Semiclassical Theory of Conductance and Noise in Open Chaotic Cavities

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Conductance and shot noise of an open cavity with diffusive boundary scattering are calculated within the Boltzmann-Langevin approach. In particular, conductance contains a nonuniversal geometric contribution, originating from the presence of open contacts. Subsequently, universal expressions for multiterminal conductance and noise, valid for all chaotic cavities, are obtained classically, based on the fact that the distribution function in the cavity depends only on energy, and using the principle of minimal correlations.

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Transport properties of quantum systems with classically chaotic dynamics (chaotic cavities) have recently become an object of intensive investigation. By far the most successful method to describe them is presently the random matrix theory (RMT), based on the assumption [1,2] that the scattering matrix of an open chaotic cavity is a member of Dyson's circular ensemble of random matrices. This hypothesis was quantified to calculate average conductance, weak localization (WL) effects, conductance fluctuations, shot noise, and other quantities (see [3] for review). It has been proven indirectly [4] by showing equivalence with Gaussian random matrix ensembles.

Despite this evident success, a number of questions remain unaddressed. Thus, by construction all RMT results are *universal* in the sense that they do not depend on the shape of the cavity (provided the dynamics is chaotic). It is clear, however, that certain *sample-specific* corrections exist. In particular, average conductance must contain *geometric* contributions, originating due to the possibility of escape. Below we show that these corrections, sensitive to the shape of the cavity, indeed, exist, and their relative magnitude is of the same order as the ratio of the cross section of the leads to the total surface of the cavity.

Another problem concerns shot noise—zero-frequency current-current fluctuations caused by discreteness of electron charge [5]. For uncorrelated transfer of classical particles shot noise assumes the *Poissonian value* $S_P = 2e\langle I\rangle$, with $\langle I\rangle$ being the average current. Fermi statistics induce correlations, and suppress shot noise below S_P. Thus, in metallic diffusive wires, one has $S = 2e\langle I\rangle/3$: The suppression factor $F = S/S_P$ equals $1/3$. A remarkable feature is that this $1/3$ suppression was derived in two different ways, quantum mechanically [6] (based on scattering approach or equivalent Green's function technique) and semiclassically [7,8], using the Boltzmann-Langevin approach [9]. The equivalence between these two is by no means evident, and the occurrence of the same result in the two approaches is even considered as a numerical coincidence [10]. Furthermore, in symmetric chaotic cavities (equal numbers of channels in both leads) RMT gives for the suppression factor [2] $F = 1/4$. An alternative, but still quantum-mechanical derivation of the same result [11], as well as a generalization to the arbitrary two-terminal case [3,11] and multiterminal effects [12] have been given. However, it is also desirable to develop a semiclassical analysis of shot noise in chaotic cavities, based on the Boltzmann-Langevin approach. Comparison of classical and quantum results may shed new light onto the physics of shot noise.

Recently, it has been realized [13,14] that a model of a circular billiard with diffusive scattering at the boundary (which models surface disorder on the scale of the wave length) exemplifies an "extremely chaotic" system, with typical relaxation time of the order of the flight time, and, on the other hand, can be treated analytically beyond RMT. Reference [13] used this example to study statistical properties of closed cavities. Below we use this model to study conductance and shot noise semiclassically in cavities connected to two open leads. In particular, we find corrections to the conductance which depend on the position of the leads, and reproduce the $1/4$-suppression of shot noise for the fully chaotic regime. The crucial observation is that the distribution function of electrons in the cavity in the leading order is a function only of the energy.

Subsequently, assuming that this is the case for a *generic* chaotic cavity, and using the principle of *minimal correlations* (representing an alternative to the Boltzmann-Langevin method [9]), supporting the current conservation, we derive the expressions for the multiterminal conductance and noise. It is conceptually important that these expressions obtained semiclassically reproduce the RMT results available in the literature.

