Charge Fluctuation Instability of the Dust Lattice Wave

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Taking into account the statistical charge fluctuations on microspheres embedded in a low-pressure gas discharge, a stochastic differential equation for the one-dimensional dust lattice wave is derived. Using this equation, the nonstochastic differential equations for the mean particle displacement and for the second moments of the displacement and velocity are obtained. The analysis of the equation for the second moments shows that for sufficiently small gas pressure the charge fluctuations can result in exponential growth of the average energy of lattice oscillations.

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In a dusty plasma there are two types of low-frequency oscillations—dust acoustic (DA) and dust lattice (DL) waves. The DA waves can be excited in a plasma with weak electrostatic coupling between the charged dust particles. This mode was theoretically predicted by Rao *et al.* [1]. DL waves are a result of oscillations of dust particles with strong electrostatic coupling, when particles form crystal-like structures. The DL waves were investigated in many experiments in a radio-frequency discharge plasma [2–7]. In these experiments, particles were suspended in the sheath at the lower electrode, where the electrostatic force on the particle balances the force of gravity. The first theoretical considerations of the DL mode were based on the usual approximation of the one-dimensional (1D) particle string [2,8] that have been employed in solid state theory for the description of the elastic vibrations. In later publications the same approach was extended to the 2D plasma crystal [7].

Usually, the theoretical investigations of oscillations in a dusty plasma are based on the assumption that all the dust particles carry an equal constant charge. However, the real distribution function of the charge has a finite dispersion [9,10], and the charge of any particle continuously fluctuates around an equilibrium mean value. In the Langevin approach for particle charging, the distribution is a stationary Gaussian and the dispersion is directly proportional to the mean charge [11]. In the present paper we study the influence of particle charge fluctuations on the traveling DL wave and consider the conditions, when these fluctuations can result in the instability of oscillations.

In order to describe the DL wave propagation in a plasma crystal we choose the simplified model of the 1D particle string [8]. Below we assume a weak correlation of fluctuations on the neighbor particles, so that taking into account the real geometry of the plasma crystal in the sheath does not yield qualitative changes of the subsequent results. The electrostatic potential of each particle is assumed to be screened Debye potential. The energy of the electrostatic coupling between nth and mth particle of the string is of the form

$$\mathcal{W}_{n,m} = \frac{Q_n Q_m}{|x_n - x_m|} \exp\left(-\frac{|x_n - x_m|}{\lambda_{\mathrm{D}e}}\right), \tag{1}$$

where Q_n and Q_m are the charges of the particles, x_n and x_m are their coordinates in the string, and $\lambda_{\mathrm{D}e}$ is the electron Debye length. The corresponding force acting on the nth particle is $F_{n,m} = -\partial \mathcal{W}_{n,m}/\partial x_n$. Introducing the average interparticle distance in the string, Δ, and the particle displacement from the steady state, $\delta x_n = x_n - n\Delta$, we obtain the following equation for the dimensionless displacement $y_n = \delta x_n/\Delta$:

$$\ddot{y}_n + 2\gamma \dot{y}_n = \frac{1}{M\Delta} \sum_{m \neq n} F_{n,m}(y_n - y_m), \tag{2}$$

where M is the particle mass and γ is the damping rate due to neutral gas friction [12]. Normally, the interparticle distance in the crystal exceeds the screening length ($\Delta/\lambda_{\mathrm{D}e} \simeq 1.5$–$2$), so that for the present problem we take into account only the influence of the nearest neighbors. Let us present the charge of each particle as $Q_n(t) = \langle Q \rangle + \delta Q_n(t)$, where $\delta Q_n(t)$ corresponds to the random fluctuation of the charge around the mean $\langle Q \rangle$ (angle brackets denote average over ensemble). Assuming $|y_n - y_{n\pm1}| \ll 1$ and using Eq. (1) we have

