

Spin-Orbit Coupling Effect on Quantum Hall Ferromagnets with Vanishing Zeeman Energy

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We present the phase diagram of a ferromagnetic $\nu = 2N + 1$ quantum Hall liquid in a narrow quantum well with vanishing single-particle Zeeman splitting, ε_Z , and a pronounced spin-orbit coupling. Upon decreasing ε_Z the spin-polarization field of a liquid takes, first, the easy-axis configuration, followed by the formation of a helical state which affects the transport and NMR properties of a liquid and the form of topological defects in it.

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The concept of quantum Hall ferromagnets (QHF) [1–3] is now broadly accepted as an approach to treat the 2D electron systems in the vicinity of odd-integer filling factors, $\nu = 2N + 1$, in particular, due to the success in the theory [3,4] and experimental observations [5–7] of quantum Hall effect skyrmions, which are electrically charged topological textures in the ferromagnetic order parameter in two dimensions. The attempts to stabilize skyrmions [3,4], as compared to electrons and holes on the top of a filled spin-split Landau level (LL) have triggered the studies [8,9] of semiconductor structures with a reduced value of the single-electron Zeeman energy, ε_Z .

The latter is possible to realize using GaAs/AlGaAs structures, due to a strong spin-orbit (SO) coupling in this zinc-blend-type semiconductor [10,11]. In particular, the conduction band electron Lande factor, g in GaAs may reduce its absolute value or even change sign under a hydrostatic pressure, and also due to the electron confinement in a narrow quantum well (QW). Both theory [12] and experiment [13,14] agree on that in a nonstrained 55-Å-wide GaAs/Al_{0.33}Ga_{0.67}As QW, $g = 0$, and the value $g = -0.1$ has been recently measured in a nonstrained $w = 68$ Å QW [8]. As a result, a narrow quantum well is a system, where Zeeman energy can be swept through zero, and where the tendency of interacting electrons at $\nu = (2N + 1)$ to form a ferromagnetic state (with a large exchange energy $\mathfrak{S} \gg \varepsilon_Z$) polarized along the external magnetic field confronts the effects of the SO coupling [15–17] itself, as an alternative source for choosing the spin-polarization direction for 2D electrons.

In GaAs/AlGaAs quantum wells grown along the [001] facet, the effective two-dimensional SO coupling has the form of $H_{so} = \tilde{u}_{so}(p_x\sigma^x - p_y\sigma^y) + u_{so}\epsilon^{ijz}p_i\sigma_j$, where $\mathbf{p} = -i\nabla - e\mathbf{A}/c$ is the electron momentum [$\mathbf{A} = (0, xB_z, 0)$], σ_j are Pauli matrices, and $\hbar = 1$. The first term in H_{so} reflects the lack of inversion symmetry in the well-grown along the [001] facet with square 2D lattice symmetry and comes from the $(\gamma p^3\sigma)$ nonparabolicity in the conduction band of bulk GaAs [11,16], so that $\tilde{u}_{so} = \gamma\langle p_z^2 \rangle$. Experimentally [17] and theoretically [16] found values of γ range from $\gamma = 27$ to 22 eV Å^3 . The second SO term is due to the quantum

well potential asymmetry [15]. In a narrow QW, with $\psi = 2^{-1/2} \sin(z\pi/w)$ transverse part of the electron wave function, $\tilde{u}_{so} = \gamma(\pi/w)^2 \gg u_{so}$ [17]. (Directions for x and y axes are chosen in such a way that $\tilde{u}_{so}, u_{so} > 0$.)