Average distribution function.—We consider a circular billiard of a radius R, to which two ideal leads are attached, left (L) at angles $\theta_0 - \beta/2 < \theta < \theta_0 + \beta/2$ and right (R) at $-\alpha/2 < \theta < \alpha/2$ (Fig. 1). Eventually, we assume angular widths of the contacts, α and β, to be small. In the semiclassical description, the electron is characterized by the coordinate $\boldsymbol{r}$, direction of momentum $\boldsymbol{n} = \boldsymbol{p}/p$, and energy E (which is omitted where it cannot cause

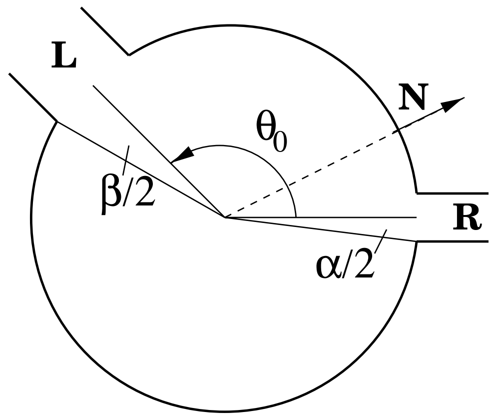

FIG. 1. Geometry of a circular cavity with diffusive boundary scattering.

confusion). Inside the cavity the motion is ballistic, and the average distribution function $f(\boldsymbol{r},\boldsymbol{n})$ obeys

$$\boldsymbol{n}\nabla f(\boldsymbol{r},\boldsymbol{n}) = 0 . \quad (1)$$

This equation must be supplemented by boundary conditions. At the surface (denoted Ω) we consider purely diffusive scattering [15] for which the distribution function of the outgoing particles is constant and fixed by flux conservation,

$$f(\boldsymbol{r},\boldsymbol{n}) = \pi \int_{(\boldsymbol{N}\boldsymbol{n}')>0} (\boldsymbol{N}\boldsymbol{n}') f(\boldsymbol{r},\boldsymbol{n}') d\boldsymbol{n}', \qquad (\boldsymbol{N}\boldsymbol{n}) < 0 .$$

Here $\boldsymbol{r} \in \Omega$, and $\boldsymbol{N}$ is an outward normal to the surface; we normalized $d\boldsymbol{n}$ so that $\int d\boldsymbol{n} = 1$. Furthermore, electrons coming from the leads are described by equilibrium distribution functions, $f_{L,R}(E) = \theta(\mu_{L,R} - E)$, $\mu_L = eV$, $\mu_R = 0$ (V is the applied voltage). We assume that these incoming particles are emitted uniformly to all directions. Thus, denoting the cross section of the right lead $\Omega_R = \{r = R; -\alpha/2 < \theta < \alpha/2\}$, and similarly Ω_L, we complete our system by

$$f(\boldsymbol{r},\boldsymbol{n}) = f_{L,R}, \qquad \boldsymbol{r} \in \Omega_{L,R}; \qquad (\boldsymbol{N}\boldsymbol{n}) < 0 . \quad (2)$$

Because of Eq. (1), the distribution function inside the cavity is equal to the distribution function $f(\theta)$ of outgoing particles, taken at the appropriate point of the surface $\{r = R; \theta\}$. For this function we obtain the integral equation

$$f(\theta)|_{\Omega} = \frac{1}{4} \int_{\Omega + \Omega_R + \Omega_L} d\theta' \left| \sin \frac{\theta - \theta'}{2} \right| f(\theta'), \quad (3)$$

which has to be solved with the condition $f(\theta)|_{\Omega_{L,R}} = f_{L,R}$ [see Eq. (2)]. This equation applies to arbitrary sizes and positions of the leads. To progress, we assume the leads to be narrow ($\alpha, \beta \ll 1$) [16], and replace integrals of the type $\int_{-\alpha/2}^{\alpha/2} F(\theta)\, d\theta$ by $\alpha F(0)$. We then obtain

$$f(\theta) = \frac{\alpha f_R + \beta f_L}{\alpha + \beta} + \frac{g(0) - g(\theta_0)}{4\pi} \frac{\alpha\beta(\beta - \alpha)}{(\alpha + \beta)^2} \times (f_L - f_R) + \frac{g(\theta) - g(\theta - \theta_0)}{4\pi} \frac{\alpha\beta}{\alpha + \beta} (f_L - f_R), \quad (4)$$

with the notation

$$g(\theta) = \sum_{l=1}^{\infty} \frac{\cos l\theta}{l^2} = \frac{1}{12}(3\theta^2 - 6\pi\theta + 2\pi^2), \quad 0 \le \theta \le 2\pi .$$

The term $(\alpha f_R + \beta f_R)/(\alpha + \beta)$ in Eq. (4) is uniform and does not depend on θ. Inside the cavity, it is responsible for a contribution which is position and momentum independent. It is also the principal term, since all others are proportional to the first power of α and β. These other terms represent *geometric corrections*, which are sensitive to the position of the leads and, thus, are sample specific.