$$F_{n,n-1} + F_{n,n+1} = \frac{\langle Q \rangle^2}{\Delta^2} \mathrm{e}^{1-\eta}(1 + \delta\tilde{Q}_n)\{(1 + \delta\tilde{Q}_{n-1})[\eta - (1 + \eta^2)(y_n - y_{n-1})] - (1 + \delta\tilde{Q}_{n+1})[\eta + (1 + \eta^2)(y_n - y_{n+1})]\}, \tag{3}$$

where $\delta\tilde{Q}_n = \delta Q_n/\langle Q \rangle$ and $\eta = 1 + \Delta/\lambda_{\mathrm{D}e}$. For the traveling wave the solution of Eq. (2) with force (3) can be presented in the form $y_n = y(t)\exp(iKn\Delta)$, where K is the wave vector. Note that for the DL wave, as well as for any elastic wave in a discrete medium, the range of the wave vector values is the first Brillouin zone [13], $-\pi/\Delta \leq K \leq \pi/\Delta$.

Introducing the DL frequency scale

$$\Omega^2 = \frac{\langle Q\rangle^2}{M\Delta^3}(1+\eta^2)e^{1-\eta}, \qquad (4)$$

and substituting Eq. (3) in Eq. (2) we transform the latter to

$$\ddot{y} + 2\gamma\dot{y} + \omega^2[1+\xi(t)]y = f(t), \qquad (5)$$

where $\omega^2 = 2\Omega^2(1-\cos K\Delta)$ and the charge fluctuations are described by the following functions:

$$\xi(t) = \delta\tilde{Q} - \frac{\delta\tilde{Q}_+(e^{iK\Delta}-1) + \delta\tilde{Q}_-(e^{-iK\Delta}-1)}{2(1-\cos K\Delta)},$$
$$f(t) = \frac{\eta}{1+\eta^2}\Omega^2(\delta\tilde{Q}_- - \delta\tilde{Q}_+). \qquad (6)$$

Here $\delta\tilde{Q} \equiv \delta\tilde{Q}_n$, $\delta\tilde{Q}_+$, and $\delta\tilde{Q}_-$ denote the charge fluctuations on the "central," "right," and "left" particles, respectively. We see that the traveling DL wave is described by a damped harmonic oscillator equation (5) with a randomly varying frequency and a random "external" force. In order to study the properties of the stochastic process $y(t;[\xi])$ (where $[\xi]$ denotes all possible realizations of the random variable ξ) we use the approximation method of expansion over a small Kubo number [14]. In principle, this general method allows us to obtain *nonstochastic* differential equations for mean and second moments of any random function, which obeys the linear stochastic differential equation of arbitrary order with sufficiently weak and rapidly fluctuating coefficients.

Let us introduce new variables, $\mathcal{Y}_1 = y$ and $\mathcal{Y}_2 = \omega^{-1}\dot{y}$. Using these variables we can reduce Eq. (5) to the first-order vector differential equation,

$$\dot{\mathbf{Y}} = \omega\mathbf{A}(t)\mathbf{Y} + \omega^{-1}\mathbf{f}(t), \qquad (7)$$

where

$$\mathbf{Y} = \begin{bmatrix}\mathcal{Y}_1\\ \mathcal{Y}_2\end{bmatrix}, \qquad \mathbf{A}(t) = -\begin{bmatrix}0 & -1\\ 1+\xi(t) & 2\epsilon\end{bmatrix},$$
$$\mathbf{f}(t) = \begin{bmatrix}0\\ f(t)\end{bmatrix}, \qquad (8)$$

and $\epsilon = \gamma/\omega$. We can present the matrix $\mathbf{A}(t)$ as a sum of a constant matrix $\mathbf{A}_0$ and a random matrix $\mathbf{A}_1(t)$,

$$\mathbf{A}_0 = -\begin{bmatrix}0 & -1\\ 1 & 2\epsilon\end{bmatrix}, \qquad \mathbf{A}_1(t) = -\xi(t)\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}.$$

