Violation of Time Reversal Invariance in K^0 Decays

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A T-odd correlation observed in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ has been cited as direct evidence of time-reversal violation (TRV). Here it is argued that when CP violation is due to $K^0\overline{K^0}$ mixing it is doubtful that any decay experiment by itself can provide direct evidence for TRV.

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CP violation has been observed in several different observables in K^0 decay. It is then expected from the CPT theorem that the CP-violating interaction also violates time-reversal invariance. Recently a nonzero value of an observable that appears to be odd under time reversal has been detected [1] in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$. We address here the question of whether this may be considered as direct evidence for T violation.

The effect in the experiment is associated with $K^0\overline{K^0}$ mixing. It is known from the detailed analysis (summarized below) of the CP-violating effects that this mixing indeed violates T as expected from CPT invariance. Thus the question we ask is not whether T is violated, which is known, but a didactic question as to whether we now have direct evidence.

Defining the CP eigenstates

$$|K_1\rangle = (|K^0\rangle + |\overline{K^0}\rangle)/\sqrt{2},$$

$$|K_2\rangle = (|K^0\rangle - |\overline{K^0}\rangle)/\sqrt{2},$$

the mass matrix in the K_1-K_2 representation may be written in general without assuming CPT invariance as [2]

$$M - i\,\frac{\Gamma}{2} = \begin{pmatrix} m_1 & im' + m'' \\ -im' + m'' & m_2 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \gamma_1 & \\ & \gamma_2 \end{pmatrix}. \tag{1}$$

The off-diagonal terms are associated with CP violation in the following way:

m': CP violation, CPT invariance, T violation.

m'': CP violation, CPT violation, T invariance.

We assume that the only CP violation occurs in the mixing matrix M. It is easy to see, by returning to the $K^0\overline{K^0}$ representation, that we may interpret $2m''$ as $m(K^0) - m(\overline{K^0})$.

The decaying states are given in lowest order by

$$|K_S\rangle = \begin{pmatrix} 1 \\ \delta + \rho \end{pmatrix}, \qquad |K_L\rangle = \begin{pmatrix} -\delta + \rho \\ 1 \end{pmatrix}, \tag{2}$$

$$\varrho = \frac{-im'}{(m_S - m_L) - i(\gamma_S - \gamma_L)/2}, \tag{3}$$

$$\delta = \frac{m''}{(m_S - m_L) - i(\gamma_S - \gamma_L)/2}.$$

We can define the usual parameter ϵ

$$\epsilon = -\delta + \rho = \frac{m' - im''}{(\gamma_S - \gamma_L)/2 - i(m_S - m_L)}. \tag{4}$$

If for simplicity we use the approximate equality $(m_S - m_L) = (\gamma_S - \gamma_L)/2$ we find

$$\epsilon = |\epsilon|\exp(i\varphi_\epsilon) = \frac{\exp(i\pi/4)}{\sqrt{2}\,(m_L - m_S)}\,(m' - im''). \tag{5}$$

The phase of ϵ is equal to the phase of φ_{+-} of the CP-violating parameter η_{+-} to a high degree of accuracy given the empirical limits on ϵ'. The measured value of φ_{+-} agrees with the prediction from CPT invariance within a degree and thus produces a strong limit on m''. It has been emphasized [3] that this is the best test of CPT invariance.

One way to search for T violation is to study T-odd correlations in the final state of a weak decay. An old example [4] is the D parameter in beta decay which measures the dependence of the decay on $\vec{J} \cdot \vec{p}_e \times \vec{p}_\nu$ where J is the nuclear spin and $(\vec{p}_e, \vec{p}_\nu)$ are the (e, ν) momenta. This is a sign of T violation only if the Born approximation is valid so that one can equate the $|\text{in}\rangle$ and $|\text{out}\rangle$ states of the decay products. Thus final-state interactions can produce such a correlation in the absence of T violation [5]; this is sometimes called pseudo-TRV. In the case of nuclear beta decay the final-state interaction is electromagnetic and can be calculated accurately. For neutron beta decay the calculated D parameter is only 1.1×10^{-5} but the experimental limits are only at the level of 10^{-3}.