In the present paper, we study the effect of spin-orbit coupling on the $\nu = (2N + 1)$ QHF formed by electrons with a vanishing single-particle Zeeman energy in a narrow QW in a perpendicular magnetic field. In this analysis, we address the properties of narrow wells, since they have a more prominent SO coupling in the 2D electron Hamiltonian. In a high magnetic field providing $\omega_c = |eB_z|/mc \gg \tilde{u}_{so}/\lambda$ ($\lambda = \sqrt{c/|eB_z|}$), a weak SO coupling does not alter Landau quantization. However, it affects the evolution of ferromagnetic properties of a quantum Hall effect liquid upon sweeping ε_Z through zero. As in ordinary ferromagnets, SO coupling results in the crystalline anisotropy field [18], that deflects spin polarization \mathbf{n} from alignment in the magnetic field direction, \mathbf{l}_z . As a result, a liquid with $|\varepsilon_Z| < \varepsilon_Z^c = 2(\tilde{u}_{so}/\lambda\omega_c)^2\mathfrak{S}_{N+1}$ takes one of two twofold degenerate easy-axis magnetic states, which generates a source for formation of a domain structure in a QHF. Moreover, as a feature of the SO coupling in the 2D electron system lacking inversion symmetry, for a tinier splitting, $|\varepsilon_Z| < \varepsilon_Z^h = \sqrt{2}(\tilde{u}_{so}/\lambda\omega_c)^2\mathfrak{S}_{N+1}$, the spin polarization acquires a helically twisted texture with a mesoscopic-scale period, $\mathcal{L}/\lambda \approx 3.25\omega_c\lambda/\tilde{u}_{so}$, in the $[1\bar{1}0]$ crystallographic direction. The transition of a liquid into the helical state may manifest itself in a change of the NMR line shape from the QW structure, or in the anisotropy of dissipative transport characteristics of a QHF. We also discuss topological defects in the helical state, $\pm e/2$ -charged dislocations, which, in pairs, constitute skyrmions.

The recipe [3,4] of describing smoothly textured QHF's at odd-integer filling factors is to use the 2D sigma model, which operates with the energy functional, $\Phi\{\mathbf{n}(\mathbf{x})\}$ of 2D electrons expressed in terms of their local excess spin polarization field $\mathbf{n}(\mathbf{r}, t)$ ($|\mathbf{n}| = 1$). Locally, the QHF can be viewed as a liquid of electrons which fully occupy $(N + 1)$ LL's with spin parallel to \mathbf{n} , and N LL's with antiparallel spins. This assumes the existence of a local unitary spin transformation, $U(\mathbf{r})$, of electron wave

functions, which reduces the filling of the $(N + 1)$ st LL to a complete occupation of only states “up” with respect to the locally determined axis $\mathbf{n}(\mathbf{r})$, and which is related to the spin-polarization field as $n^i = \text{Tr}(s^i \Lambda)$, where $s^i(\mathbf{r}) = U(\mathbf{r})\sigma^i U^\dagger(\mathbf{r})$, and $\Lambda = (1 + \sigma^z)/2$ is the electron spin-density matrix in the rotated frame. Such a liquid retains incompressibility, which is guaranteed by a large exchange energy gap \mathfrak{S} . The derivation of a sigma model consists in the use of Hubbard-Stratonovich transformation and the saddle-point self-consistency equation, as a method to obtain an expansion of the thermodynamic potential of a liquid over small gradients of a polarization field, or, equivalently, over the matrix $\tilde{\Omega}(\mathbf{r}) = U(\mathbf{r})[-i\nabla U^\dagger(\mathbf{r})] \equiv [i\nabla U(\mathbf{r})]U^\dagger(\mathbf{r})$. The latter matrix appears in a local perturbation to the single-particle Hamiltonian written in the rotated spin frame,

$$H_\Omega = i\omega_c(a_+\Omega_- - \Omega_+a_-) + (\omega_c/2)[\tilde{\Omega}^2 - \alpha\nabla \times \tilde{\Omega}],$$

where $a_\pm = \pm(ip_x \mp \alpha p_y)/\sqrt{2}$ are the inter-LL opera-

tors, $\alpha = eB_z/|eB_z|$ indicates the direction of the cyclotron rotation of carriers, and $\Omega_\pm = (\Omega_x \mp i\alpha\Omega_y)/\sqrt{2}$. Further procedure (equivalent to the Hartree-Fock approximation) consists in a perturbative expansion of the saddle-point Hubbard-Stratonovich action (obtained for a given single-particle Hamiltonian, which includes $H_\Omega + H_{\text{so}}$) up to the second order both in $\tilde{\Omega}$ (assuming that $\tilde{u}_{\text{so}}/\lambda \ll \mathfrak{S}$). In the local spin-frame,

