Dynamics of a Domain Wall in Soft-Magnetic Materials: Barkhausen Effect and Relation with Sandpile Models

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The CZDE model [P. Cizeau, S Zapperi, G. Durin, and H. E. Stanley, Phys. Rev. Lett. **79**, 4669 (1997)] for the dynamics of a domain wall in soft-magnetic materials is investigated. The equation of motion for the domain wall is reduced to a dimensionless form where the control parameters are clearly identified. In this way we show that in soft-magnetic materials with low anisotropies the noise can be approximated by a columnar disorder, and perturbation theory gives a good estimate of the avalanche exponents. Moreover, the resulting exponents are found to be identical to those obtained for directed Abelian sandpile models. The analogies and differences with these models and the question of self-organized criticality in the Barkhausen effect are discussed.

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The Barkhausen effect (BE) has been taken as an experimental observation of self-organized criticality (SOC) [1]. Based on phenomenological analogies between the BE and sandpile models, such as the existence of power law distributions of avalanche size $P(s) \sim s^{-\tau}$ and duration $P(T) \sim T^{-\alpha}$, some authors have claimed that the BE exhibits SOC behavior [2,3]. On the other hand, it has been shown that the demagnetization field acts as a feedback mechanism which drives the system into the critical state [4–7], supporting the existence of SOC in the BE.

This conclusion has been criticized by other researches which have pointed out that the observation of power law distributions is not necessarily an evidence of SOC [8,9]. There are alternative approaches, like the random field Ising model [10,11], where the power law distributions are a consequence of the scaling properties of disorder. On the other hand, numerical simulations of a micromagnetic model have shown that different regimes may be observed, depending on the ratio between the exchange correlation and structural correlation lengths [12]. In order to observe criticality the tuning of the exchange correlation length is necessary, excluding the occurrence of SOC.

In the central part of the hysteresis loop the Barkhausen jumps in the magnetization are mainly due to domain walls motion, while other effects like nucleation and irreversible rotations can be neglected. Moreover, for low disorder the domain walls do not have overhangs [13], and for long rod geometries one can have long domain walls. In this situation a good model to describe the domain wall dynamics is the one introduced by Cizeau, Zapperi, Durin, and Stanley (CZDE) [5], which is a generalization of the single degree of freedom model of Alessandro *et al.* [14]. This model has several limitations: it neglects nucleation, interactions between domain walls, temperature effects, and it is valid only for low disorder. However, even with these simplifications it gives a satisfactory explanation to some of the features observed in BE experiments [5,7],

Some of the statistical properties resulting from this model have been already investigated in [5]. The upper critical dimension is $d_c = 3$ and the critical exponents were obtained by perturbation theory. However, there are still some open questions. For instance, it is not clear yet if perturbation theory gives the correct exponents or a renormalization group (RG) analysis is required.

In the present work we investigate the CZDE model. First, we reduce the CZDE equation of motion to a dimensionless form where we can clearly identify the control parameters of the model. Then we analyze the form of the noise correlator, taking into account the typical values of the different magnitudes for soft-magnetic materials with low anisotropies. We found out that in this case the noise correlator can be approximated by a columnar disorder and, hence, perturbation theory [5] gives a good estimate of the avalanche exponents. From our dimensional analysis we also explain some features observed in the BE in magnetic materials under an external tensile stress. Finally, we discuss some similarities and differences of the CZDE model and directed Abelian sandpile models, based on the evolution of the avalanche front and the type of disorder.

In the CZDE model a 180° domain wall is modeled by a $(d-1)$-dimensional interface, dividing two regions of opposite magnetization, moving in a d-dimensional environment described by its position $h(x,t)$. Considering the contribution of magnetostatic, ferromagnetic, and magnetocrystalline interactions, one obtains the following equation of motion [5]

$$\lambda \frac{\partial}{\partial t} h(x,t) = \nu_0 \nabla^2 h(x,t) + 2\mu_0 M_s H + \Delta^{1/2}\eta[x, h(x,t)] - 4\mu_0 M_s^2 \mathcal{N} \int \frac{d^{d-1}x'}{V} h(x',t) + \int d^{d-1}x' K(x-x')[h(x',t) - h(x,t)], \tag{1}$$

where λ is a viscosity coefficient, ν_0 is the surface tension of the wall, H is the magnetic field intensity, V is