We assume that the random variable $\xi(t)$ has a short autocorrelation time τ_c, so that the Kubo number is small [14],

$$\sqrt{\langle\xi^2\rangle}\,\omega\tau_c \ll 1. \qquad (9)$$

In this case we can expand the formal solution of Eq. (7) over this small parameter. Keeping the first three terms of this expansion and taking the average we obtain the following equation for the mean $\langle\mathbf{Y}\rangle$ which is valid for time scales $t \gtrsim \tau_c$:

$$\frac{d}{dt}\langle\mathbf{Y}\rangle = \omega\left(\mathbf{A}_0 + \omega\int_0^\infty \langle\mathbf{A}_1(t)e^{\mathbf{A}_0\omega\tau}\mathbf{A}_1(t-\tau)\rangle \times e^{-\mathbf{A}_0\omega\tau}\,d\tau\right)\langle\mathbf{Y}\rangle + \int_0^\infty \langle\mathbf{A}_1(t)e^{\mathbf{A}_0\omega\tau}\mathbf{f}(t-\tau)\rangle\,d\tau. \qquad (10)$$

If $\xi(t)$ is a stationary random process, then the averaged values in the integrals in Eq. (10) do not depend on t and the equation obtained has constant coefficients. For $\epsilon \ll 1$ these coefficients can be calculated using the relation

$$e^{\pm\mathbf{A}_0\omega\tau} \simeq e^{\mp\gamma\tau}\begin{bmatrix}\cos\omega\tau \pm \epsilon\sin\omega\tau & \pm\sin\omega\tau\\ \mp\sin\omega\tau & \cos\omega\tau \mp \epsilon\sin\omega\tau\end{bmatrix},$$

which directly follows from the exact solution of Eq. (7) with $\xi(t) = f(t) \equiv 0$. Hence, with the accuracy $O(\epsilon)$ we have from Eq. (10)

$$\frac{d}{dt}\langle\mathbf{Y}\rangle = \omega(\mathbf{A}_0 + \omega\hat{\mathbf{A}}_1)\langle\mathbf{Y}\rangle + \hat{\mathbf{f}}, \qquad (11)$$

where elements of the matrix $\hat{\mathbf{A}}_1$ and vector $\hat{\mathbf{f}}$,

$$\hat{\mathbf{A}}_1 = \frac{1}{2}\begin{bmatrix}0 & 0\\ C^{(1)}_{\xi\xi} & -C^{(2)}_{\xi\xi}\end{bmatrix}, \qquad \hat{\mathbf{f}} = -\begin{bmatrix}0\\ C_{\xi f}\end{bmatrix}$$

are the following constants:

$$C^{(1)}_{\xi\xi} = \int_0^\infty R_{\xi\xi}(\tau)\sin 2\omega\tau\,d\tau,$$
$$C^{(2)}_{\xi\xi} = \int_0^\infty R_{\xi\xi}(\tau)(1-\cos 2\omega\tau)\,d\tau, \qquad (12)$$
$$C_{\xi f} = \int_0^\infty R_{\xi f}(\tau)e^{-\gamma\tau}\sin 2\omega\tau\,d\tau.$$

The integrals in Eq. (12) contain correlation functions $R_{\xi\xi}(\tau) = \langle\xi(t)\xi(t-\tau)\rangle$ and $R_{\xi f}(\tau) = \langle\xi(t)f(t-\tau)\rangle$. We obtain these functions using Eq. (6),

$$R_{\xi\xi}(\tau) = \langle\delta\tilde{Q}(t)\delta\tilde{Q}(t-\tau)\rangle\frac{1-2\cos K\Delta}{1-\cos K\Delta} + \frac{\langle\delta\tilde{Q}_+(t)\delta\tilde{Q}_-(t-\tau)\rangle}{1-\cos K\Delta} + 2\langle\delta\tilde{Q}(t)\delta\tilde{Q}_+(t-\tau)\rangle, \qquad (13)$$

$$R_{\xi f}(\tau) = \frac{\eta}{1+\eta^2}\Omega^2(\langle\delta\tilde{Q}(t)\delta\tilde{Q}(t-\tau)\rangle - \langle\delta\tilde{Q}_+(t)\delta\tilde{Q}_-(t-\tau)\rangle)\frac{\sin K\Delta}{1-\cos K\Delta}.$$