$$H_{\text{so}} = (\tilde{u}_{\text{so}}/\lambda)[i(a_+s^+ - s^-a_-) + \Omega_+s^+ + s^-\Omega_-] + (\alpha u_{\text{so}}/\lambda) \times [a_+s^- + s^+a_- - i(\Omega_+s^- - s^+\Omega_-)],$$

where $s^\pm = (s^x \mp \alpha is^y)/\sqrt{2}$, and the relevant part of H_{so} is off diagonal with respect to the LL number [19].

The calculation, which leads us to the sheet density of thermodynamic potential of a QHF, $\Phi\{\mathbf{n}(\mathbf{x})\}$ differs from earlier Hartree-Fock calculations [3,20] only by taking into account the SO-coupling term H_{so} , along with H_Ω . It results in

$$\Phi = \frac{\mathfrak{S}_{N+1}}{2\pi} \left\{ \sum_\beta \frac{(\nabla n^\beta)^2}{8} - \left[\frac{\tilde{u}_{\text{so}}^2 + u_{\text{so}}^2}{2\omega_c^2\lambda^4} \mathbf{n}_\parallel^2 + \frac{2\tilde{u}_{\text{so}}u_{\text{so}}}{\omega_c^2\lambda^4} n^x n^y \right] + \frac{\varepsilon_Z n^z}{2\lambda^2} - \frac{\tilde{u}_{\text{so}}\tilde{\mathbf{n}}_\parallel - u_{\text{so}}\mathbf{n}_\parallel}{\omega_c\lambda^2} \cdot \nabla n^z \right\} + E_{\text{sk}}, \quad (1)$$

where \mathbf{n}_\parallel is the planar component of the spin polarization field, and $\tilde{n}^x = n^y$, $\tilde{n}^y = n^x$. To obtain Φ in Eq. (1), we have extended perturbative expansion up to the terms $\mathfrak{S}\nabla^2$, $\mathfrak{S}(\tilde{u}_{\text{so}}/\lambda\omega_c)\nabla$, and $\mathfrak{S}(\tilde{u}_{\text{so}}/\lambda\omega_c)^2$. The exchange factor,

$$\mathfrak{S}_{N+1} = \int_0^\infty dz V(\sqrt{2}z) e^{-z} \left\{ \sum_{M=N,N+1} M[L_M(z)L_N^1(z) - L_{M-1}(z)L_{N-1}^1(z)] \right\} = \frac{e^2\sqrt{2}\pi}{\chi\lambda} \theta_{N+1},$$

was calculated for each odd-integer filling $\nu = 2N + 1$ ($N = 0, 1, 2, \dots$); $L_N^M(z)$ are the generalized Laguerre polynomials. For $V(r) = e^2/r\chi$, which is a reasonable approximation for the 2D electron interaction in a narrow QW ($w \ll \lambda$), $\theta_1 = \frac{1}{4}$, $\theta_2 = \frac{7}{16}$, $\theta_3 \approx 0.57$, $\theta_4 \approx 0.67$, and $\theta_5 \approx 0.76$. Since the relevant part of the SO coupling is off-diagonal with respect to the Landau level number, it appears only in the second order of a perturbation theory, or due to its mixing with the H_Ω term [21]. The single-particle spin-splitting, ε_Z in Eq. (1) is corrected by the effect of the 2D single-particle SO coupling: $\varepsilon_Z = \mu g B - \alpha\nu(\tilde{u}_{\text{so}}^2 - u_{\text{so}}^2)/\omega_c\lambda^2$. We also include into $\Phi(\mathbf{n}, \nabla\mathbf{n})$ the topological term and the Coulomb energy of additional charges, $\rho(\mathbf{x})$, in order to discuss charged skyrmion-type textures,

$$E_{\text{sk}} = \frac{\mathfrak{S}_{N+1}}{2} \rho + \int d\mathbf{x}' \rho(\mathbf{x}) \frac{V(\mathbf{x} - \mathbf{x}')}{2} \rho(\mathbf{x}');$$

$$\rho(\mathbf{x}) = \frac{-\alpha}{8\pi} \epsilon^{\beta\gamma\delta} \epsilon^{ij} n^\beta \partial_i n^\gamma \partial_j n^\delta. \quad (2)$$

