Precision Observables and Electroweak Theories

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We compute the bounds from precision observables on alternative theories of electroweak symmetry breaking. We show that a cutoff as large as 3 TeV can be accommodated by the present data, without any new particles or unnatural fine tuning.

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During the past few years, precision measurements of electroweak observables have probed the standard model of particle physics to the 0.1% level. They now give a 95% C.L. upper bound of 230 GeV on the mass of the standard model Higgs boson [1]. Precision measurements have also constrained many alternative theories to the standard model. For example, they have ruled out many of the most naive technicolor theories [2].

The theory of effective Lagrangians provides a convenient way to describe the low-energy effects of new physics beyond the minimal standard model. One approach is to take the standard model with a fundamental Higgs boson and add a set of $SU(3) \times SU(2) \times U(1)$ invariant higher-dimensional operators, suppressed by a scale Λ. These operators are generated by new physics at the scale Λ, beyond that of the usual standard model. Because the effective theory includes a fundamental Higgs boson, triviality gives the only upper bound on the scale Λ. This approach has recently been used to study the Higgs mass limit that comes from precision measurements. It was shown that the new operators can raise the limit on the Higgs mass as high as 400–500 GeV, barring unnatural cancellations [3].

A second approach is to eliminate the Higgs entirely and parametrize the present data in terms of the standard model fields that have been discovered to date. In this approach, there are no new particles below the scale Λ, which defines the scale of the physics responsible for electroweak symmetry breaking. At low energies, all effects of this physics can be described by a gauge invariant chiral Lagrangian, in which the higher-dimensional operators are suppressed by Λ. This approach is valid for energies $E \lesssim \Lambda$. General unitarity considerations restrict $\Lambda \lesssim 3$ TeV [4].

In this Letter we pursue this second approach and focus on the physics of electroweak symmetry breaking. We will use the precision measurements to constrain the coefficients of the leading higher dimensional operators in the chiral Lagrangian, as a function of the scale Λ. We will find that even for $\Lambda \approx 3$ TeV the present precision data can be accommodated without any new particles or unnatural fine tuning.

If $\Lambda \simeq 3$ TeV, the physics of electroweak symmetry breaking lies outside the reach of the CERN LEP and Fermilab Tevatron colliders. Our analysis indicates that this possibility remains open, despite the 230 GeV upper limit on the mass of the standard model Higgs. We shall see that the data are perfectly consistent with theories in which there are *no* new particles below 3 TeV. Of course, it is an open question whether such theories can actually be constructed, consistent with the data. Nevertheless, our results point to a loophole in the common assertion that the precision data require a Higgs boson or other new physics to be close at hand.

The plan of this Letter is as follows: We start by presenting the gauged chiral Lagrangian associated with electroweak symmetry breaking. We then focus on the two operators that are most important for precision measurements on the Z pole. We compute the effects of these operators on experimental observables and derive limits on their coefficients as a function of the scale Λ. Finally, we discuss our results in the context of alternative scenarios for electroweak symmetry breaking.

The gauged chiral Lagrangian provides a model-independent description of the physics that underlies electroweak symmetry breaking [5,6]. It is valid for energies $E \lesssim \Lambda$, where the new physics becomes manifest.

The Lagrangian is constructed from the Goldstone bosons w^a associated with breaking $SU(2) \times U(1) \rightarrow U(1)$. The fields w^a are assembled into the group element $\Sigma = \exp(2iw^a\tau^a/v)$, where the τ^a are Pauli matrices, normalized to 1/2, and $v = 246$ GeV is the scale of the symmetry breaking. The fields w^a transform nonlinearly under $SU(2) \times U(1)$ transformations, $\Sigma \longrightarrow L\Sigma R^\dagger$, where $L \in SU(2) \equiv SU(2)_L$ and $R \in U(1) \subset SU(2)_R$. The gauge bosons appear through their field strengths, $W_{\mu\nu} = W^a_{\mu\nu}\tau^a$ and $B_{\mu\nu} = B^3_{\mu\nu}\tau^3$, as well as through the covariant derivative, $D_\mu\Sigma = \partial_\mu\Sigma + igW^a_\mu\tau^a\Sigma - ig'\Sigma B^3_\mu\tau^3$.