Returning to the initial variable y, we finally have the following equation for the mean displacement $\langle y\rangle$:

$$\frac{d^2}{dt^2}\langle y\rangle + 2\gamma_{f1}\frac{d}{dt}\langle y\rangle + \omega_{f1}^2\langle y\rangle = -\omega C_{\xi f}, \qquad (14)$$

where $\gamma_{f1} = \gamma(1 + \omega^2 C^{(2)}_{\xi\xi}/4\gamma)$ and $\omega^2_{f1} = \omega^2(1 - \omega C^{(1)}_{\xi\xi}/2)$. We see that the charge fluctuations cause a change in the DL frequency and the damping rate.

Let us make further assumptions about the stochastic properties of $\delta\tilde{Q}(t)$: The correlation of fluctuations on the neighbor particles is negligible in comparison with the autocorrelation; the fluctuations of the particle charges are determined by fluctuations of the electron and ion fluxes in the macroscopically *equilibrium* plasma. In this case we can use the results obtained for the single particle from the orbit motion limited theory [15,16],

$$\begin{aligned} &\langle \delta\tilde{Q}(t)\delta\tilde{Q}(t - \tau)\rangle \simeq \tilde{\sigma}^2 \exp(-\Omega_{U_0}\tau), \\ &\langle \delta\tilde{Q}(t)\delta\tilde{Q}_{\pm}(t - \tau)\rangle \simeq 0, \end{aligned} \tag{15}$$

where $\tilde{\sigma}^2$ is the dimensionless dispersion of the charge distribution (normalized to $\langle Q\rangle^2$) and $\Omega_{U_0} \equiv \tau_c^{-1}$ is the steady-state charging frequency [17].

In laboratory experiments the plasma crystal is located near the edge of the sheath, and the ion drift velocity towards the electrode u is of the order of the ion acoustic velocity c_s (and much greater than the ion thermal velocity). Using the expressions for $\tilde{\sigma}^2$ and Ω_{U_0} from Ref. [15], we obtain for these conditions

$$\begin{aligned} \tilde{\sigma}^2 &\simeq \frac{\alpha + u^2/c_s^2}{\alpha(1 + \alpha + u^2/c_s^2)}\left(\frac{e}{|\langle Q\rangle|}\right) \sim \frac{e}{|\langle Q\rangle|}, \\ \Omega_{U_0} &\simeq \frac{1 + \alpha + u^2/c_s^2}{\sqrt{2\pi}}\left(\frac{\alpha\omega_{pi}^2}{u}\right) \sim \frac{ac_s}{\lambda_{De}^2}, \end{aligned} \tag{16}$$

where $\alpha = e|\langle Q\rangle|/aT_e$ is a coefficient of the order of unity, a is the particle radius, T_e is the electron temperature, and ω_{pi} is the ion plasma frequency. Substituting Eq. (15) in Eq. (13) and using Eqs. (12) and (14) we obtain for ω^2_{f1} and γ_{f1}

$$\begin{aligned} \frac{\omega_{f1}^2}{\omega^2} &\simeq 1 - 2\tilde{\sigma}^2\frac{\Omega^2}{\Omega_{U_0}^2}(1 - 2\cos K\Delta), \\ \frac{\gamma_{f1}}{\gamma} &\simeq 1 + 4\tilde{\sigma}^2\frac{\Omega^4}{\gamma\Omega_{U_0}^3}(1 - \cos K\Delta)(1 - 2\cos K\Delta). \end{aligned} \tag{17}$$

We see that the variation of both the frequency and the damping rate can be either positive or negative, depending on the value of the wave vector. However, for autocorrelation function (15) the condition (9) takes the form $\tilde{\sigma}(\Omega/\Omega_{U_0}) \ll 1$, so that $\omega_{f1} \simeq \omega$. In principle, for sufficiently small values of γ, the coefficient γ_{f1} can be negative at $K\Delta < \pi/3$; i.e., Eq. (14) can be unstable. But as shown below, γ_{f1} always remains positive for real conditions of experiments.