From a phenomenological point of view, thermodynamic potential $\Phi\{\mathbf{n}(\mathbf{x})\}$ in Eq. (1) describes an easy-axis ferromagnet with square 2D Bravais lattice and broken inversion symmetry, and in a perpendicular magnetic field. It contains all terms in the magnetization energy allowed by the crystalline symmetry of the [001]-grown

quantum well in a zinc-blend-type semiconductor [22]. The sketched above microscopic derivation of $\Phi\{\mathbf{n}(\mathbf{x})\}$ was necessary to obtain the values of coefficients in front of the phenomenologically allowed invariants composed of \mathbf{n} and ∇ . The first term in Φ describes spin stiffness. The second term determines an easy-axis anisotropy along $\mathbf{l}_+ = [110]/\sqrt{2}$. For structures with $\tilde{u}_{\text{so}} \gg u_{\text{so}}$, it rather defines a weakly anisotropic easy plane for spin polarization, which competes with the Zeeman energy term. Such a competition resumes itself in the deviation of polarization from a fully \mathbf{l}_z -aligned state at $|\varepsilon_Z| < \varepsilon_Z^e$.

$$\varepsilon_Z^e = 2(u_+/\lambda\omega_c)^2 \mathfrak{S}_{N+1}, \quad u_\pm = \tilde{u}_{\text{so}} \pm u_{\text{so}}. \quad (3)$$

As a function of a varying ε_Z , this can be viewed as a second order phase transition into the easy-axis state,

$$\mathbf{n}_\pm = -(\varepsilon_Z/\varepsilon_Z^e)\mathbf{l}_z \pm \mathbf{l}_+ \sqrt{1 - (\varepsilon_Z/\varepsilon_Z^e)^2}, \quad (4)$$

across which the symmetry between $\pm\mathbf{l}_+$ magnetic directions gets spontaneously broken [18].

Since for $|\varepsilon_Z| < \varepsilon_Z^e$ both easy-axis configurations, \mathbf{n}_+ and \mathbf{n}_- , have the same energy density,

$$\Phi_e = -\frac{\mathfrak{S}_{N+1}}{4\pi\lambda^2} \left(\frac{u_+/\lambda}{\omega_c} \right)^2 \left[1 + \left(\frac{\varepsilon_Z}{\varepsilon_Z^e} \right)^2 \right], \quad (5)$$

the easy-axis state of a QHF tends to acquire a domain structure: by splitting dynamically into the set of mesoscopic-size regions with opposite polarization projections onto the $[110]$ axis. The latter possibility has to affect the skyrmion-dominated dissipative transport properties of a liquid. The matter is that the activation energy of a skyrmion-antiskyrmion pair confined to the domain wall is lower than in the 2D bulk [21]. In fact, the larger the difference in the polarization between two domains, the more skyrmion is energetically confined to it. Since \mathbf{n}_{\pm} -polarized states are degenerate, the areas covered by \mathbf{n}_{+} and \mathbf{n}_{-} domains are statistically equal, so that the network of better conducting domain walls (with a lower activation energy for thermally excited carriers) forms a percolation cluster, thus resulting in a continuous decline in the activation energy for the macroscopic σ_{xx} which would follow the decrease of ε_Z .

