Bulk Tunneling at Integer Quantum Hall Transitions

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The tunneling into the *bulk* of a 2D electron system in a strong magnetic field is studied near integer quantum Hall transitions. We present a nonperturbative calculation of the tunneling density of states (TDOS) for both Coulomb and short-ranged electron-electron interactions. In the case of the Coulomb interaction, the TDOS exhibits a 2D quantum Coulomb gap behavior, $\nu(\varepsilon) = C_Q|\varepsilon|/e^4$, where C_Q is a nonuniversal coefficient of quantum mechanical origin. For short-ranged interactions, we find that the TDOS at low bias follows $\nu(\varepsilon)/\nu(0) = 1 + (|\varepsilon|/\varepsilon_0)^\gamma$, where γ is a universal exponent determined by the scaling dimension of short-ranged interactions.

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The integer quantum Hall transition (IQHT) refers to the continuous, zero-temperature phase transition between two consecutive integer quantum Hall states in a 2D electron system (2DES) [1,2]. It happens when the Fermi energy of the 2DES moves across a critical energy located near the center of each disorder-broadened Landau level. On both sides of the transition, while the Hall conductivity σ_{xy} is integer quantized, the dissipative conductivity σ_{xx} vanishes at low temperatures. At the transition, however, both σ_{xx} and σ_{xy} remain finite, supporting a critical, conducting state in two dimensions.

Theoretically, the IQHT has been studied extensively in terms of its important, *noninteracting analog* in which electron-electron interactions are ignored [2]. The extent to which this disordered, free electron model describes the IQHT in real materials depends on the effects of electronic interactions. Recently, the stability of the noninteracting theory has been analyzed by calculations of the scaling dimensions of the interactions [3]. It was found that short-ranged interactions are irrelevant in the renormalization group sense, but the long-ranged $1/r$-Coulomb interaction is a relevant perturbation. In the latter case, the universality class (e.g., the critical exponents) of the transition is expected to change based on the standard theory of critical phenomena. The experimentally measured value of the dynamical scaling exponent, $z = 1$, is consistent with Coulomb interaction being relevant at the transitions [4]. However, the precise role played by Coulomb interaction and the mechanism by which it governs the scaling behavior have not been understood.

The tunneling density of states (TDOS) is a simple and useful probe of the nature and the effects of electronic interactions. Recent tunneling experiments in the integer quantum Hall regime discovered, remarkably, that the TDOS vanishes linearly on approaching the Fermi energy [5]. Since, at the transition, σ_{xx} is finite and the localization length ξ is very large, the linear suppression of the TDOS at criticality is expected to have a different origin than the 2D classical Coulomb gap behavior deep in the insulating phases. The results of numerical calculations of the TDOS using Hartree-Fock approximation (HFA) of Coulomb interaction also show a linear Coulomb gap at *all* Fermi energies in the lowest Landau level [6,7]. Since, HFA does not include screening of the exchange interaction, the correct behavior of the TDOS remained unclear, especially at the transition.

In this paper, we present a nonperturbative calculation of the TDOS *at* the IQHT, taking into account the dynamical screening of Coulomb interaction by the diffusive electrons. We show that the TDOS is given by

$$\nu(\varepsilon) = C_Q|\varepsilon|/e^4, \qquad (1)$$

at low energy ε (bias). We shall refer to Eq. (1) as the 2D quantum Coulomb gap behavior. The coefficient C_Q is not a universal number as in the 2D classical Coulomb gap expression [8], but rather a quantity of quantum mechanical origin. In general, it depends on magnetic field and the microscopic details of the sample such as the mobility. We find, at the IQHT,

$$C_Q = \sqrt{1/\pi g}\,[1 + \Phi(\sqrt{g})]e^{g+(1/4g)\ln^2\Delta}, \qquad (2)$$

where $g = 2\pi^2\sigma_c$ with σ_c the critical conductivity in units of $e^2/\hbar$, and $\Delta = (k_f a_B)^2/2\pi^2 g k_f l$ with a_B the Bohr radius and l the zero-field mean free path. Φ is the error function. For large ε, $\nu(\varepsilon)$ crosses over to the perturbative diagrammatic result in strong magnetic fields [9,10]. We also study the case of short-ranged interactions corresponding to experimental situations when the presence of nearby ground planes or the tunneling electrodes themselves screen the long-ranged Coulomb interaction between electrons are large distances. In this case, we find that the zero-bias TDOS is suppressed from the noninteracting value but remains finite. The corrections at finite ε follow a universal power law: $[\nu(\varepsilon) - \nu(0)]/\nu(0) = (|\varepsilon|/\varepsilon_0)^\gamma$ with $\gamma = x/z$, where $x \simeq 0.65$ is the scaling dimension of short-ranged interactions and $z = 2$ [3].