Conductance.—The average current through the cavity, $I = e \int_{\Omega_R} d\boldsymbol{r} \sum_{\boldsymbol{p}} \boldsymbol{v} f(\boldsymbol{r},\boldsymbol{n})$, is expressed through the distribution function of outgoing particles, $f(\theta)$, up to linear order in α and β,

$$I = \frac{eN_R}{2\pi\hbar} \int dE \left\{ \frac{1}{4} \int_{\Omega} d\theta \left| \sin\frac{\theta}{2} \right| f(\theta) - f_R \right\}, \quad (5)$$

where we defined the numbers of transverse channels in the left and right leads, $N_L = p_F R\beta/\pi\hbar$ and $N_R = p_F R\alpha/\pi\hbar$ ($N_L, N_R \gg 1$). From Eqs. (4) and (5) we obtain the conductance

$$G = \frac{e^2}{2\pi\hbar} \frac{N_L N_R}{N_L + N_R} \left[1 + \frac{\alpha\beta}{8\pi(\alpha + \beta)} \theta_0(2\pi - \theta_0) \right].$$

The first term is precisely the result given by RMT, and the second one represents a nonuniversal correction, which depends on the position of the leads θ_0. It is positive and reaches a maximum for $\theta_0 = \pi$. We stress that this correction is not due to WL effects (which cannot be treated classically at all), but is really a geometric correction, originating from the fact that the escape time from an open cavity is finite. For $\alpha \sim \beta > (p_F R/\hbar)^{-1/2}$ the geometric corrections dominate the WL effects, which are of order $1/N_L \sim 1/N_R$. To the best of our knowledge, these corrections have never been discussed [17].

Shot noise.—Now we give a semiclassical theory of noise suppression. We keep only the leading terms in α and β, though the Boltzmann-Langevin method [8,9], employed below, can also give geometric corrections. We express the distribution function as a sum of average [given by Eq. (4)] and fluctuating parts. The latter one, $\delta f(\boldsymbol{x}, t)$, obeys the Boltzmann equation,

$$(\partial/\partial t + v_F \boldsymbol{n}\nabla)\delta f(\boldsymbol{x}, t) = j(\boldsymbol{x}, t),$$

where $\boldsymbol{x} \equiv \{\boldsymbol{r}, \boldsymbol{n}, E\}$. The random Langevin source j is zero on average, and its correlations are

$$\langle j(\boldsymbol{x}, t) j(\boldsymbol{x}', t')\rangle = \nu^{-1} \delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t') \times \delta(E - E') J(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{n}'), \quad (6)$$

with $\nu = m/(2\pi\hbar^2)$ being the density of states, and

$$J(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{n}'') = \int d\boldsymbol{n}' [\delta(\boldsymbol{n} - \boldsymbol{n}'') - \delta(\boldsymbol{n}' - \boldsymbol{n}'')] \times [W_{\boldsymbol{n},\boldsymbol{n}'} f(1 - f') + W_{\boldsymbol{n}',\boldsymbol{n}} f'(1 - f)],$$

where $W_{\boldsymbol{n},\boldsymbol{n}'}$ is the probability of scattering from $\boldsymbol{n}$ to $\boldsymbol{n}'$ per unit time at the point $\boldsymbol{r}$. We introduced the notations

$f \equiv f(\boldsymbol{r}, \boldsymbol{n})$ and $f' \equiv f(\boldsymbol{r}, \boldsymbol{n}')$. Note that $\int J\, d\boldsymbol{n} = \int J\, d\boldsymbol{n}' = 0$, due to the conservation of the number of particles.

The probability $W_{\boldsymbol{n},\boldsymbol{n}'}(\boldsymbol{r})$ can be found from the following considerations. It is only nonzero for $(\boldsymbol{n}\boldsymbol{N}) > 0$, $(\boldsymbol{n}'\boldsymbol{N}) < 0$, and under these conditions does not depend on $\boldsymbol{n}'$. Thus, $W_{\boldsymbol{n},\boldsymbol{n}'} = 2\tilde{W}_{\boldsymbol{n},\boldsymbol{r}}$, where $\tilde{W}_{\boldsymbol{n},\boldsymbol{r}}$ is the probability of the particle $\boldsymbol{n}$ to be scattered per unit time at the point $\boldsymbol{r}$. During the time interval Δt the particles which are closer to the surface then $v_F(\boldsymbol{n}\boldsymbol{N})\Delta t > 0$ are scattered with the probability one, and others are not scattered at all. Taking the limit $\Delta t \to 0$, we obtain