The gauged chiral Lagrangian is built from these objects. It can be organized in a derivative expansion,

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots, \tag{1}$$

where

$$\mathcal{L}^{(2)} = \frac{v^2}{4}\,\mathrm{Tr}D_\mu\Sigma D_\mu\Sigma^\dagger + \frac{g'^2 v^2}{16\pi^2}\,b_1\,(\mathrm{Tr}T\,\Sigma^\dagger D_\mu\Sigma)^2 + \frac{gg'}{16\pi^2}\,a_1\,\mathrm{Tr}B_{\mu\nu}\Sigma^\dagger W_{\mu\nu}\Sigma\,, \tag{2}$$

and $T = \Sigma^\dagger \tau^3 \Sigma$. The Lagrangian is invariant under SU(2) × U(1) gauge transformations. In the unitary gauge, with $\Sigma = 1$, the terms in $\mathcal{L}^{(2)}$ give rise to the W and Z masses. The terms in $\mathcal{L}^{(4)}$ give rise to "anomalous" three- and four-gauge boson self-couplings; they are of higher order in the derivative expansion, so we do not consider them here.

The coefficients a_1 and b_1 are important because they contain information about the physics of electroweak symmetry breaking. Note that the operator proportional to a_1 preserves weak isospin in the limit $B_\mu^3 \to 0$, while the one proportional to b_1 does not. The coefficients are obtained by matching Green functions in the effective theory with those of the underlying fundamental theory, just below the scale Λ. The coefficients a_1 and b_1 are normalized so that they are naturally $\mathcal{O}(1)$ for a strongly interacting sector with $\Lambda \simeq 3$ TeV. They can be much smaller if the symmetry-breaking sector is weakly coupled; they can be larger if the fundamental theory contains many particles charged under SU(2) × U(1).

In what follows we will study the effects of a_1 and b_1 on the W and Z propagators. These coefficients are closely related to the parameters S and T [2]. The relation is found by renormalizing the coefficients from Λ to the scale M_Z, where S and T are defined. One finds

$$\begin{aligned} S &= S_0 + \frac{1}{6\pi}\log\left(\frac{\Lambda}{M_Z}\right), \\ T &= T_0 - \frac{3}{8\pi c^2}\log\left(\frac{\Lambda}{M_Z}\right), \end{aligned} \tag{3}$$

where $c = \cos\theta_W$, and S_0 and T_0 are fixed in terms of a_1 and b_1 at the scale Λ,

$$S_0 = -\frac{a_1}{\pi}, \qquad T_0 = \frac{b_1}{\pi c^2}. \tag{4}$$

Note that the logarithms are exactly calculable because they come from standard model loops. (We assume explicitly that there are no light particles, such as pseudo-Goldstone bosons, with masses between Λ and M_Z [7].) Equation (3) connects the new physics at the scale Λ with precision measurements at the scale M_Z.

We are now ready to find the constraints imposed by precision electroweak measurements on S and T and, consequently, on the scale Λ and the coefficients a_1 and b_1.

Most global analyses of precision electroweak data are carried out in the context of the standard model with a fundamental Higgs boson. Fortunately, these analyses can be easily converted to the case at hand. One simply subtracts the contributions to S and T from a standard model Higgs boson, evaluated at a reference mass, $M_H^{\rm ref}$, and then adds back the contribution from Eq. (3). In this way one can readily compute the values of S and T that come from the gauged chiral Lagrangian.

The contributions to S and T from a heavy Higgs have been computed in the literature [6]. They are

$$\begin{aligned} S &= -\frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{M_H}{M_Z}\right)\right], \\ T &= \frac{3}{8\pi c^2}\left[\frac{5}{12} - \log\left(\frac{M_H}{M_Z}\right)\right], \end{aligned} \tag{5}$$

where the constant is computed in the $\overline{\rm MS}$ (modified minimal-subtraction) scheme. Note that the logarithmic dependence on the Higgs mass is exactly the same as the logarithmic dependence on Λ in Eq. (3). This is no surprise, because M_H plays the role of Λ, and the standard model renormalization is exactly the same in each case.