The presence of the random force $f(t)$ in Eq. (5) results in the appearance of a stationary mean displacement $\langle y\rangle_\infty$ at large time scales. Assuming that $\gamma_{f1} > 0$, we have from Eq. (14) at $t \gg \gamma_{f1}^{-1}$

$$\langle y\rangle_\infty \simeq -\omega^{-1}C_{\xi f} = -\frac{\eta\tilde{\sigma}^2}{1 + \eta^2}\frac{\Omega^2}{\Omega_{U_0}^2}\frac{\sin K\Delta}{1 - \cos K\Delta}. \tag{18}$$

The stationary displacement increases as $\langle y\rangle_\infty \propto K^{-1}$ at small $K\Delta$. Expression (18) is valid for $K\Delta \gg \gamma/\Omega$, because Eq. (11) is obtained under the assumption $\omega \gg \gamma$.

Let us now find the equations for the second moments, in particular, for the mean squared displacement $\langle yy^*\rangle$ and velocity $\langle \dot{y}\dot{y}^*\rangle$. These equations allow us to evaluate how the average wave energy depends on time. From Eq. (7) we derive the equation for the products $\mathcal{Y}_j\mathcal{Y}^*_{j'}$ written in the tensor form,

$$\frac{d}{dt}(\mathcal{Y}_j\mathcal{Y}^*_{j'}) = \omega\mathcal{B}^{ll'}_{jj'}\mathcal{Y}_l\mathcal{Y}^*_{l'} + \omega^{-1}(f_j\mathcal{Y}^*_{j'} + f^*_{j'}\mathcal{Y}_j). \tag{19}$$

Here all the indexes change from 1 to 2 and tensor $\mathcal{B}^{ll'}_{jj'} = A^l_j\delta^{l'}_{j'} + A^{*l'}_{j'}\delta^l_j$, where A^l_j are elements of the matrix $\mathbf{A}(t)$ [see Eq. (8)]. In order to check the "energy stability" of the DL wave we can omit in Eq. (19) inhomogeneous terms containing random force $f(t)$, because these terms alone cannot cause instability when the homogeneous equation (19) (after averaging) is stable. In the vector form, the homogeneous equation (19) is

$$\dot{\mathbf{W}} = \omega\mathbf{B}(t)\mathbf{W}, \tag{20}$$

where $\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t)$ and

$$\mathbf{W} = \begin{bmatrix} \mathcal{Y}_1\mathcal{Y}_1^* \\ \mathcal{Y}_1\mathcal{Y}_2^* \\ \mathcal{Y}_2\mathcal{Y}_1^* \\ \mathcal{Y}_2\mathcal{Y}_2^* \end{bmatrix}, \qquad \mathbf{B}_0 = -\begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 2\epsilon & 0 & -1 \\ 1 & 0 & 2\epsilon & -1 \\ 0 & 1 & 1 & 4\epsilon \end{bmatrix},$$

$$\mathbf{B}_1(t) = -\begin{bmatrix} 0 & 0 & 0 & 0 \\ \xi^*(t) & 0 & 0 & 0 \\ \xi(t) & 0 & 0 & 0 \\ 0 & \xi(t) & \xi^*(t) & 0 \end{bmatrix}.$$

