

Can ϵ'/ϵ be Supersymmetric?

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Supersymmetric contributions to ϵ'/ϵ have been generally regarded as small. We point out, however, that this is based on specific assumptions, such as universal scalar mass, and in general need not be true. Based on (1) hierarchical quark Yukawa matrices protected by flavor symmetry, (2) generic dependence of Yukawa matrices on Polonyi/moduli fields as expected in many supergravity/superstring theories, (3) Cabibbo rotation originating from the down-sector, and (4) phases of order unity, we find the typical supersymmetric contribution to ϵ'/ϵ to be of order 3×10^{-3} for $m_{\tilde{q}} = 500$ GeV, possibly dominating the reported KTeV Collaboration value $\epsilon'/\epsilon = (28 \pm 4.1) \times 10^{-4}$. If so, the neutron electric dipole moment is likely to be within the reach of the currently planned experiments.

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CP violation is the least understood aspect in the properties of the fundamental particles besides the mechanism of the electroweak symmetry breaking. The so-called “indirect CP violation” ϵ in the neutral kaon system has been known for three decades as the only evidence that there is a fundamental distinction between particles and antiparticles. This year, however, produced two new manifestations of CP violation: $\sin 2\beta$ from $B \rightarrow \psi K_s$ at the Collider Detector at Fermilab (CDF) [1], even though the evidence is still somewhat weak, and a beautiful measurement of “direct CP violation” ϵ'/ϵ in a neutral kaon system from the KTeV Collaboration [2]. The latter confirmed the previous evidence reported by NA31 [3] at a much higher accuracy and excludes the so-called superweak model of CP violation. The reported number, $\epsilon'/\epsilon = (28 \pm 4.1) \times 10^{-4}$ [2] was, however, somewhat surprisingly large. The standard model prediction is currently controversial (see Table I) and is dominated by theoretical uncertainties in quantities such as the nonperturbative matrix elements and the strange quark mass m_s . Given this situation, one cannot interpret the KTeV data reliably; in particular, it is not clear if the data is consistent with the standard model (see also, for a recent discussion, Ref. [7]).

On the other hand, the standard model is believed to be only an effective low-energy approximation of fundamental physics. This is largely because it lacks a dynamical explanation of the mechanism of electroweak symmetry breaking and suffers from a serious hierarchy problem that the electroweak scale is unstable against radiative corrections. The best available simultaneous solution to both of these problems is supersymmetry. Therefore, it is a natural question to ask if supersymmetry gives a sizable contribution to ϵ'/ϵ given a precise measurement. The experimental sensitivity to a possible supersymmetric

(SUSY) contribution is currently plagued by the theoretical uncertainties mentioned above, but we can expect them to be resolved, or at least alleviated, eventually by improvements, in particular, in lattice quantum chromodynamics (QCD) calculations. It is therefore timely to reconsider the supersymmetric contribution to ϵ'/ϵ .

In this Letter, we revisit the estimate of ϵ'/ϵ in supersymmetric models [8]. The common lore in the literature is that the supersymmetric contribution to ϵ'/ϵ is in general rather small. We point out, however, that this lore is largely based on the specific choice of supersymmetry breaking effects sometimes called minimal supergravity framework [9]. A more general framework of flavor structure tends to give a relatively large contribution to ϵ'/ϵ in a wide class of models. The assumptions are as follows: (1) hierarchical quark Yukawa matrices are protected by flavor symmetry, (2) generic dependence of Yukawa matrices on Polonyi/moduli fields, as expected in many supergravity/superstring theories, (3) Cabibbo rotation originating from the down sector, and (4) phases of order unity. In fact, there is even an intriguing possibility that the observed ϵ'/ϵ is mostly or entirely due to the supersymmetric contribution.

TABLE I. Various estimates of ϵ'/ϵ in the literature. Two Buras' estimates use different values of the strange quark mass, $m_s(2 \text{ GeV}) = 130 \pm 20 \text{ MeV}$ (QCD sum rule) and $110 \pm 20 \text{ MeV}$ (lattice QCD).