With this result, we are ready to make contact with the data. Let $S(m_t, M_H; m_t^{\rm ref}, M_H^{\rm ref})$ and $T(m_t, M_H; m_t^{\rm ref}, M_H^{\rm ref})$ be the standard model S and T parameters, presented as a function of the physical top quark and Higgs boson masses, defined with respect to reference values $m_t^{\rm ref}$ and $M_H^{\rm ref}$, respectively. The values of S and T from the gauged chiral Lagrangian are then given by

$$\begin{aligned} S(m_t, S_0, \Lambda) &= S(m_t, M_H^{\rm ref}; m_t^{\rm ref}, M_H^{\rm ref}) \\ &\quad + S_0 + \frac{5}{72\pi} + \frac{1}{6\pi}\log\left(\frac{\Lambda}{M_H^{\rm ref}}\right), \\ T(m_t, T_0, \Lambda) &= T(m_t, M_H^{\rm ref}; m_t^{\rm ref}, M_H^{\rm ref}) \\ &\quad + T_0 - \frac{5}{32\pi c^2} - \frac{3}{8\pi c^2}\log\left(\frac{\Lambda}{M_H^{\rm ref}}\right). \end{aligned} \tag{6}$$

The physically allowed region of S-T space is determined from a χ^2 fit to fourteen precisely measured electroweak observables. Each observable $\mathcal{O}_i$ is represented by a four-parameter linearized function,

$$\begin{aligned} \mathcal{O}_i &= \mathcal{O}_i^{\rm ref} + s_i S + t_i T + x_i(\alpha_s - \alpha_s^{\rm ref}) \\ &\quad + y_i(\Delta\alpha_{\rm had}^5 - \Delta\alpha_{\rm ref}^5), \end{aligned} \tag{7}$$

where $\mathcal{O}_i^{\rm ref}$ is the standard model value of the observable at the reference values of top quark and Higgs boson masses. The strong coupling α_s is evaluated at the scale M_Z; we take $\alpha_s^{\rm ref} = 0.12$ as the corresponding reference point. In this expression, $\Delta\alpha_{\rm had}^5$ is the five-flavor, hadronic portion of the vacuum polarization correction to the electromagnetic coupling constant at the scale M_Z, and $\Delta\alpha_{\rm ref}^5 = 277.5 \times 10^{-4}$ is its reference point. The coefficients s_i and t_i are computed from the standard model [2]. The coefficients x_i, y_i and the reference values $O_i^{\rm ref}$ are computed using the ZFITTER 6.11 computer code [8]. All coefficients are insensitive to the choice of the reference points.

The fourteen observables are the following: the width Γ_Z of the Z boson [9]; the e^+e^- pole cross section of the Z [9]; the ratio of the hadronic and leptonic partial widths of the Z [9]; the Z-pole forward-backward asymmetries for final-state leptons, b quarks, and c quarks [9];

Z-pole left-right coupling asymmetries for electrons and τ leptons as determined from final-state τ polarization measurements [9]; the Z-pole hadronic charge asymmetry [9]; the left-right cross section asymmetry for Z production [9]; the mass M_W of the W boson [9]; R_-, a quantity constructed from the ratios of neutral- and charged-current ν and $\overline{\nu}$ cross sections [10]; the weak charge of the cesium nucleus [11]; and the weak charge of the thallium nucleus [12]. The fit is performed with $\Delta\alpha^5_{\rm had}$ constrained to the value $(277.5 \pm 1.7) \times 10^{-4}$, as determined by a recent analysis [13]. The χ^2 weight matrix includes correlated errors for the LEP Z line shape parameters. The resulting two-dimensional 68.3% confidence region in S-T space is shown in Fig. 1 for the reference point $(m_t^{\rm ref}, M_H^{\rm ref}) = (175, 500)$ GeV. The one-dimensional 68% confidence intervals for the parameters are

$$S = -0.13 \pm 0.10, \qquad T = 0.13 \pm 0.11\,,$$
$$\alpha_s(M_Z) = 0.119 \pm 0.003\,, \tag{8}$$
$$\Delta\alpha^5_{\rm had}(M_Z) = (277.6 \pm 1.7) \times 10^{-4}\,.$$

Note that the S and T confidence regions (one and two dimensional) implicitly incorporate the uncertainties resulting from the imprecise knowledge of $\alpha_s(M_Z)$ and $\Delta\alpha^5_{\rm had}(M_Z)$.