Taking the average in Eq. (20) we obtain the equation for the second moments $\langle\mathbf{W}\rangle$ in the form of (homogeneous) Eq. (10) with substitution $\langle\mathbf{Y}\rangle \to \langle\mathbf{W}\rangle$ and $\mathbf{A}_{0,1} \to \mathbf{B}_{0,1}$. In this equation we can reduce the number of independent variables and present the resulting equation to an accuracy $O(\epsilon)$ in the following form:

$$\frac{d}{dt}\langle\mathbf{W}\rangle_r = \omega(\mathbf{B}_{0r} + \omega\hat{\mathbf{B}}_{1r})\langle\mathbf{W}\rangle_r, \tag{21}$$

where the reduced vector $\langle\mathbf{W}\rangle_r$ and matrices $\mathbf{B}_{0r}$, $\hat{\mathbf{B}}_{1r}$ are

$$\langle\mathbf{W}\rangle_r = \begin{bmatrix} \langle\mathcal{Y}_1\mathcal{Y}_1^*\rangle \\ \frac{1}{2}\langle\mathcal{Y}_1\mathcal{Y}_2^* + \mathcal{Y}_2\mathcal{Y}_1^*\rangle \\ \langle\mathcal{Y}_2\mathcal{Y}_2^*\rangle \end{bmatrix},$$

$$\mathbf{B}_{0r} = -\begin{bmatrix} 0 & -2 & 0 \\ 1 & 2\epsilon & -1 \\ 0 & 2 & 4\epsilon \end{bmatrix},$$

$$\hat{\mathbf{B}}_{1r} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ C^{(1)}_{\xi\xi} + C^{(1)}_{\xi\xi^*} & -(C^{(2)}_{\xi\xi} + C^{(2)}_{\xi\xi^*}) & 0 \\ 2C^{(3)}_{\xi\xi^*} & 2(C^{(1)}_{\xi\xi} - C^{(1)}_{\xi\xi^*}) & -2C^{(2)}_{\xi\xi} \end{bmatrix}.$$

In addition to Eq. (12), we introduce here the constant $C^{(3)}_{\xi\xi^*} = \int_0^\infty R_{\xi\xi^*}(\tau)\,(1 + \cos 2\omega\tau)\,d\tau$, where $R_{\xi\xi^*}(\tau)$ is the correlation function of $\xi(t)$ and its complex conjugate $\xi^*(t)$,

$$R_{\xi\xi^*}(\tau) = \langle \delta\tilde{Q}(t)\delta\tilde{Q}(t-\tau)\rangle \frac{2-\cos K\Delta}{1-\cos K\Delta} - \langle \delta\tilde{Q}_+(t)\delta\tilde{Q}_-(t-\tau)\rangle \frac{\cos K\Delta}{1-\cos K\Delta} + 2\langle \delta\tilde{Q}(t)\delta\tilde{Q}_+(t-\tau)\rangle . \quad (22)$$

The solution of the differential equation (21) is determined from the roots of the characteristic equation: $\det[\omega(\mathbf{B}_{0\mathrm{r}} + \omega\hat{\mathbf{B}}_{1\mathrm{r}}) - \lambda\mathbf{I}] = 0$, where $\mathbf{I}$ is the unit matrix. In explicit form we obtain

$$\lambda^3 + [6\gamma + \tfrac{1}{2}\omega^2(3C^{(2)}_{\xi\xi} + C^{(2)}_{\xi\xi^*})]\lambda^2 + 2\omega^2(2 - \omega C^{(1)}_{\xi\xi})\lambda + 2\omega^2[4\gamma + \omega^2(C^{(2)}_{\xi\xi} - C^{(3)}_{\xi\xi^*})] = 0 . \quad (23)$$

Equation (21) loses stability when any one of the coefficients in Eq. (23) becomes negative. Since $\omega \gg \gamma$, it is most likely to expect that the last term in Eq. (23) may be negative for sufficiently small γ. Hence, the criterion for the average energy of the DL wave to grow exponentially with time is

$$\tilde{\sigma}^2 \frac{\Omega^2}{\gamma\Omega_{U_0}}(2 - \cos K\Delta) > 1 . \quad (24)$$