$$W_{\boldsymbol{n},\boldsymbol{n}'} = \begin{cases} v_F(\boldsymbol{n}\boldsymbol{N})\delta(R - r), & (\boldsymbol{n}\boldsymbol{N}) > 0, (\boldsymbol{n}'\boldsymbol{N}) < 0\,, \\ 0, & \text{otherwise.} \end{cases}$$

Now we define the noise spectral power at zero frequency $\omega = 0$ in the usual way, $S_{ij} = 2\int dt\, \langle \delta I_i(t) \delta I_j(0) \rangle$, where $i, j = L, R$, and $\delta I_i(t)$ is the fluctuating current at lead i, associated with the function δf. After standard transformations [8], we obtain, for $S \equiv S_{RR}$,

$$S = 2e^2\nu \int dE \int d\boldsymbol{n}\, d\boldsymbol{n}'\, d\boldsymbol{r}\, T_R(\boldsymbol{r}, \boldsymbol{n}) T_R(\boldsymbol{r}, \boldsymbol{n}') \times J(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{n}')\,,$$

where $T_R(\boldsymbol{r}, \boldsymbol{n})$ is the probability that the particle at $(\boldsymbol{r}, \boldsymbol{n})$ will exit through the right lead. It obeys Eq. (1) with the boundary condition of diffusive scattering; in addition, it equals 1 at Ω_R and 0 at Ω_L. In the leading order $T_R = \alpha/(\alpha + \beta)$; this order, however, does not contribute to S, since $\int J\, d\boldsymbol{n} = \int J\, d\boldsymbol{n}' = 0$. The next order is that for $\boldsymbol{n}$ pointing out to the right (left) contact, $T_R = 1(0)$. Substituting the average distribution function $f = (\alpha f_R + \beta f_R)/(\alpha + \beta)$ into J, we obtain $S = 2eGVF$, with the noise suppression factor

$$F = \frac{N_L N_R}{(N_L + N_R)^2}\,, \tag{7}$$

coinciding with the quantum-mechanical result [3,11]. In particular, for symmetric system $N_L = N_R$ we recover 1/4-suppression, as found previously in Ref. [2].

Multiterminal conductance.—The rest of the paper, motivated by Eq. (4), is based on the assumption that in the leading order the distribution function inside an *arbitrary* chaotic cavity depends only on energy, but not on a coordinate or direction of momentum [18]. This assumption should be considered as an alternative to the assumption that the scattering matrix of the cavity is taken from Dyson's circular ensemble [1,2]. We show that this assumption, together with the requirement of current conservation, determines the conductance in the multiterminal case.

Consider a chaotic cavity of an arbitrary shape, connected through N ideal leads of widths $W_n \gg \hbar/p_F$ to the reservoirs, described by equilibrium distribution functions $f_n(E) = f_F(E - eV_n)$, f_F being the Fermi distribution function. It is instructive to define the outgoing current $J_n(E)dE$ through the nth lead in the energy interval between E and $E + dE$, so that the total current is $I_n = \int J_n\, dE$. Denoting the distribution function in the cavity as $f_C(E)$, we easily obtain $J_n(E) = e^{-1}G_n(f_C - f_n)$, where $G_n = e^2 N_n/(2\pi\hbar)$ is the Sharvin conductance of the nth contact and N_n is the number of transverse channels. Without inelastic scattering inside the cavity, the current in each energy interval must be conserving. From the condition $\sum J_n = 0$, we immediately find the distribution function f_C,

$$f_C = \sum_n \alpha_n f_n, \qquad \alpha_n \equiv G_n \Big/ \sum_n G_n\,. \tag{8}$$

Then, defining the conductance matrix G_{mn} by $I_m = \sum_n G_{mn} V_n$, we obtain

$$G_{mn} = (\alpha_m - \delta_{mn})G_n\,. \tag{9}$$

This conductance matrix is symmetric, and for the two-terminal case becomes $G_{LR} = (e^2/2\pi\hbar)[N_L N_R/(N_L + N_R)]$, which is the RMT result.