To test the consistency of our approach, we perform a chi-square fit of the measured values of S and T to the standard model functions $S(m_t, M_H; m_t^{\rm ref}, M_H^{\rm ref})$ and $T(m_t, M_H; m_t^{\rm ref}, M_H^{\rm ref})$, which are calculated with ZFITTER 6.11. The χ^2 weight matrix is obtained from the inverse of the S-T error matrix. We add an additional term to the χ^2 function to include a constraint on the top quark mass [14], $m_t = 174.3 \pm 5.1$ GeV. We then compare the result of this fit with that of a direct fit to the standard model using the same fourteen observables with the same constraints. The standard model fit yields a central value for M_H of 106.3 GeV and a 95% upper limit of 228.5 GeV. The S-T fit yields very consistent values of 107.4 and 228.8 GeV, respectively.

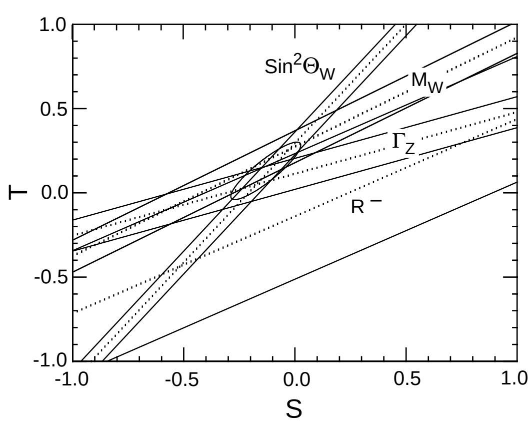

FIG. 1. Fit to S and T from electroweak observables, with $M_H^{\rm ref} = 500$ GeV and $m_t^{\rm ref} = 175$ GeV.

In what follows, we use a similar procedure to derive confidence intervals for the parameters S_0, T_0, and Λ, which characterize the electroweak symmetry breaking sector. We fit the measured values of S and T to the functions defined in equations Eqs. (6), with the same reference masses as above. In addition, we add the same m_t-constraining term to the χ^2 function.

Of course, it is not possible to determine all three of S_0, T_0, and Λ using just two measurements. Indeed, for any fixed Λ, it is always possible to adjust the matching coefficients S_0 and T_0 to fit the low energy data. However, the situation $S \ll S_0$ and $T \ll T_0$ would be unnatural, since it would suggest finely tuned cancellations in Eqs. (3). Indeed, there is no reason to expect any correlation between chiral Lagrangian parameters generated directly at the scale Λ and logarithmic radiative corrections generated in running the theory from Λ down to M_Z. We will see that even for $\Lambda \approx 3$ TeV no such tuning is required.

The result of our fit is shown in Fig. 2. We plot the allowed region for S_0 and T_0 for $\Lambda = (3, 2, 1, 0.5, 0.1)$ TeV. The $\Lambda = 100$ GeV point is shown, although our chiral Lagrangian description is not valid for such a low cutoff. The 68% and 95% C.L. ellipses are shown for $\Lambda = 3$ TeV; the fit yields the central values $(S_0, T_0) = (-0.27, 0.46)$ and the 68% C.L. ranges

$$-0.37 < S_0 < -0.17, \qquad 0.34 < T_0 < 0.58\,. \tag{9}$$

For smaller Λ, the central values for S_0 and T_0 become smaller, as shown in Fig. 2 while the error ellipse retains its size and orientation.