In the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ the term observed in the decay angular distribution is proportional to

$$(\vec{n}_e \times \vec{n}_\pi) \cdot \hat{z}(\vec{n}_e \cdot \vec{n}_\pi). \tag{6}$$

This is clearly CP violating where $(\vec{n}_e, \vec{n}_\pi)$ are the normals to the planes of the (lepton pair, pion pair) and $\hat{z}$ is a unit vector in the direction of the center of mass of the pion pair. It is also a T-odd observable since it involves an odd number of momenta. However, there is a large final-state interaction between the two pions so that we expect a nonzero effect even in the absence of T violation.

The analysis of the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ shows the decay to involve primarily two CP-conserving decay

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amplitudes: (i) the pair conversion of a bremsstrahlung $E1$ photon from K_1 decay and (ii) the pair conversion of a virtual $M1$ photon from K_2 decay. The CP violation is entirely due to the admixture of K_1 in K_L. The resulting asymmetry associated with the term (6) is given to a good approximation by [6]

$$A = 15\% \times \sin(\phi_\epsilon + \Delta)\,,$$

where $\Delta \approx 30°$ is the difference between the $\pi\pi$ phase shifts in s and p waves.

It is of didactic interest to consider the limiting case in which $\Delta = 0$. Assuming CPT invariance we set $m'' = 0$ and find

$$A = 15\% \times \sin(\pi/4)\,.$$

If we now consider the opposite possibility of maximal CPT violation so that the CP-violating term is T invariant we set $m' = 0$ and find (with an appropriate sign for m'')

$$A = 15\% \times \sin(3\pi/4)\,.$$

Thus, in the absence of final state interactions, we get the same asymmetry when we assume there is no T violation; in this case we clearly have pseudo-TRV. The almost exact equality of the two asymmetries is due to the chance that the phase in Eq. (5) is $\pi/4$.

The explanation lies in what we might call "initial state interaction" associated with $(\gamma_S - \gamma_L)$. Pseudo-TRV can occur whenever the calculation of the decay amplitude involves an on-shell intermediate state resulting in a phase factor unrelated to TRV. The term $(\gamma_S - \gamma_L)$ is seen to be the source of the phase $\pi/4$ in Eq. (5). If $(\gamma_S - \gamma_L)$ is set to zero we see that $\varphi_\epsilon = \pi/2$ for the CPT-invariant case and $\varphi_\epsilon = 0$ or π for the T-invariant case. Thus the vanishing of $(\gamma_S - \gamma_L)$ and of Δ is required to rule out pseudo-TRV in this case.

In the present case we can consider the K^0 decay to the state F as involving the sum of three diagrams: (a) $K^0 \rightarrow F$, (b) $K^0 \rightarrow \overline{K^0}$ via a point interaction, giving ΔM and the CP-violating m', followed by $\overline{K^0} \rightarrow F$, and (c) $K^0 \rightarrow \overline{K^0}$ via a loop with an absorptive part giving $\Delta\Gamma$ followed by $\overline{K^0} \rightarrow F$. The third of these is what we call the initial state interaction.

Another way of saying this is that the state $|K_L(\text{in})\rangle$ is not equivalent to $|K_L(\text{out})\rangle$ as is implicitly assumed in applying the T transformation to the decay process. The time reverse of a decaying state is not a physical state. It might seem that this argument would rule out all tests of T invariance in decays such as the D parameter discussed above. However, in most cases the decay width is totally irrelevant until one includes higher-order weak effects, but the presence of $\Delta\Gamma$ in the expression for the phase of ϵ shows that it is very relevant in the case of K^0 decays.

In conclusion, it is very clear that $K^0 - \overline{K^0}$ mixing involves CP violation and T violation and the analysis of the phase φ_{+-} provides a strong limit on CPT violation. However, we also conclude from our analysis of T-odd correlations that any direct tests of TRV where $K^0 - \overline{K^0}$ mixing is involved must be viewed with extreme caution.

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