traces in Fig. 2 may be "contaminated" with as much as 25% nonevanescent waves. Nonevanescent waves have transmission close to unity for any gap, whereas the evanescent waves decay exponentially. Thus, for any *average* angle of incidence $\theta > \theta_c$, the beam still has components with $\theta < \theta_c$, which will end up dominating the signal for sufficiently large gap. Therefore, superluminal communication with FTIR may, strictly speaking, be possible. However, when the gap becomes large enough to make this superluminal communication scheme useful, the signal will be dominated by nonevanescent waves. The response functions derived above are not physical, as they do not include the unavoidable distribution of incidence angles. It is possible that if the (noncausal) time-response functions Eqs. (5) and (6) are integrated over the distribution of incidence angles, the resulting response function will be causal. However, the solution to this problem is far from obvious and may require a more subtle application of electromagnetic theory.

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*Electronic address: klaas.wynne@phys.strath.ac.uk

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