Transition to Turbulence in the Wake of a Sphere

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Experimental investigation of the wake of a sphere is reported, namely, the change from steady to periodic flow in the Reynolds number range [100, 360]. First, visualizations have been carried out in a water channel to make precise the geometry of downstream bubble and shedding vortices behind the sphere. Thereafter, measurements have been performed in a wind tunnel and results have been compared successfully to the predictions of the Landau model.

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The wake of the sphere has been the object of a large number of studies (see [1] for a review); however, previous studies have dealt with experiments on a large range of Reynolds numbers ($\mathrm{Re} = Ud/\nu$) about [100, 10^4, or 10^5], with few measurements per decade. The Strouhal-Reynolds number curve ($\mathrm{St} = fd/U$) exhibits, for instance, scattered data which makes very difficult comparisons between different experiments (cf. Fig. 3 of [1]). Previous visualizations at low Reynolds numbers, performed by Magarvey and Bishop [2], Levi [3], and Taneda [4], allow one to distinguish different regimes: axisymmetrical steady flow for $\mathrm{Re} < 150$, loss of symmetry of the flow for $150 < \mathrm{Re} < \mathrm{Re}_c$ and unsteady flow when $\mathrm{Re} > \mathrm{Re}_c$. We believe that a detailed investigation of a narrow Reynolds number range [100, 360] will give the precise description of the periodic regime which appears after the transition from steady to unsteady wake. In this paper we address the following points: what is the variation of the velocity fluctuation with the Reynolds number, and does the bifurcation from steady to periodic flow follow a Landau-Hopf scenario? We give precisely the Strouhal-number–Reynolds-number law on a small range and find that it is reliable within the margin of accuracy of measurements. To our knowledge, it is the first quantitative study of the amplitude of velocity fluctuation as a function of the Reynolds number, just above the threshold. Here we bring information concerning the intrinsic features of spatial structures observed and the nonlinear character of the unsteady fluctuation. First, we describe preliminary visualizations which are in good agreement with previous publications [2–4] and confirm the existence of the three regimes. Moreover, we report quantitative measurements of the periodic regime in the wind tunnel, which confirm the scenario predicted from numerical simulations by Natarajan and Acrivos [5] and allow one to define a more precise determination of the threshold.

Visualizations have been carried out in a vertical circular water channel (diameter 80 mm). The sphere (diameter 10 mm) was held by a thin upstream metallic pipe (diameter 2 mm) which fed the 1% diluted purple ink through three regularly disposed holes. Among the difficulties of our study are the way of holding the sphere and the full three-dimensional character of its wake. The Reynolds number is determined thanks to a calibration with $\pm 5\%$ accuracy. Between $\mathrm{Re} = 180$ and the critical value Re_c (about 280), two steady trails appear downstream of the bubble (Fig. 1). The plane of these trails does not include the center of the sphere. Its location has been changed in any orientation of the water channel by a very slight inclination of the upstream pipe. This last result is in good agreement with the report of Sakamoto and Haniu [6] concerning the shape of the wake of a sphere in a small upstream gradient of velocity. The

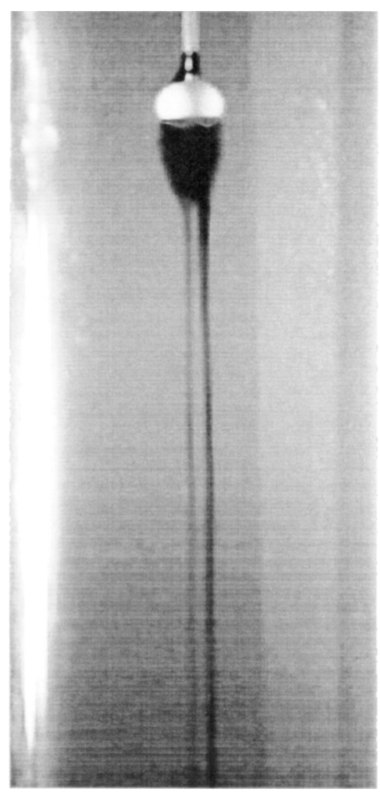

FIG. 1. Two steady trails obtained by visualization in the water channel, for $\mathrm{Re} = 270$.