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the sample volume, M_s is the saturation magnetization, and Δ is the strength of the pinning centers. Long-range demagnetization effects are described by the fourth term in the right-hand side, where $\mathcal{N}$ is the demagnetization factor. Dipolar interactions are characterized by the fifth term where the kernel $K(x)$ has Fourier transform $\tilde{K}(k) \propto \mu_0 M_s^2 |k| \cos^2\theta$, θ being the angle between k and the magnetization. $\eta(x,h)$ is a Gaussian uncorrelated noise due to lattice defects or other factors, with zero mean and noise correlator

$$\langle \eta(x,h)\eta(x',h')\rangle = \delta^d(x - x')R(h - h'). \tag{2}$$

$R(h)$ has the asymptotic behaviors $R(h) \approx 1$ for $h \ll a_\perp$ and $R(h) \ll 1$ for $h \gg a_\perp$, where $a_\perp$ is a characteristic correlation length of the disorder, in general of the order of the distance between the pinning centers.

For the analysis developed below it is appropriate to express the CZDE equation of motion in dimensionless variables. We take as a characteristic length

$$l_M = \frac{\nu_0}{4\mu_0 M_s^2}, \tag{3}$$

which is the characteristic length above which demagnetization effects become relevant [5]. As characteristic time we take $t_M = \lambda l_M^2/\nu_0$. Then x and h are expressed in units of l_M and t is expressed in units of t_M. Moreover, we express H in units of $2M_s$, $K(x)$ in units of $4\mu_0 M_s^2$, and Δ in units of ν_0^2/l_M^{3-d}. Using the dimensionless variables Eq. (1) can be written as

$$\begin{aligned}\frac{\partial}{\partial t} h(x,t) = {} & \nabla^2 h(x,t) + H \\ & + \Delta^{1/2}\eta[x, h(x,t)] - \mathcal{N} \int \frac{d^{d-1}x'}{V} h(x',t) \\ & + \int d^{d-1}x' K(x - x')[h(x',t) - h(x,t)],\end{aligned} \tag{4}$$

and the noise correlator is still given by Eq. (2), but now $R(h)$ has the asymptotic behaviors

$$R(h) \approx \begin{cases} 1 & \text{for } h \ll r, \\ 0 & \text{for } h \gg r. \end{cases} \tag{5}$$

where $r = a_\perp/l_M$.

In the dimensionless equation of motion we identify the control parameters H, $\mathcal{N}$, Δ, and r. The influence of the control parameters H, $\mathcal{N}$, and Δ has already been investigated in [5]. The attention is thus focused in the remaining control parameter r. From Eq. (5) it is clear that this parameter has a strong influence on the form of the noise correlator. Depending on r, the noise can be approximated by an uncorrelated noise ($r \ll 1$) or by a columnar disorder ($r \gg 1$). The order of magnitude of l_M can be computed, taking into account that $\nu_0 \sim K\delta_w$, where K is the anisotropy constant and δ_w is the domain wall width. Moreover, $\nu_0 M_s \sim 1$ T and $M_s \sim 10^6$ A/m for soft magnetic materials resulting in $l_M \sim 10^{-6} K\delta_w$. In materials with low anisotropies ($K \ll 10^6$ J/m^3) $l_M \ll \delta_w$. Besides, for low disorder, $\delta_w \ll a_\perp$ and, therefore, $l_M \ll a_\perp$ ($r \gg 1$). Thus, the noise is well approximated by a columnar disorder.

In such a case $R(h)$ can be approximated by the first term of its expansion around $h = 0$, which is exactly what one does in perturbation theory. In other words, in soft magnetic materials with low anisotropies and low disorder, perturbation theory will give a good estimate of the scaling exponents. Hence, the dynamic, roughness and correlation length exponents, below $d_c = 3$, are given by [5]

$$z = 1, \qquad \zeta = \frac{3 - d}{2}, \qquad \nu = \frac{1}{d - 1}, \tag{6}$$

respectively. On the other hand, the avalanche scaling exponents τ and α [$P(s) \sim s^{-\tau}$, $P(T) \sim T^{-\alpha}$] can be obtained using some scaling relations derived in [5] resulting

$$\tau = 2 - \frac{1}{\alpha}, \qquad \alpha = \frac{d + 1}{2}. \tag{7}$$

Now we proceed to discuss these results in comparison with experiments.