From the relation (4) between (S_0, T_0) and (a_1, b_1), we see that chiral Lagrangian coefficients of order 1 or smaller are needed to fit the precision data, for all reasonable values of Λ. As a measure of the tuning which is required to fit the data, we compute the ratio of the constant term to the logarithm in Eqs. (3); the deviation of this ratio from one

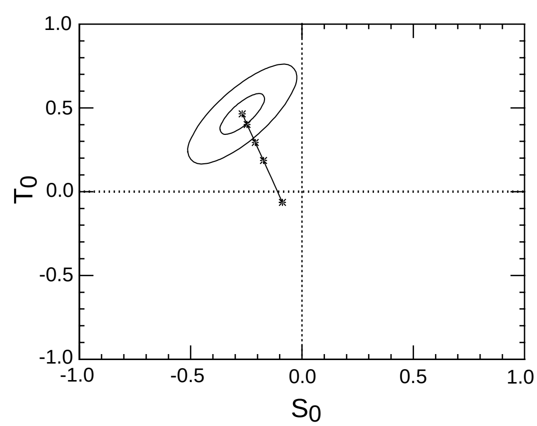

FIG. 2. Fit to S_0 and T_0 from electroweak observables, for $\Lambda = (3, 2, 1, 0.5, 0.1)$ TeV. (The values decrease from the upper left to the lower right.) Both 68% and 95% C.L. ellipses are shown for $\Lambda = 3$ TeV.

is an indication of the degree to which each constant must be adjusted to cancel the logarithm and fit the data at M_Z. Taking the central values $(a_1, b_1) = (0.85, 1.11)$ from the fit at $\Lambda = 3$ TeV, we find a ratio of 1.4 for S and 0.85 for T. Even without including the experimental uncertainties, we see that no significant tuning of a_1 and b_1 is required.

Precision electroweak measurements place a strong upper limit on about 230 GeV on the mass of the Higgs boson in the context of the standard model of particle physics. In this Letter, we have seen that these measurements do *not* rule out alternative theories. Indeed, we find that they permit strongly interacting theories with no new particles up to a scale of 3 TeV. Our results have implications for the design of potential new high energy colliders.

Nevertheless, we have seen that precision measurements place significant constraints on these alternative theories. They constrain the parameters a_1 and b_1 to be of order unity, and for $\Lambda \gtrsim 1$ TeV, they completely fix their signs. It is, of course, an urgent and open question to determine whether a reasonable model can be constructed with these parameters. For example, it has previously been observed that it is difficult to obtain $a_1 > 0$ in naive technicolor theories [2]. In such models, S receives a small positive contribution of approximately 0.1 for each weak doublet in the fundamental theory.

More generally, we would argue that the data disfavor models in which fermion masses are generated directly by the electroweak symmetry breaking dynamics. Fermion masses arise from interactions of the form

$$\Phi_U^{ij}\,\overline{Q}_L^i u_R^j + \Phi_D^{ij}\,\overline{Q}_L^i d_R^j + \Phi_L^{ij}\,\overline{L}_L^i e_R^j\,, \qquad (10)$$

where $i, j = 1, 2, 3$ are flavor indices and Φ_a^{ij}, $a = U, D, L$, are (possibly composite) fields which assume nonzero vacuum expectation values. In the standard model, $\Phi_U^{ij} = \lambda_U^{ij}\Phi$, $\Phi_D^{ij} = \lambda_D^{ij}\Phi^*$, and $\Phi_L^{ij} = \lambda_L^{ij}\Phi$, where Φ is the single Higgs boson and the λ_a^{ij} are 27 Yukawa couplings which break the $U(3)^5$ flavor symmetry. In theories in which these symmetries are dynamically broken, the fields Φ_a^{ij} are dynamical degrees of freedom that carry representations of the flavor symmetry group. When the Φ_a^{ij} are integrated out, they give a contribution to a_1 which includes a trace over a large number of fields. Generically, we expect the trace to be large: in the unrealistically minimal scenario in which the trace is 27 times the contribution of a single scalar, we find $|a_1| = 27 \times (5/72) = 1.9$. A more realistic model would require significant cancellations to achieve the observed value of a_1.

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Note added.—After this work was completed, we became aware of Ref. [15]. In this paper the authors show that the upper bound on Λ is very close to the upper bound on M_H in the standard model when a_1 and b_1 are near zero.

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