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two trails oscillate above the critical Reynolds number, and the periodic modulation is clearer and clearer when the Reynolds number is increased. This modulation leads to a three dimensional pattern where vortex loops are periodically shed (Fig. 2).

Quantitative laser-Doppler and hot-wire anemometer measurements have been performed in a wind tunnel. In the square test section ($100 \times 25 \times 25$ cm^3), the free-stream turbulence level was close to 0.1% and flow uniformity better than 0.5% over 80% of the tunnel width. The free-stream velocity U was measured by laser Doppler velocimetry (LDV) on the center of the tunnel cross section, 50 mm upstream of the sphere. Corrections were made for blockage effects and the downstream evolution of U due to the growth of the boundary layer [7]. The kinematics viscosity ν was deduced from the temperature measurement by a calibrated platinum resistance. The uncertainty upon the Reynolds number has been estimated as equal to $\pm 2\%$. The sphere (diameter 10 mm) was held by four thin metal wires (diameter 0.08 mm) which were fixed on a frame outside the test section tightened by weight, similar to the experiments on the torus wake [8]. Although the body-to-wire diameter ratio was larger than 100, this configuration froze the spatial mode making possible repetitive experiments. A laser-photo diode was used to monitor the vibrations of the sphere. The amplitude of the measured oscillations, mainly longitudinal in the flow direction, remained below 5% of the diameter of the sphere. No influence of sphere oscillation on the vortex shedding frequency could be detected. In the following, Cartesian coordinates (O_x, O_y, O_z) will be used with the origin O at the center of the sphere, O_x is the streamwise horizontal direction, O_z is the vertical one, and O_y is the horizontal transverse one.

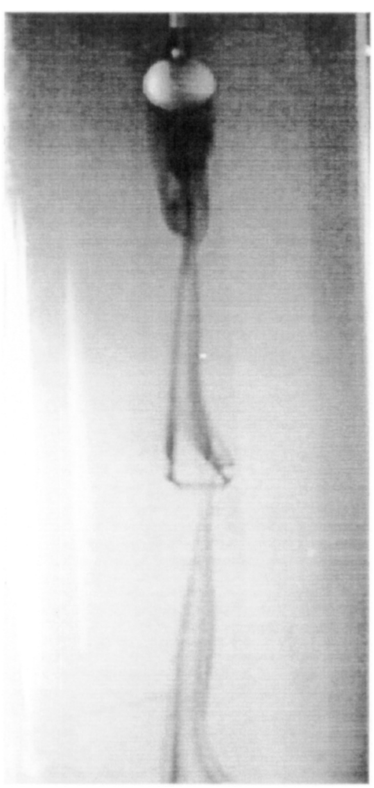

FIG. 2. Periodic vortex shedding obtained by visualization in the water channel, for Re = 330.

Spectral analysis of the streamwise velocity fluctuations shows the appearance of a peak (and eventually its harmonics) when the Reynolds number is above the critical value $\mathrm{Re}_c = 280$ (Fig. 3). Below the threshold, it was impossible to detect any precise peak in the spectrum. Above the Reynolds number 360, the velocity fluctuations are irregular and their spectrum exhibits a low frequency part. We have plotted (Fig. 4) the energy of the velocity fluctuations $u_x'^2$ as a function of the Reynolds number for the same location of the measurement point. Near the threshold, this variation is linear in agreement with the Landau model which describes the transition from steady to periodic flow as a supercritical bifurcation [9]. $M(t; x, y, z)$ is the real amplitude of the oscillation, M_{sat} is the amplitude for saturated regime,

$$dM/dt = \sigma M - lM^3, \tag{1}$$

$$E = M_{\mathrm{sat}}^2 = \sigma/l \propto (\mathrm{Re} - \mathrm{Re}_c). \tag{2}$$