We begin with the partition function,

$$Z = \int \mathcal{D}[\bar\psi, \psi]\mathcal{D}[\phi]\mathcal{D}[V]P[V]e^{-S}, \qquad (3)$$

where the imaginary time action $S = S_0 + S_I$,

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$$S_0 = \int_0^\beta dt\, d^2r\, \bar{\psi}\left[\partial_t - \frac{1}{2m}(\partial_\alpha - ieA_\alpha)^2 + V\right]\psi,$$
$$S_I = \frac{1}{2}\int_0^\beta dt\, d^2r\, d^2r'\, \phi(r)U^{-1}(r - r')\phi(r') \quad (4)$$
$$+ i\int_0^\beta dt\, d^2r\, \phi(r)\bar{\psi}(r)\psi(r).$$

Here, $\bar{\psi}$ and ψ are the electron Grassmann fields, ϕ is a scalar potential coupled to electron density, and A_α, $\alpha = x, y$ is the external vector potential, $\epsilon_{\alpha\beta}\partial_\alpha A_\beta = B\hat{z}$. Integrating out ϕ in Eq. (3) gives rise to the electron-electron interaction $U(r - r')$ in Coulomb gauge. Quench average over the short-ranged impurity potential V is represented in Eq. (3) by integrating over a Gaussian distribution $P[V]$ using the standard replica method.

The TDOS can be obtained from the impurity averaged Green's function $G(\tau) = \langle\psi(r,\tau)\bar{\psi}(r,0)\rangle$,

$$\nu(\varepsilon) = -\frac{1}{\pi}\,\mathrm{Im}\int d\tau\, e^{i\omega_n\tau}G(\tau)\,|_{i\omega_n\to\varepsilon+i\eta}. \quad (5)$$

In the imaginary time path integral, $G(\tau)$ is given by

$$G(\tau) = Z^{-1}\int \mathcal{D}[\bar{\psi},\psi]\mathcal{D}[\phi]\mathcal{D}[V]P[V]\psi(\tau)\bar{\psi}(0)e^{-S}. \quad (6)$$

Consider a U(1) gauge rotation, $\psi \to \psi e^{i\theta}$, $\bar{\psi} \to e^{-i\theta}$ with $\theta = \theta(r,\tau)$. In the rotated frame, $G(\tau)$ has the same form as in Eq. (6), but with a transformed action

$$S_\tau = S - i\int_0^\beta d^2r\, dt(j_0^\tau a_0 - j_\mu a_\mu), \quad (7)$$

where $a_\mu = \partial_\mu\theta$, $\mu = \tau, x, y$, is the longitudinal U(1) gauge field coupled to the fermion three-current j_μ, and

$$j_0^\tau = \delta(r)[\Theta(t) - \Theta(t - \tau)] \quad (8)$$

is the source density (current) disturbance due to the tunneling electrons. Because of the latter, the ground state develops a nonvanishing charge and current density, and the saddle point of the tunneling action S_τ is shifted from that of S in Eq. (6). Minimizing the action, $\partial S/\partial\theta = 0$, one finds that the induced charge and current density, $\rho = \langle j_0\rangle$ and $\mathbf{J} = \langle\mathbf{j}\rangle$, satisfy the continuity equation,