A further analysis of the functional in Eq. (1) extended onto the limit of $\varepsilon_Z = 0$ shows that, apart from the domain structure formation, there is another reason for the field $\mathbf{n}(\mathbf{r})$ to be inhomogeneous. The fourth term [22] in Eq. (1) tends to twist the polarization field of a QHF into helical texture,

$$\mathbf{n}(\mathbf{r}) = \mathbf{l} \sin[\phi(\mathbf{r})] + \mathbf{l}_z \cos[\phi(\mathbf{r})]. \quad (6)$$

The latter is characterized by the helicity plane built upon two unit vectors, $\mathbf{l} = (l^x, l^y, 0)$ and \mathbf{l}_z , spatial orientation \mathbf{m} , and period \mathcal{L} , $\phi(\mathbf{r} + \mathbf{m}\mathcal{L}) = \phi(\mathbf{r}) + 2\pi$. As a variational approximation, one can use $\mathbf{n}(\mathbf{r})$ in Eq. (6) with $\phi(\mathbf{r}) = \mathbf{m}\mathbf{r}/\mathcal{L}$, treating \mathbf{l} , \mathbf{m} , and \mathcal{L} as minimization parameters. The resulting texture can be viewed as an image of a spoke in a wheel rolling in the direction of \mathbf{m} , with ϕ being the integral angle encircled by a spoke, so that we shall call ϕ the “helicity phase.” The energy density of an optimal variational state,

$$\mathbf{m} = \mathbf{l} = \mathbf{l}_{-} \equiv [1\bar{1}0]/\sqrt{2}, \mathcal{L}^v = \pi\lambda^2\omega_c/u_{+}, \quad (7)$$

$$\Phi_h^v = -\frac{\mathfrak{N}_{N+1}}{4\pi\lambda^2} \left\{ \left(\frac{u_{+}/\lambda}{\omega_c} \right)^2 + \frac{1}{2} \left(\frac{u_{-}/\lambda}{\omega_c} \right)^2 \right\}, \quad (8)$$

is lower than the energy $\Phi_e(\varepsilon_Z = 0)$ of a homogeneous easy-axis configuration in Eq. (5). The optimal variational state provides us with values of Φ_h and \mathcal{L} which are very close to the \mathbf{n} -field distribution that really minimizes [23] the functional Φ in Eq. (1). For the experimentally relevant example of $u_{+} \approx u_{-}$ (i.e., $\tilde{u}_{so} \gg u_{so}$), $\Phi_h^v = -\frac{3}{2}(\tilde{u}_{so}/\lambda\omega_c)^2(\mathfrak{N}_{N+1}/4\pi\lambda^2)$ and \mathcal{L}^v in Eq. (7) should be compared [23] with $\Phi_h \approx -1.53(\tilde{u}_{so}/\lambda\omega_c)^2(\mathfrak{N}_{N+1}/4\pi\lambda^2)$ and $\mathcal{L} \approx 3.25\omega_c\lambda^2/\tilde{u}_{so}$. We, therefore, assess the stability of a helical state, against the easy-axis one, on the basis of the energetics of variational helical texture with parameters given by Eq. (7).

Because of the difference in symmetry of helical and easy-axis states, which cannot be continuously transformed one into another, the transformation between

them can be viewed as a first-order phase transition. The condition for such a transition, $\Phi_h = \Phi_e(\varepsilon_Z)$, determines critical value of Zeeman splitting, ε_Z^h estimated as

$$\varepsilon_Z^h = \sqrt{2}(u_{+}u_{-}/\omega_c^2\lambda_B^2)\mathfrak{N}_{N+1} \approx \varepsilon_Z^e/\sqrt{2}. \quad (9)$$