The extrapolation of zero energy provides a quantitative criterion for the determination of the critical Reynolds number more accurate than the observation of a peak by spectral analysis or from the oscillation of the flow in visualizations. This value $\mathrm{Re}_c = 280$ (with $\pm 2\%$ accuracy) is independent of the position of the probe [10]. Moreover, it was impossible to detect any kind of hysteresis when increasing and decreasing the upstream

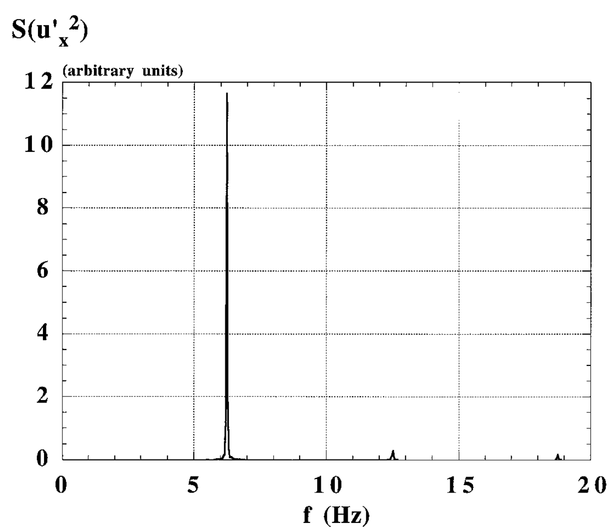

FIG. 3. Spectrum of the streamwise velocity fluctuations $S_{u_x'^2}$ in periodic regime, measured by LDV. Re = 329, $f = 6.24$ Hz; $x/d = 4.8$, $y/d = -0.2$, $z/d = 0$.

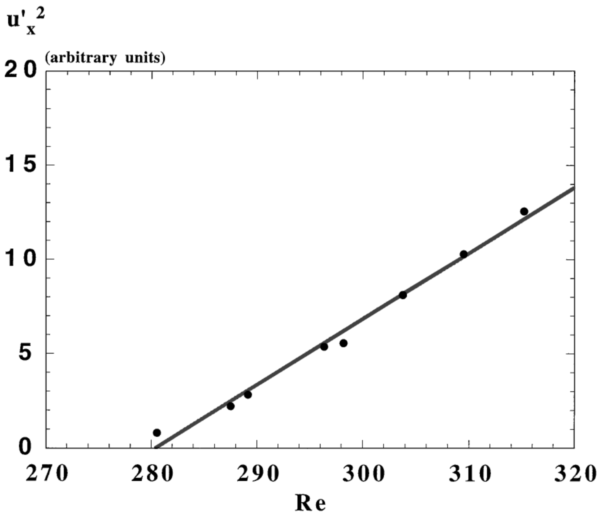

FIG. 4. Energy of the velocity fluctuations $u_x'^2$ versus Reynolds number, measured by LDV. $x/d = 4.8$, $y/d = -0.2$, $z/d = 0$.

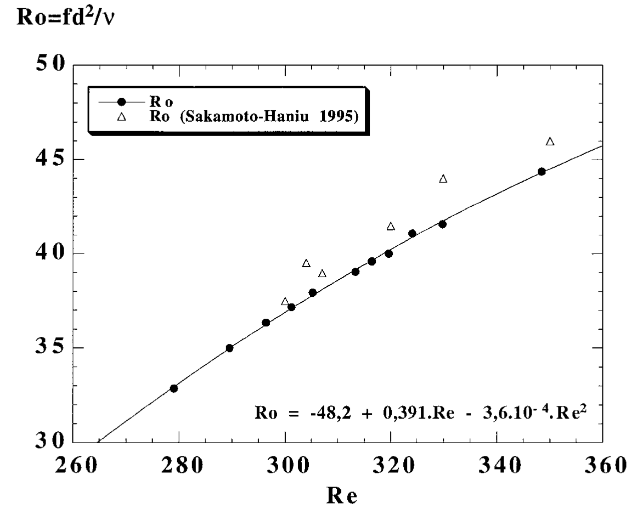

FIG. 5. Variation of the vortex shedding frequency (Ro $= fd^2/\nu$ Roshko number) with the Reynolds number.