Reference	ϵ'/ϵ
Ciuchini [4]	$(4.6 \pm 3.0 \pm 0.4) \times 10^{-4}$
Buras [5]	$(5.7 \pm 3.6) \times 10^{-4}$
Buras [5]	$(9.1 \pm 5.7) \times 10^{-4}$
Bertolini <i>et al.</i> , [6]	$(17_{-10}^{+14}) \times 10^{-4}$

To discuss the CP violating effects induced by loops of supersymmetric particles, it is convenient to introduce the mass insertion formalism [10]. The Yukawa matrices are couplings in the superpotential $W = Y_{ij}^u Q_i U_j H_u + Y_{ij}^d Q_i D_j H_d$, where H_d, H_u are Higgs doublets and i, j are flavor indices. The expectation values $\langle H_u \rangle = v \sin\beta/\sqrt{2}$ and $\langle H_d \rangle = v \cos\beta/\sqrt{2}$ generate quark mass matrices $M^u = Y^u v \sin\beta/\sqrt{2}$ and $M^d = Y^d v \cos\beta/\sqrt{2}$. They are diagonalized by bi-unitary transformations $M^u = V_L^{u*} \text{diag}(m_u, m_c, m_t) V_R^{u\dagger}$ and $M^d = V_L^{d*} \text{diag}(m_d, m_s, m_b) V_R^{d\dagger}$, and the Cabibbo–Kobayashi–Maskawa matrix is given by $V_L^{u\dagger} V_L^d$. The squarks have chirality-preserving mass-squared matrices $\mathcal{L} \supset -\tilde{Q}_i^* (M_{\tilde{Q}}^2)_{ij} \tilde{Q}_j - \tilde{U}_i^* (M_{\tilde{U}}^2)_{ij} \tilde{U}_j - \tilde{D}_i^* (M_{\tilde{D}}^2)_{ij} \tilde{D}_j$ and chirality-violating trilinear couplings $\mathcal{L} \supset -\tilde{Q}_i (A^d)_{ij} \tilde{D}_j H_d - \tilde{Q}_i (A^u)_{ij} \tilde{U}_j H_u$, where H_d, H_u are Higgs doublets. The Higgs expectation values generate the left-right (LR) mass-squared matrix $M_{LR}^{2,d} = A^d v \sin\beta/\sqrt{2}$ and $M_{LR}^{2,u} = A^u v \cos\beta/\sqrt{2}$. The convenient basis to discuss flavor-changing effects in the gluino loop diagrams is the so-called super-CKM basis [10]. In this basis the relevant quark mass matrix is diagonalized (say, M^d) and the squarks are also rotated in the same way, $M_{\tilde{Q}}^2 \rightarrow V_L^{d\dagger} M_{\tilde{Q}}^2 V_L^d$, $M_{\tilde{D}}^2 \rightarrow V_R^{d\dagger} M_{\tilde{D}}^2 V_R^d$, and $M_{LR}^{2,d} \rightarrow {}^t V_L^d M_{LR}^{2,d} V_R^d$. Flavor-changing effects can be estimated by insertion of flavor-off-diagonal components of the mass-squared matrices in this basis. By normalizing the off-diagonal components by average squark mass-squared $m_{\tilde{q}}^2$, we define $(\delta_{LL}^d)_{ij} = (V_L^{d\dagger} M_{\tilde{Q}}^2 V_L^d)_{ij}/m_{\tilde{q}}^2$, $(\delta_{RR}^d)_{ij} = (V_R^{d\dagger} M_{\tilde{D}}^2 V_R^d)_{ij}/m_{\tilde{q}}^2$, and $(\delta_{LR}^d)_{ij} = ({}^t V_L^d M_{LR}^{2,d} V_R^d)_{ij}/m_{\tilde{q}}^2$.

The supersymmetric contributions due to gluino loops to neutral Kaon parameters $(\Delta m_K)_{\text{SUSY}}$, ϵ_{SUSY} , and $(\epsilon'/\epsilon)_{\text{SUSY}}$ have been calculated, and have been used to place bounds on mass insertion parameters [11]. The values of the mass insertion parameters which saturate the observed numbers of Δm_K , ϵ , and ϵ'/ϵ are tabulated in Table II, after updating the numbers in Ref. [12]. These numbers are subject to theoretical uncertainties in QCD corrections and matrix elements of order of at least a few tens of percent (this is at least what is obtained for the $\Delta S = 2$ transitions [13]). Barring possible cancellations with the standard-model amplitudes as well as with the other SUSY contributions (i.e., chargino and charged Higgs exchanges), the mass insertion parameters have to be smaller than or, at most, comparable to the entries in the Table. Stringent bounds on $(\delta_{12}^d)_{LL}$ from Δm_K and ϵ have been regarded as a problem in supersymmetric models. A random mass-squared matrix of squarks would lead to a large $(\delta_{12}^d)_{LL}$ which overproduce Δm_K or ϵ . Usually an assumption is invoked that the squark mass-squared matrix is proportional to the identity matrix (universality), at least for the first and second generations (alternatively, one can invoke an alignment of the quark and squark mass matrices [14]). Even when