The samples used in BE experiments are usually three dimensional (here we exclude thin films where the CZDE does not apply [5]), with a three-dimensional array of domains separated by two-dimensional domain walls. Since the upper critical dimension is $d_c = 3$ we thus expect to measure the mean field (MF) exponents $\tau = 1.5$ and $\alpha = 2$. This is actually observed in some experimental setups [5]. However, there are magnetic materials where the avalanche exponents are smaller that those predicted by the MF theory, with values around $\tau = 1.3$ and $\alpha = 1.5$ [4,6,7]. These values are observed, for instance, in Perminvar [4], a soft-magnetic material with high anisotropy, and in soft-magnetic materials with high anisotropies induced by an applied uniaxial stress [6,7].

A possible explanation of this fact is the following. In some magnetic materials, due to anisotropy, the domain wall is practically flat along the direction of the magnetization, but rough perpendicular to the magnetization [15]. In this case the fluctuations of the domain wall are effectively two dimensional [16], and the avalanche exponents are obtained setting $d = 2$ in Eq. (7) resulting $\tau \approx 1.33$ and $\alpha = 1.5$. These values are actually in very good agreement with those reported in [4,6,7]. However, we should take into account that for high anisotropies the noise correlator may not be approximated by a columnar disorder and, therefore, perturbation theory may not give a good estimate of the scaling exponents. In this case RG corrections yield [17] $\tau = 1.25$ and $\alpha = 1.43$, which are, nevertheless, smaller than those measured in experiments.

Our analysis is not limited to the power law exponents, but we also make some predictions about the scaling of the cutoffs. The power laws for the avalanche size and duration are valid only up to certain cutoffs s_c and T_c. In the same way the Barkhausen jump amplitude v also follows a power law up to a cutoff v_c. If these cutoffs are a consequence of a finite size effect, then we can affirm that there is SOC in the BE; otherwise we can exclude this possibility.

Recent experiments on the BE in soft magnetic materials under an external stress [6,7] have drawn attention over the stress dependency of these cutoffs. When an external tensile stress σ is applied to a positive magnetostrictive the domain wall length L increases [18]. Hence, monitoring the stress dependency of the cutoffs with σ, we can investigate if the cutoffs are actually due to a finite size effect. Bahiana *et al.* [6] observed that v_c increases with increasing σ, concluding that this cutoff is due to a finite size effect. However, more extensive measurements by Durin and Zapperi [7] reveal that, while v_c increases, T_c decreases in such a way that s_c remains constant.

This apparent contradiction can be explained analyzing the stress dependency of our choice of characteristic length l_M [see Eq. (3)] and time t_M. The experimental evidence tells us that the characteristic avalanche size is independent of σ and, therefore, the same behavior is expected for the characteristic linear size of the avalanches. In other words, l_M should be independent of σ. On the other hand, $\nu_0 \sim \sqrt{AK_\sigma}$ and $K_\sigma \sim (3/2)\lambda_s \sigma$, where A is the exchange constant and K_σ is the induced anisotropy [19], yielding $t_M = \lambda l_M^2/\nu_0 \sim \sigma^{-1/2}$ and $v_c \sim s_c/t_M \sim \sigma^{1/2}$. These scaling dependencies (that of t_M and v_c with σ) are in very good agreement with those reported from the experimental measurements [6,7], clearly supporting our dimensionless analysis. Moreover, they also reveal that the increase of v_c with increasing σ, observed in recent experiments on Barkhausen effect in soft magnetic materials under applied stress [6,7], is not a finite size effect, but a change in the time scale and, therefore, cannot be taken as evidence of SOC in the Barkhausen effect.

As already pointed out in [5], and within the CZDE model, the critical state is obtained only after the demagnetization constant $\mathcal{N}$ and the driving rate c are fine tuned to zero. Hence, the cutoffs observed in these experiments can be attributed to the correlation length associated either to the demagnetization field or to the driving field. A similar behavior is observed in sandpile models where criticality is obtained after the driving and dissipation rates are fine tuned to zero [20]. Thus, even in sandpile models, the prototype of the SOC system, we cannot speak about SOC criticality in a strict sense.

The analogy between the CZDE model with sandpile models is not limited to the control parameters, but it is also found in other aspects. The dynamic scaling exponent $z = 1$ and the avalanche exponents in Eq. (7) are identical to those obtained for directed Abelian sandpile models (DASM) [21].

DASM are sandpile models where toppling takes place following a preferential direction, which in principle may be determined by the action of certain external fields—gravity, for instance. The toppling rule is in general deterministic and the only source of noise is the driving field [21]. There is a time scale separation between the characteristic time of the driving field and avalanche duration: the system is perturbed by the driving field only after all sites becomes stable. Hence, in the internal time scale of the evolution of an avalanche the driving field does not act, i.e., the noise introduced by the driving field acts as a columnar disorder [22]. In analogy, we have obtained that in soft magnetic materials with low anisotropies the noise can be approximated by a columnar disorder. Further similarities and also some differences are observed analyzing the evolution of the avalanche front.