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\mathbf{J} = \Gamma(r,t), \quad (9)$$

where $\Gamma(r,t) = \partial_\tau j_0^\tau = \delta(r)[\delta(t) - \delta(t - \tau)]$ corresponds to the process of injecting an electron at $r = 0$ and time $t = 0$ and removing it at $t = \tau$. Since the critical conductivity is finite at the IQHT, the charge spreading is expected to be described by (anomalous) diffusion, we have $\mathbf{J} = -D(\nabla - \gamma_H\hat{z}\times\nabla)\rho$, where D is the field-dependent diffusion coefficient and $\gamma_H = \sigma_{xy}/\sigma_{xx}$ is the Hall angle. Inserting this into Eq. (9), we obtain the diffusion equation,

$$\frac{\partial}{\partial t}\rho - D\nabla^2\rho = \Gamma(r,t). \quad (10)$$

Notice that γ_H does not enter because the transverse force does not affect the charge spreading *in the bulk* of the sample. Solving Eq. (10) leads to

$$\rho(q,\omega_n) = \frac{\Gamma(\tau,\omega_n)}{|\omega_n| + D(q,\omega_n)q^2}, \quad (11)$$

where $\Gamma(\tau,\omega_n) = 1 - \exp(-i\omega_n\tau)$ and ω_n is the boson Matsubara frequency.

Next, we go back to the tunneling action in Eq. (7) and expand the currents around the expectation values: $j_0 = \rho + \delta j_0$ and $\mathbf{j} = \mathbf{J} + \delta\mathbf{j}$. The fluctuating parts now satisfy the *homogeneous* continuity equation: $\partial_t\delta j_0 + \nabla\cdot\delta\mathbf{j} = 0$. For convenience, we choose the unitary gauge by setting $\phi(r,t) = a_0(r,t)$. Using Eq. (9), the tunneling action [11] for charge spreading becomes,

$$S_\tau = S_u - i\int_0^\beta dt\, d^2r\, \rho a_0, \quad (12)$$

where the unitary gauge action S_u is given by

$$S_u = S_0 + i\int_0^\beta dt\, d^2r\, \delta j_\alpha a_\alpha$$
$$+ \frac{1}{2}\int_0^\beta dt\, d^2r\, d^2r'\, a_0(r)U^{-1}a_0(r'). \quad (13)$$

The one-particle Green's function in Eq. (6) becomes

$$G(\tau) = Z^{-1}\int \mathcal{D}[\bar{\psi},\psi]\mathcal{D}[\theta]\mathcal{D}[V]P[V]\psi(\tau)\bar{\psi}(0)e^{-S_\tau}. \quad (14)$$

We now quench average over the impurity potential and integrate out the fermions for a fixed gauge configuration [12,13]. Equation (14) becomes

$$G(\tau) = \int \mathcal{D}[\theta]G_\theta(\tau)e^{i\int dt\, d^2r\, \rho a_0 - S_{\mathrm{eff}}[a_\mu]}, \quad (15)$$

where $G_\theta(\tau) \equiv \langle\psi(\tau)\bar{\psi}(0)\rangle_\theta$ is given by

$$G_\theta(\tau) = \frac{\int \mathcal{D}[\psi,\bar{\psi}]\mathcal{D}[V]P[V]\psi(\tau)\bar{\psi}(0)e^{-S_u[\psi,\bar{\psi},a_\mu]}}{e^{-S_{\mathrm{eff}}[a_\mu]}}, \quad (16)$$

with the gauge field effective action,

$$e^{-S_{\mathrm{eff}}[a_\mu]} = \int \mathcal{D}[\psi,\bar{\psi}]\mathcal{D}[V]P[V]e^{-S_u[\psi,\bar{\psi},a_\mu]}. \quad (17)$$

Thus far, all evaluations have been formal and exact, and explicitly displayed the structure of the theory in the U(1) sector.

To calculate $G(\tau)$ in Eq. (15), one has to make systematic approximations to obtain $S_{\mathrm{eff}}[a_\mu]$ and $G_\theta(\tau)$. Following essentially the formalism of Finkelshtein [13–15], we obtain S_{eff} to quadratic order in a,

$$S_{\mathrm{eff}} = T\sum_n\int d^2q\, a_0(q,\omega_n)\Pi(q,\omega_n)a_0(-q,-\omega_n). \quad (18)$$

Here Π is the polarization function,

$$\Pi^{-1}(q,\omega_n) = \frac{U(q)}{1 + U(q)\frac{dn}{d\mu}\frac{Dq^2}{|\omega_n| + Dq^2}}, \quad (19)$$

which coincides with the dynamical screened Coulomb interaction [9,10]. In Eq. (19), $dn/d\mu$ is the compressibility. The bare diffusion constant in the self-consistent Born approximation (SCBA) is $D = \frac{1}{2} r_c^2 \tau_0^{-1}$, where $r_c = (2N + 1)^{1/2} l_B$, with l_B the magnetic length and N the Landau-level index.