such an assumption is made at the Planck scale, radiative effects can induce $(\delta_{12}^d)_{LL}$ and hence overproduce Δm_K or ϵ . Once the bounds are satisfied, however, the supersymmetric contribution to ϵ' tends to be rather small: Δm_K and ϵ require $|(\delta_{12}^d)_{LL}| = ([\text{Re}(\delta_{12}^d)_{LL}]^2 + [\text{Im}(\delta_{12}^d)_{LL}]^2)^{1/4} \lesssim 0.019\text{--}0.092$, which is much smaller than the corresponding bounds from ϵ'/ϵ , $|\text{Im}(\delta_{12}^d)_{LL}| \lesssim 0.10\text{--}0.27$ [15]. This fact led to a common wisdom that the supersymmetric contribution to ϵ' is in general small. For the rest of this Letter, we simply assume that $(\delta_{ij}^d)_{LL}$ parameters are under control by some mechanisms such as a flavor symmetry and do not go into any further discussions.

However, the contribution from $(\delta_{12}^d)_{LR}$ can be important; even $|\text{Im}(\delta_{12}^d)_{LR}| \sim 10^{-5}$ gives a significant contribution to ϵ'/ϵ while the bounds on $(\delta_{12}^d)_{LR}$ from Δm_K and ϵ are only about 3×10^{-3} and 3×10^{-4} , respectively. Actually, one can even imagine saturating both ϵ and ϵ'/ϵ at the borderline of the current limits [16]. Therefore, whether the supersymmetric contribution to ϵ'/ϵ can be important is an issue of how large $(\delta_{12}^d)_{LR}$ is expected to be in supersymmetric models rather than that of phenomenological viability.

What is the general expectation on the size of $(\delta_{12}^d)_{LR}$? The common answer in the literature to this question is that it is very small in general, and hence the supersymmetric contribution to ϵ' has been regarded small as well. This is indeed the case if one assumes that all soft supersymmetry breaking parameters are universal at Planck or GUT scale, where $(\delta_{12}^d)_{LR}$ is induced only radiatively at higher orders in small Yukawa coupling constants of first

TABLE II. The values of the mass insertion parameters which saturate the observed numbers $(\Delta m_K)_{\text{SUSY}} = 3.521 \times 10^{-12}$ MeV, $(\epsilon)_{\text{SUSY}} = 2.27 \times 10^{-3}$, and $(\epsilon'/\epsilon)_{\text{SUSY}} = 2.8 \times 10^{-3}$ for three values of $x \equiv m_g^2/m_{\tilde{q}}^2$. They are based on Ref. [12] scaled to the current observed numbers. The squark mass is taken to be 500 GeV and have theoretical uncertainties of at least a few tens of percent. Barring possible cancellations, the mass insertion parameters must be smaller than or, at most, comparable to the entries given here.

Δm_K	x	$ \text{Re}(\delta_{12}^d)_{LL} ^{1/2}$	$ \text{Re}(\delta_{12}^d)_{LR} ^{1/2}$	$ \text{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} ^{1/2}$
	0.3	0.019	0.0078	0.0025
	1.0	0.040	0.0044	0.0028
	4.0	0.092	0.0053	0.0040
ϵ	x	$ \text{Im}(\delta_{12}^d)_{LL} ^{1/2}$	$ \text{Im}(\delta_{12}^d)_{LR} ^{1/2}$	$ \text{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} ^{1/2}$
	0.3	0.0015	0.00063	0.00020
	1.0	0.0032	0.00035	0.00022
	4.0	0.0075	0.00042	0.00032
ϵ'/ϵ	x	$ \text{Im}(\delta_{12}^d)_{LL} $	$ \text{Im}(\delta_{12}^d)_{LR} $	
	0.3	0.10	0.000011	
	1.0	0.50	0.000021	
	4.1	0.27	0.000065	

and second generation particles [9]. However, the universal breaking is a strong assumption and is known not to be true in many supergravity and string-inspired models [17]. On the other hand, the LR mass matrix has the same flavor structure as the fermion Yukawa matrix and both, in fact, originate from the superpotential couplings. Our theoretical prejudice is that there is an underlying symmetry (flavor symmetry) which restricts the form of the Yukawa matrices to explain their hierarchical forms. Then the LR mass matrix is expected to have a form very similar to the Yukawa matrix. More precisely, we expect the components of the LR mass matrix to be roughly the supersymmetry breaking scale (e.g., $m_{3/2}$) times the corresponding component of the quark mass matrix. However, there is no reason for them to be simultaneously diagonalizable based on this general argument. In general, we expect the size of $(\delta_{12}^d)_{\text{LR}}$ to be

$$(\delta_{12}^d)_{\text{LR}} \sim \frac{m_{3/2} M_{12}^d}{m_{\tilde{q}}^2}. \quad (1)$$