In the DASM the evolution of the avalanche is similar to directed percolation, although it does not belong to the directed percolation universality class [23]. The preferential direction is that indicated by the toppling rule, usually referred to as depth l. With increasing time the avalanche front advances to more depth layers and, therefore, the evolution in time is equivalent to the evolution in depth, i.e., $t \sim l^z$ with $z = 1$. Hence, the mean squared displacement along the preferential direction scales as $\langle l^2(t)\rangle \sim t^{2/z} \sim t^2$. On the other hand, if ζ is the anisotropy exponent for the average transverse extent $x_\perp \sim l^\zeta$ of an avalanche, then $\langle x_\perp^2(t)\rangle \sim t^{2\zeta/z}$. In two dimensions the exponent ζ is obtained exactly [21] resulting $\zeta = 1/2$ yielding $\langle x_\perp^2(t)\rangle \sim t$. Thus, the motion along the preferential direction is ballistic and diffusive along $x_\perp$.

A similar analysis can be performed for the evolution of the avalanche front in the CZDE model. There are, nevertheless, some differences. Now the transverse space (the direction of advance of the interface h) is one dimensional. On the other hand, there is no preferential dimension in the $d - 1$ dimensional substrate, labeled by the position x. In this case using simple scaling arguments one obtains that $\langle x^2(t)\rangle \sim t^{2/z} \sim t^2$ and $\langle \Delta h^2(t)\rangle \sim t^{2\zeta/z}$, where Δh is the interface advance during an avalanche. In two dimensions using Eqs. (6) and (7), we obtain $\langle \Delta h^2(t)\rangle \sim t$.

Then we conclude that the evolution of the avalanche front is very similar in both models. In the transverse direction we have normal diffusion, while in the other sub-space the motion is ballistic type. One may ask how we also obtain a ballistic type motion in the CZDE model, taking into account that there is no preference direction in the substrate. The answer is very simple: the action of an external field is not the only possibility to obtain ballistic motion—Lévy flights also give rises to this type of motion [24].

If we have a system of noninteracting particles which follow diffusionlike motion and we apply a force F along one direction, x for instance, then $\langle x(t)\rangle = vt$ and $\langle x^2(t)\rangle = v^2t^2$, where v is linear in F. The other possibility is to have a system of free noninteracting particles, but following Lévy type diffusion. In this case, contrary to the classical diffusion, the distribution of displacements between collisions is not narrow, but follows a Lévy distribution, with the asymptotic behavior for large jumps l given by $P(l) \sim l^{-1-\alpha}$. α is the characteristic exponent and when $0 < \alpha < 1$ the ballistic type motion results [24].

The other question is how the ballisticlike motion appears in the CZDE model. The answer is found in the long-range nature of dipolar interactions. The existence of long-range interactions carries as a consequence that the advance of the domain wall at a given position disturbs the domain wall energy configuration at points far away from this position, which may then advance. One thus expects that the avalanche front is, in this case, characterized by a wide distribution of jump distances (Lévy flights). This conclusion has been obtained using naive arguments, but it is clearly corroborated by the ballistic motion.

In summary, we have investigated the CZDE in soft-magnetic materials with low anisotropies. After expressing the equation of motion in a dimensionless form we have obtained that the noise correlator is approximated by a columnar disorder. In this way we have shown that perturbation theory gives a good estimate of the avalanche exponents. Moreover, we have found that the evolution of the avalanche front in the CZDE model has certain similarities with DASM. Although there is no preference direction in the CZDE model, due to long-range dipolar interactions, the motion is ballistic.

We conclude that the cutoffs of the distributions of avalanche size, duration, and amplitude observed in recent Barkhausen experiments in soft magnetic materials under an applied stress are not due to finite size effects, and therefore these systems are not in a critical state. On the contrary, the models proposed to explain the different features observed in Barkhausen experiments show that criticality is obtained after some control parameter is fine tuned, e.g., the degree of disorder in the random-field Ising model (RFIM), the ratio between the exchange and structural correlation lengths in micromagnetic models, or the demagnetization factor in the CZDE model. This variety of control parameters not only reveals the complex nature of this phenomena, but also rules out the occurrence of SOC. The competition of disorder and demagnetization effects against the long range interactions drives the system out from the critical state.

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