The helical state formation can manifest itself in several observations. For example, the local value of the Knight shift [6], δ_{hf} in the spin-splitting of Ga and As nuclei located in the QW acquires an alternate coordinate-dependent sign, thus modifying the NMR lineshape, $I(\delta)$. Locally, the NMR shift $\delta = (\omega - \omega_0)/\delta_{hf}$ is due to the hyperfine interaction of nuclear spins with fully polarized electrons, with δ_{hf} being its maximal value just in the QW center. In a homogeneously polarized gas, the NMR line from the QW has a double-peak structure [6], $I(1 > \delta > 0) \propto [\delta(1 - \delta)]^{-1/2}$, with a distinct satellite at $\delta = 1$ split by the hyperfine coupling. In the easy-axis configuration, Eq. (4), the double-peak structure persists, with a reduced maximal splitting: $\delta_{hf} \rightarrow (\varepsilon_Z/\varepsilon_Z^e)\delta_{hf}$, as far as $n^z = -\varepsilon_Z/\varepsilon_Z^e$. On the contrary, in a helical phase, this has to transform into a single broadly tailed resonance with a non-Lorentzian shape approximated by $I(\delta) \propto |\delta|^{-1/2}$ for $1 > \delta > -1$.

The anisotropy of transport characteristics of a QHF with respect to $[110]$ and $[1\bar{1}0]$ crystallographic directions may be another feature of the helical state. Speaking about dissipative conductivity formed by thermally activated electron-hole pairs at the spin-split LL's, this can be understood after taking into account that charge-carrying excitations determined in a locally rotated spin frame [adjusted to $\mathbf{n}(\mathbf{r})$] are subjected to a smooth ($\mathcal{L} \gg \lambda$) potential due to Zeeman energy, $(\varepsilon_Z/2)\cos[(x - y)/\sqrt{2}\mathcal{L}]$. Therefore, at low temperatures, $T < \varepsilon_Z$ the dissipative conductivity $\sigma_{-,-}$ along $\mathbf{m} \parallel [1\bar{1}0]$ would be suppressed, as compared to $\sigma_{+,+}$ (across \mathbf{m}).

For the dissipative transport dominated by skyrmions [5], the difference between $\sigma_{-,-}$ and $\sigma_{+,+}$ is to be the result of the anisotropy of a skyrmion itself. In fact, the form of a skyrmion in a helically twisted texture is quite complex: In a periodic system, these are dislocations which represent the very elementary topological defects, rather than skyrmions. The periodicity of a helical texture in Eq. (6) is controlled by the helicity phase $\phi_0(\mathbf{m}\mathbf{r}/\mathcal{L})$ in Eqs. (6) and (7). One missing (or extra) period in one half of a plane, as compared to the other half (a dislocation) is equivalent to the phase shift of $\pm 2\pi$ accumulated at large distances from the dislocation core, thus resulting in the winding number $D = \pm 1$. On the other hand, we assume that the dislocation core is not singular. To illustrate the topology of a nonsingular core, let us draw a large-radius circle around a dislocation. At large distances, where helical structure is not perturbed, such a contour maps into the equator of a unit sphere and encircles it N or $(N - 1)$ times, depending on which one of two semicircles is retraced: drawn above, or below the dislocation.

Upon moving the upper half of a contour down through the dislocation, an extra loop encircling the unit sphere equator should continuously disappear. The latter is possible if the contour image slips through either the $+1_+$, or, alternatively, -1_+ pole of a unit sphere, which can be modeled by such a field configuration $\mathbf{n}(\mathbf{r}) = \mathbf{l}_z n^z + \mathbf{l}_+ n^+ + \mathbf{l}_- n^-$, that

$$n^z + i n^- = e^{i\phi + iD\varphi} \sqrt{1 - (n^+)^2}, \begin{cases} n^+(0) = \pm 1, \\ n^+(r \rightarrow \infty) = 0, \end{cases}$$

where (r, φ) are polar coordinates calculated from the dislocation center, $r = 0$. The image of a 2D plane provided by $\mathbf{n}(\mathbf{r})$ maps onto only one half of a sphere $|\mathbf{n}| = 1$, so that the dislocation core is characterized by an additional topological number, $\vartheta \equiv n^+(0) = \pm 1$ distinguishing between “left” and “right” semispheres. Using $\rho(\mathbf{x})$ in Eq. (2), we find that the core of a dislocation ($D = 1$) or antidislocation ($D = -1$) carry a half-integer electric charge $\int d\mathbf{x} \rho(\mathbf{x}) = \frac{1}{2} \vartheta D$. However, an isolated dislocation has a logarithmically large energy,