To be more concrete, one can imagine a string-inspired theory where the Yukawa couplings in the superpotential $W = Y_{ij}^d(T) Q^i D^j H_d$ are, in general, complicated functions of the moduli fields T . The moduli fields have expectation values of order string scale $\langle T \rangle$ which describe the geometry of the compactified extra six dimensions. The low-energy Yukawa couplings are then given by their expectation values $Y_{ij}^d(\langle T \rangle)$. On the other hand, the moduli fields in general also have couplings to fields in the hidden sector and acquire supersymmetry-breaking F -component expectation values $F_T \sim m_{3/2}$ in the Planck unit $M_{\text{Pl}} = \sqrt{8\pi}$. This generates trilinear couplings given by [17]

$$\mathcal{L} \supset \frac{\partial Y_{ij}^d}{\partial T} \langle F_T \rangle \tilde{Q}^i \tilde{D}^j H_d, \quad (2)$$

which depend on a different matrix $\partial Y_{ij}^d / \partial T$. Because of holomorphy, flavor symmetry is likely to constrain Y_{ij}^d and its derivative to be similar while they in general do not have to be exactly proportional to each other and hence are not simultaneously diagonalizable.

In order to proceed to numerical estimates of $(\delta_{12}^d)_{\text{LR}}$, we need to specify if the quark mixings come from an up or down sector. In general, attributing mixing to up sector gives smaller flavor-changing effects and receives less constraints [14]. On the other hand, historically, the Cabibbo angle has been often attributed to the down sector because of a numerical coincidence $V_{us} = \sin\theta_C = 0.22 \sim \sqrt{m_d/m_s}$. For our purpose, we pick the latter choice, which fixes the form of the mass matrix for the first and second generations to be

$$M^d \simeq \begin{pmatrix} m_d & m_s V_{us} \\ & m_s \end{pmatrix}, \quad (3)$$

where the (2,1) element is unknown due to our lack of knowledge on the mixings among right-handed quarks. Based on the general considerations on the LR mass

matrix above, we expect

$$m_{\text{LR}}^{2,d} \simeq m_{3/2} \begin{pmatrix} a m_d & b m_s V_{us} \\ & c m_s \end{pmatrix}, \quad (4)$$

where a , b , and c are constants of order unity. Unless $a = b = c$ exactly, M_d and $m_{\text{LR}}^{2,d}$ are not simultaneously diagonalizable, and we find

$$\begin{aligned} (\delta_{12}^d)_{\text{LR}} &\simeq \frac{m_{3/2} m_s V_{us}}{m_{\tilde{q}}^2} \\ &= 2 \times 10^{-5} \left(\frac{m_s(M_{\text{Pl}})}{50 \text{ MeV}} \right) \left(\frac{m_{3/2}}{m_{\tilde{q}}} \right) \left(\frac{500 \text{ GeV}}{m_{\tilde{q}}} \right). \end{aligned} \quad (5)$$

It is interesting to see that $(\delta_{12}^d)_{\text{LR}}$ of this naive dimensional estimate gives the saturation of the bound from ϵ'/ϵ (see Table II) if it has a phase of order unity.

The key point of the above example is that the large value of ϵ'/ϵ of the KTeV and NA31 experiments can be accounted for in the supersymmetric context without particularly contrived assumptions on the size of the $(\delta_{12}^d)_{\text{LR}}$ mass insertion, with the exception of taking it to have a large CP violating phase [18].

One may wonder if typical off-diagonal elements in $(\delta_{ij}^d)_{\text{LR}}$ may already be excluded from other flavor-changing processes. For instance, $(\delta_{23}^d)_{\text{LR}}$ is constrained by $b \rightarrow s\gamma$ to be less than $1-3 \times 10^{-2}$ [12]. This is to be compared to the estimate $(\delta_{23}^d)_{\text{LR}} \sim m_{3/2} m_b V_{cb} / m_{\tilde{q}}^2 \sim 2 \times 10^{-5}$. The constraint from $b \rightarrow s l^+ l^-$ is similarly insignificant [19]. Constraints from the up sector are much weaker.