In laboratory experiments ($a \sim 1$–$10\ \mu$m, $\Delta \sim \lambda_{De} \sim 10^2$–$10^3\ \mu$m) the typical values of frequencies (4) and (16) are $\Omega \sim 30$–3×10^3 s^{-1} and $\Omega_{U_0} \sim 10^3$–10^5 s^{-1}. For large particles, the charging frequency Ω_{U_0} considerably exceeds Ω, but for $a \lesssim 1\ \mu$m these frequencies can be comparable. Therefore, criterion (24) can be satisfied for a plasma crystal of sufficiently small particles. For example, the DL wave can be unstable energywise for particles of a size $a \sim 1\ \mu$m at a pressure of about 1 Pa, when the interparticle distance Δ is about 100 μm or less. This is not the usual, but a physically realistic condition for a plasma crystal, and we can expect that the instability due to the charge fluctuations may be observed in experiments. In accordance with Eq. (4), criterion (24) has sharp dependence on Δ. Thus, even a small increase of particle density in the crystal can result in the appearance of the described instability.

An instability in the average energy does not necessarily imply an instability of Eq. (14) for the average displacement $\langle y\rangle$. In a plasma crystal, the damping rate γ_{f1} is positive for all physically reasonable parameters [see Eq. (17)]. Therefore, even if the DL wave is unstable energywise, the value of $\langle y\rangle$ decreases with time and tends to the stationary value $\langle y\rangle_\infty$.

Note that in a dusty plasma the charge fluctuations can cause an instability of oscillations only in the crystal phase, where the discrete structure is essential and wave propagation is determined by the charge of each separate particle. In the gaseous phase, all the variables in the initial equations are averaged over space, so that the dispersion of the charge distribution tends to zero and the fluctuations can be neglected.

In the present paper we consider only one kind of random fluctuation which results in the appearance of the random term in the expression for the interparticle coupling force. Of course, there are also fluctuations of the local electron temperature and electrostatic fluctuations in the plasma. But it is noteworthy that for any kind of fluctuation the resulting equation for the traveling DL wave has the form of Eq. (5) [with the specific functions $\xi(t)$ and $f(t)$], so that we always can use the described method for the subsequent investigations.

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[1] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[2] G. E. Morfill, H. M. Thomas, and M. Zuzic in *Advances in Dusty Plasmas*, edited by P. K. Shukla, D. A. Mendis, and T. Desai (World Scientific, Singapore, 1997), pp. 99–142.
[3] M. Zuzic, H. M. Thomas, and G. E. Morfill, J. Vac. Sci. Technol. A **14**, 496 (1996).
[4] J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
[5] S. Peters, A. Homann, A. Melzer, and A. Piel, Phys. Lett. A **223**, 389 (1996).
[6] A. Homann, A. Melzer, S. Peters, and A. Piel, Phys. Rev. E **56**, 7138 (1997).
[7] A. Homann, A. Melzer, R. Madani, and A. Piel, Phys. Lett. A **242**, 173 (1998).
[8] F. Melandsø, Phys. Plasmas **3**, 3890 (1996).
[9] T. K. Aslaksen and O. Havnes, J. Plasma Phys. **51**, 271 (1994).
[10] B. F. Gordiets and C. M. Ferreira, J. Appl. Phys. **84**, 1231 (1998).
[11] T. Matsoukas, M. Russell, and M. Smith, J. Vac. Sci. Technol. A **14**, 624 (1996).
[12] P. Epstein, Phys. Rev. **23**, 710 (1924).
[13] C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1976).
[14] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (Elsevier, Amsterdam, 1981).
[15] T. Matsoukas and M. Russell, Phys. Rev. E **55**, 991 (1997).
[16] H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1989).
[17] F. Melandsø, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).