$$\mathcal{E}(D = \pm 1) = \int \frac{d\mathbf{x}}{2\pi} \frac{\mathfrak{S}_{N+1}}{8} (\nabla\varphi)^2 \approx \frac{\mathfrak{S}_{N+1}}{8} \ln(rk_0),$$

and, at low temperatures, dislocations and antidislocations have to form pairs bound by a long-range (logarithmic) attraction, except, maybe, for a possible Kosterlitz-Thouless melting effect. Since both dislocation and antidislocation in a bound pair may be equally charged, such a pair, (D, ϑ) and $(-D, -\vartheta)$ together constitute a skyrmion.

The result of the above analysis of phases of a ferromagnetic quantum Hall effect liquid in a narrow QW can be summarized as follows. Upon decreasing the single-particle Zeeman splitting, $|\varepsilon_Z|$ (e.g., by pressure), spin polarization of a liquid starts to acquire at $|\varepsilon_Z| = \varepsilon_Z^c$ the easy-axis configuration, which is followed by the abrupt fall into a helical state at $|\varepsilon_Z| = \varepsilon_Z^h \approx \varepsilon_Z^c/\sqrt{2}$. Using the bulk SO coupling parameter $\gamma = 25 \text{ eV } \text{\AA}^3$, as a reference, we estimate for the $\nu = 1$ liquid in a 68- \AA -wide GaAs/AlGaAs quantum well structure with a carrier density $2.8 \times 10^{11} \text{ cm}^{-2}$ studied by Maude *et al.* [8] that $\varepsilon_Z^c/(e^2/\chi\lambda) \approx 1.5 \times 10^{-3}$, and $\varepsilon_Z^h/(e^2/\chi\lambda) \sim 1 \times 10^{-3}$, which roughly fits into the range of a variable Zeeman energy in Ref. [8], where the dissipative transport activation has been drastically affected by pressure. The helical texture period estimated for the same parameters is $\mathcal{L} \sim 5 \times 10^3 \text{ \AA}$. Note that, according to Eqs. (9) and (3), the parametric range of pressures where helical and easy-axis phases are stable is broader for higher odd-integer filling factors in the same density structure.

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 - [23] To minimize the energy, we first find such a helicity phase, $\phi_0(\mathbf{r} + \mathbf{m}\mathcal{L}) = \phi_0(\mathbf{r}) + 2\pi$ that optimizes Φ for a given period \mathcal{L} . Then, we minimize $\Phi(\mathcal{L})$ with respect to \mathcal{L} . The optimal ϕ_0 for $\varepsilon_Z = 0$ is determined by the equation $-(1/4)\partial_{\eta}^2\phi_0 - (u_-/\lambda^2\omega_c)^2\sin\phi_0\cos\phi_0 = 0$, with first integral $A = (\lambda^2\omega_c/2u_-)^2(\partial_{\eta}\phi_0)^2 - \cos^2\phi_0$, $\eta = \mathbf{r} \cdot \mathbf{m}$, related to the period as $2\mathcal{L}u_-/\lambda^2\omega_c = \int_0^{2\pi} d\phi\sqrt{A + \cos^2\phi}$. Using these relations, we find the energy dependence $\Phi_h(\mathcal{L})$ and formally minimize it, $d\Phi_h(\mathcal{L})/d\mathcal{L} = 0$, generating one more integral relation, $\int_0^{2\pi} d\phi\sqrt{A + \cos^2\phi} = 2\pi u_+/u_-$. For $u_+ \approx u_- \approx \tilde{u}_{so}$, we find $A \approx 0.532$, $\mathcal{L} \approx 3.25\omega_c\lambda^2/\tilde{u}_{so}$, and $\Phi_h \approx -1.53(\tilde{u}_{so}/\lambda\omega_c)^2(\mathfrak{S}_{N+1}/4\pi\lambda^2)$.