Multiterminal shot noise.—The principal difficulty of the application of the standard Boltzmann-Langevin method to the noise of the chaotic cavity is that this approach [9] uses the collision integral explicitly. Although we managed above to avoid this difficulty for the circular cavity with surface scattering, it can hardly be overcome for an arbitrary ballistic cavity with chaotic dynamics. To resolve this problem, we formulate the principle of minimal correlations, which is described below.

The actual origin of shot noise can be viewed semiclassically as a result of partial occupation of the states by electrons. In the equilibrium state with finite temperature T the partial occupation causes fluctuations of the distribution function with the equal time correlator [19]

$$\langle \delta f(\boldsymbol{x}, t) \delta f(\boldsymbol{x}', t) \rangle = \nu^{-1}\delta(\boldsymbol{x} - \boldsymbol{x}') f(\boldsymbol{x})[1 - f(\boldsymbol{x})]\,. \tag{10}$$

The δ function on the right-hand side of this equation means that the cross correlations are completely suppressed. We note that in the chaotic cavity the cross correlations should also be suppressed because of multiple random scattering inside the cavity. Therefore, we assume now that Eq. (10) is valid for fluctuations of the nonequilibrium state of the cavity, where the function f_C plays the role of $f(\boldsymbol{x})$ in Eq. (10). Taking into account that for $t \neq t'$ the correlator obeys the kinetic equation, $(\partial_t + v_F \boldsymbol{n}\nabla)\langle \delta f(\boldsymbol{x}, t) \delta f(\boldsymbol{x}', t') \rangle = 0$ [19], we obtain the formula

$$\begin{aligned} \langle \delta f(\boldsymbol{x}, t) \delta f(\boldsymbol{x}', t') \rangle = \nu^{-1}&\delta[\boldsymbol{r} - \boldsymbol{r}' - v_F \boldsymbol{n}(t - t')] \\ &\times \delta(\boldsymbol{n} - \boldsymbol{n}')\delta(E - E') \\ &\times f_C(1 - f_C)\,, \end{aligned} \tag{11}$$

which describes strictly ballistic motion and is therefore valid only at the time scales below the time of flight. On the other hand, after a time of the order of the dwell time τ_d the electron becomes uniformly distributed and leaves the cavity through the nth contact with the probability α_n. For times $t \gg \tau_d$ (which are of interest here) this can be described by an instantaneous fluctuation of the isotropic distribution $\delta f_C(E,t)$, which is not contained in Eq. (11). The requirement of the conservation of the number of electrons in the cavity leads to *minimal correlations* between $\delta f_C(E,t)$ and $\delta f(\boldsymbol{x},t)$ [20]. Thus, for the fluctuation of the current at the contact n, we write

$$\delta I_n = \frac{e p_F}{2\pi\hbar^2} \int_{\Omega_n} d\boldsymbol{x}\, (\boldsymbol{n}\boldsymbol{N}_n) [\delta f(\boldsymbol{x},t) + \delta f_C(E,t)], \tag{12}$$

where Ω_n and $\boldsymbol{N}_n$ denote the surface of the contact n and the outward normal to this contact. The requirement that current is conserved *at every instant of time*, $\sum_n \delta I_n = 0$, eliminates fluctuations δf_C. After straightforward calculations with the help of Eq. (11) we arrive at the expression

$$\begin{aligned} S_{mn} &= -2G_{mn}(T + T_C), \\ T_C &= \int dE\, f_C(1 - f_C), \end{aligned} \tag{13}$$

where Eq. (13), together with Eq. (9), constitutes a finite-temperature multiterminal expression for the noise power in a chaotic cavity. From Eq. (8) we obtain $T_C = (e/2)\sum_{n,m} \alpha_n \alpha_m (V_n - V_m) \coth[e(V_n - V_m)/2T]$. At zero temperature this reproduces the noise suppression factor (7) and the multiterminal RMT results for shot noise [12]. At equilibrium, $S_{mn} = -4TG_{mn}$, in accordance with fluctuation-dissipation theorem.

In conclusion, we have presented a semiclassical theory of transport in chaotic cavities. For the model of a circular billiard with diffusive boundary scattering, we reproduced the RMT results for the average conductance and shot noise and, in addition, found geometrical corrections to the average conductance, which originate due to the finite probability of escape. Subsequently, we showed how the RMT results for multiterminal conductance and noise in an arbitrary chaotic cavity can be obtained semiclassically by using the principle of minimal correlations.

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