Next, we turn to G_θ in Eq. (16). In the unitary gauge, the important interference effects between the phase of the electron wave functions have been accounted for in the tunneling action in Eq. (15). The amplitude fluctuations are small for the slowly varying gauge potentials that dominate the dynamically screened Coulomb potential in Eq. (19) [15]. This is a unique feature of the slow diffusive dynamics of the electrons. We therefore neglect the θ dependence and write $G_\theta(\tau) \approx G_0(\tau)$. The functional integral over the gauge potential in Eq. (15) can thus be carried out explicitly using the effective action in Eq. (18), leading to

$$G(\tau) = G_0(\tau) e^{W(\tau)}. \tag{20}$$

For Coulomb interaction, $U(q) = 2\pi e^2/q$ in Eq. (19). Making use of Eq. (11), $W(\tau)$ has the form

$$W(\tau) = -\frac{T}{2} \sum_n \int d^2q \, \frac{|\Gamma(\tau, \omega_n)|^2}{(|\omega_n| + Dq^2)^2} \, \frac{2\pi e^2}{q + \frac{\kappa D q^2}{|\omega_n| + Dq^2}} \,. \tag{21}$$

where $\kappa = 2\pi e^2 dn/d\mu$ is the inverse screening length at the transition. Notice that $W(\tau)$ has a similar structure as the leading correction to the TDOS in the diagrammatic perturbation theory in both zero [16] and strong magnetic field [9]. The main contribution to the q integral in Eq. (21) comes from the region $|\omega_n|/D\kappa < q < (|\omega_n|/D)^{1/2}$. In this region, the diffusion coefficient D is a constant. The anomalous diffusion [17] appears only in the opposite limit $Dq^2 \gg |\omega_n|$. As a result, $W(\tau)$ becomes, in the $T = 0$ limit,

$$W(\tau) = \frac{1}{4\pi^2} \int \frac{d\omega}{\omega} \, \frac{1}{\sigma_{xx}} \ln(\omega\tau_s)(1 - \cos\omega\tau), \tag{22}$$

where $\tau_s = 1/D\kappa^2$, and the conductivity $\sigma_{xx} = D dn/d\mu$ in units of $e^2/\hbar$ following Einstein's relation. The upper limit of the integral in Eq. (22) is $\hbar/\tau_0$ since one can show $\tau_s \ll \tau_0$ near Landau level centers. The long-time behavior of W is therefore given by $W(\tau) = (-1/8\pi^2\sigma_{xx}) \ln(\tau/\tau_0) \ln[\tau/(\tau_s^2/\tau_0)]$ [18]. Substituting this result into Eq. (20), we obtain

$$G(\tau) = G_0(\tau) \exp\left[-\frac{1}{8\pi^2\sigma_{xx}} \ln\left(\frac{\tau}{\tau_0}\right) \ln\left(\frac{\tau\tau_0}{\tau_s^2}\right)\right]. \tag{23}$$

The asymptotic behavior of $G_0(\tau)$ for large τ is $G_0(\tau) \simeq -\nu_0/\tau$, where ν_0 is the corresponding DOS [13]. In the high-field limit of the SCBA, it is well known that $\nu_0 = (1/2\pi l_B^2)(2\tau_0)/\hbar$ and $\sigma_{xx} = \nu_0 D \simeq (2N + 1)/2\pi^2$ in the center of the Nth Landau level. After analytical continuation to real time [13], we obtain the final result for the TDOS defined in Eq. (5):

$$\nu(\varepsilon) = \frac{2\nu_0}{\pi} \int_0^\infty dt \, \frac{\sin|\varepsilon|t}{t} \, e^{-[1/(8\pi^2\sigma_{xx})]\ln(t/\tau_0)\ln(t\tau_0/\tau_s^2)}. \tag{24}$$