It is tempting to speculate that the observed ϵ'/ϵ may be dominated by $(\delta_{12}^d)_{\text{LR}}$ contribution. The estimate in Eq. (5) requires $m_{3/2} \sim m_{\tilde{q}}$ and an $O(1)$ phase. Because $m_{\tilde{q}}^2$ acquires a positive contribution from gluino mass in the renormalization-group evolution while off-diagonal components in LR mass matrix don't, such a scenario would prefer models where the gaugino mass is somewhat smaller than scalar masses [assumed also to be $O(m_{3/2})$]. An important implication of supersymmetry-dominated ϵ'/ϵ is that the neutron electric dipole moment (EDM) is likely to be large. The current limit on neutron EDM $d_n < 11 \times 10^{-26} e \text{ cm}$ constrains $|\text{Im}(\delta_{11}^d)_{\text{LR}}| < (2.4, 3.0, 5.6) \times 10^{-6}$ for $m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 0.3, 1.0, 4.0$, respectively, with a theoretical uncertainty of at least a factor of 2, while our estimate gives $(\delta_{11}^d)_{\text{LR}} \sim m_{3/2} m_d / m_{\tilde{q}}^2 \sim 3 \times 10^{-6}$. It would be interesting to see results from near-future experiments which are expected to improve the limit on d_n by 2 orders of magnitude.

One may extend the discussion to the lepton sector. Let us consider $m_{\tilde{l}} \sim m_{3/2} \sim 500 \text{ GeV}$ for our discussions. The constraints from $\mu \rightarrow e\gamma$ and the electron EDM are $|(\delta_{12}^l)_{\text{LR}}| < 0.7-1.9 \times 10^{-5}$ and $|\text{Im}(\delta_{11}^l)_{\text{LR}}| < 1.5-3.5 \times 10^{-6}$ for $0.4 < m_{\tilde{\nu}}^2/m_{\tilde{l}}^2 < 5.0$ [12]. Our estimates on these mass insertion parameters are $(\delta_{12}^l)_{\text{LR}} \sim m_{3/2} m_{\mu} V_{\nu_e \mu} / m_{\tilde{l}}^2 \sim 2.1 \times 10^{-4} V_{\nu_e \mu}$ and

$(\delta_{11}^l)_{LR} \sim m_{3/2} m_e / m_{\tilde{l}}^2 \sim 1.0 \times 10^{-6}$. Because of our lack of knowledge of the lepton mixing angles, we cannot draw a definite conclusion on the $\mu \rightarrow e\gamma$ process. One possible choice is what is suggested by the small angle Mikheyev-Smirnov-Wolfenstein solution to the solar neutrino problem, $V_{\nu_e\mu} \sim \sqrt{m_e/m_\mu} \sim 0.05$. It is interesting that these estimates of $|(\delta_{12}^l)_{LR}|$ and $|\text{Im}(\delta_{11}^l)_{LR}|$ nearly saturate the bounds.

In summary, we have reconsidered the possible supersymmetric contribution to ϵ'/ϵ . Contrary to the lore in the literature, we find that generic supersymmetric models give an interesting contribution to ϵ'/ϵ , and it is even possible that it dominates in the observed value. We expect the neutron EDM to be within the reach of near-future experiments in that case.

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- [15] It, however, leaves a possibility that there is some cancellation in ϵ between the gluino loop discussed here and other possible contributions, such as a chargino loop, to allow a larger supersymmetric contribution to ϵ' .
- [16] For instance, take the case $m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1.0$ and saturate both Δm_K and ϵ . Assuming $|\text{Re}(\delta_{12}^d)_{LR}| \gg |\text{Im}(\delta_{12}^d)_{LR}|$, Δm_K requires $|\text{Re}(\delta_{12}^d)_{LR}| = 4.4 \times 10^{-3}$, while $\epsilon = |\text{Re}(\delta_{12}^d)_{LR} \text{Im}(\delta_{12}^d)_{LR}| = 3.5 \times 10^{-4}$. We then find $|\text{Im}(\delta_{12}^d)_{LR}| = 1.4 \times 10^{-5}$, which saturates ϵ'/ϵ within theoretical uncertainties. A.M. thanks G. Colangelo, G. Isidori, and L. Silvestrini for drawing our attention to this point.
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