We now discuss the behavior of $\nu(\varepsilon)$ in different regimes, after making a few remarks. (i) The quantity in the exponential in the above equation did not follow from an expansion in $1/\sigma_{xx}$, but rather resulted from the leading contribution dominated by the anomalously divergent $\ln^2$ term at long times. Next order corrections to the latter are of the order $\{1/\sigma_{xx}, 1/\sigma_{xx}^2\} \ln t/\tau_0$. (ii) While Eq. (24) correctly captures the double-log contributions, we have assumed that the conductivity σ_{xx} does not depend on frequency in Eq. (22). However, σ_{xx} can be renormalized by localization effects [of leading order $(1/\sigma_{xx}) \ln \omega\tau_0$ in the unitary ensemble] and interaction effects (of leading order $\ln \omega\tau_0$ in strong magnetic field [9,10]). Thus σ_{xx} assumes the frequency-independent SCBA value only if $-\ln(\varepsilon\tau_0) \ll \sigma_{xx}$. In this regime, and for $-\ln(\varepsilon\tau_0) \gg \sqrt{\sigma_{xx}}$, the integral in Eq. (24) gives

$$\nu(\varepsilon) = \nu_0 \exp\left[-\frac{1}{8\pi^2\sigma_{xx}} \ln(|\varepsilon|\tau_0) \ln(|\varepsilon|\tau_s^2/\tau_0)\right]. \tag{25}$$

Expanding the exponential to leading order in $1/\sigma_{xx}$, Eq. (25) reproduces the high-field diagrammatic perturbative result of Girvin *et al.* [9]. In the case of $B = 0$, such nonperturbative resummation of the perturbative double-log divergences was pointed out by Finkelshtein [14], and recently reexamined using different approaches [11,15,19]. The result of Eq. (25) can be regarded as an extension of the latter to high magnetic fields.

(iii) As stated in (ii), at low bias, i.e., for $-\ln(\varepsilon\tau_0) \gg \sigma_{xx}$, the localization and interaction effects lead to, in general, a frequency-dependent conductivity $\sigma_{xx}(\omega)$ in Eq. (22). However, at the IQHT, the critical conductivity σ_c is finite and of the order of e^2/h [6,7,20,21]. Thus, the structure of the double-log divergence at long times, can be extended into the regime of small ε, provided that σ_{xx} in Eq. (24) is replaced by the critical conductivity σ_c. It is important to note that, because of the double-log term, the exponential factor in Eq. (24) converges very fast such that the TDOS becomes analytic in small $|\varepsilon|$. Expanding to leading order in $|\varepsilon|$,

$$\nu(\varepsilon) = \nu_0 |\varepsilon| \frac{2}{\pi} \int_0^\infty dt \, e^{-[1/(8\pi^2\sigma_c)]\ln(t/\tau_0)\ln(t\tau_0/\tau_s^2)}. \tag{26}$$

Performing this integral, and using the fact that the compressibility is only weakly renormalized, i.e., $dn/d\mu \simeq \nu_0$, we obtain the results given in Eqs. (1) and (2) with $\Delta = \tau_s/\tau_0$. We thus conclude that *for the $1/r$-Coulomb interaction, the bulk TDOS at IQHT exhibits a linear Coulomb gap behavior of quantum mechanical origin, i.e., the 2D quantum Coulomb gap*. As a consequence, the level spacing at the transition becomes $\Delta E \sim 1/L$, where L is the size of the system, leading to a dynamical scaling exponent $z = 1$. This behavior of the TDOS is qualitatively different from those obtained in the clean case [22] and in a weak magnetic field [23].

We emphasize that the linear *quantum* Coulomb gap behavior results from the combined effects of (i) two dimensionality, (ii) $1/r$-Coulomb potential, and (iii) quantum diffusion, i.e., a finite conductivity at $T = 0$. It pertains therefore to other metal-insulator transitions in 2D amorphous electron systems, provided that the critical conductivity is finite [19]. An example is the recently discovered 2D $B = 0$ metal-insulator transition [24], although the asymptotic low temperature behavior of the critical conductivity extracted from the experimental data in this case is still controversial.

Finally, we turn to short-ranged interactions. It has been shown that interacting potentials $1/r^p$ that decay faster than $1/r^2$ are irrelevant and scale to zero with scaling dimensions $x = p - 2$ for $2 < p < 2 + x_{4s}$, $x_{4s} \simeq 0.65$, and $x = x_{4s}$ for $p > 2 + x_{4s}$ at the IQHT [3]. For simplicity, we will consider $U(r - r') = u\delta(r - r')$, thus $U(q) = u$ in the screened interaction in Eq. (19). Inserting the latter into Eq. (21), we find [13]

$$W_{\rm sr}(\tau) = -\int_{1/\tau}^{1/\tau_0} \frac{\alpha}{|\omega|}\, d\omega\,, \qquad (27)$$

where α is a nonuniversal quantity dependent on the interacting strength. It is given by

$$\alpha = \frac{1}{8\pi^2\sigma_c}\, \lambda \frac{2 + \lambda}{(1 + \lambda)^2} [C + \ln\sqrt{1 + \lambda}\,], \qquad (28)$$

with $\lambda = u \frac{dn}{d\mu}$, $C = 1/2 + 1/(2 - 3\eta/2)$, and η the anomalous diffusion exponent [17]. Once again, in this case, the critical conductivity σ_c is finite at the transition such that Eqs. (27) and (28) represent the leading contribution to $W_{\rm sr}(\tau)$ in the asymptotic limit. If the interaction u was a marginal perturbation, then one would obtain, as in the Luttinger liquid case, $W_{\rm sr} = -\alpha \ln(\tau/\tau_0)$, $G_{\rm sr}(\tau) \sim \tau^{-(1+\alpha)}$, and $\nu(\varepsilon) \sim \nu_0|\varepsilon|^\alpha$. However, this is not true, because u is an irrelevant perturbation, and the effective interaction scales to zero according to $u_{\rm eff} \sim u\omega^{x/z}$ with $z = 2$ at the noninteracting fixed point [3]. As a result, α obeys a scaling form $\alpha(u, \omega) = \mathcal{A}(u\omega^{x/z})$. The fact that $\mathcal{A}(u \to 0, \omega) = 0$ implies, together with Eq. (28), the leading scaling behavior $\alpha \simeq A\lambda(\omega\tau_0)^{x/z}$ with $A = C/4\pi^2\sigma_c$. Substituting this into Eq. (27), one finds that $W_{\rm sr}(\tau) = A\lambda\gamma^{-1}[(\tau_0/\tau)^\gamma - 1]$, where $\gamma = x/z \simeq 0.32$. That $W_{\rm sr}(\tau)$ converges in the large-τ limit is a consequence of the interaction being irrelevant, i.e., $\gamma > 0$. Thus, we find for large τ,

$$G_{\rm sr}(\tau) = \nu(0)\, \frac{1}{\tau} \exp\left[\frac{A\lambda}{\gamma}\left(\frac{\tau}{\tau_0}\right)^{-\gamma}\right], \qquad (29)$$

where $\nu(0) = \nu_0 \exp(-A\lambda/\gamma) < \nu_0$. After analytic continuation, the TDOS at small ε is given by

$$\nu_{\rm sr}(\varepsilon) = \nu(0)\left[1 + \left(\frac{|\varepsilon|}{\varepsilon_0}\right)^\gamma\right], \qquad (30)$$

where $\varepsilon_0 = \tau_0^{-1}(A\lambda/\gamma)^{-1/\gamma}$. This result leads to several interesting predictions: (i) For short-ranged interactions, the TDOS is finite and nonuniversal at zero bias $\nu_{\rm sr}(\varepsilon = 0) = \nu(0) \neq 0$. (ii) Since $\nu(0) \ll \nu_0$ for large λ, interactions still lead to strong density of states suppression at low bias. (iii) The plus sign in Eq. (30) indicates that the TDOS increases with bias ε according to a universal power law with an initial cusp singularity for our value of γ. These predictions can, in principle, be tested experimentally by deliberately screening out the long-ranged Coulomb interaction using metallic gates.

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