velocity. Our result $\mathrm{Re}_c = 275$ to 285 is close to the previous determination, 300, deduced from visualizations by Sakamoto and Haniu [6] and in good agreement with other experimental data, between 270 and 290, reported by Magarvey and Bishop [2] and Levi [3]. The older values of Taneda [4] are in a wider range [130, 300]. Preliminary experiments and various configurations have shown that small inhomogeneities and perturbation in the upstream flow due to the way of holding the sphere and the method of visualization can induce slight changes in the value of the threshold. Furthermore, our critical Reynolds number is pretty close to the threshold value of 277 deduced by Natarajan and Acrivos [5], and in the range [250–280] predicted from direct numerical simulation by Tomboulidès *et al.* [11] or in the range [270–280] reported recently by Johnson *et al.* [12]. In these studies, the authors have inquired about the existence of a Landau-Hopf bifurcation for the transition from steady to periodic flow. Our measurements of the energy of velocity fluctuation $u_x'^2$ confirm the validity of this scenario.

The vortex shedding frequency f was determined by spectral analysis, and used to calculate the Roshko number $\mathrm{Ro} = fd^2/\nu$ and the Strouhal number $\mathrm{St} = fd/U$. The use of the Roshko number instead of the Strouhal number allows one to avoid the main uncertainty due to the determination of the velocity (Fig. 5). The periodic regime is limited to a narrow Reynolds number range, $[\mathrm{Re}_c, 360]$, where the Roshko number satisfies the relation

$$\mathrm{Ro} = \mathrm{St} \times \mathrm{Re} = A + B \times \mathrm{Re} + C \times \mathrm{Re}^2 \qquad (3)$$

with $A = -48.2$, $B = 0.391$, and $C = -3.6 \times 10^{-4}$.

Our fit is in agreement (within 5%) with the renewed values extracted from the plot of Sakamoto and Haniu [6]. In the periodic regime, the amplitude of the oscillation varies strongly in the streamwise direction. We have determined the coordinates of the main maximum in different transverse planes, for a constant Reynolds number. The energy measured by LDV at every maximum position varies along the axis of the flow (Fig. 6). In the near wake of the sphere, the energy increases linearly from the zero boundary value. This behavior is coherent with the growth of an absolute unstable perturbation in the near wake where the transverse mean-velocity profile exhibits a strong deficit [13]. The decrease of the amplitude of fluctuation along the downstream direction is linked to the streamwise evolution of the velocity profile showing a flat curve in the far wake. This curve is quite similar to the observations [14] of velocity fluctuation in the wake of a cylinder. We have also investigated the location of the maximum in amplitude (Fig. 7) along the streamwise direction and its variation with the Reynolds

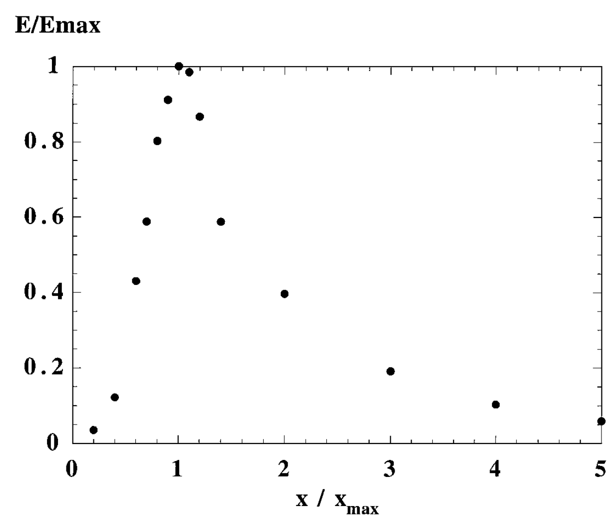

FIG. 6. Variation of the renormalized energy ($E/E_{\max}$) along the flow axis, Re = 324.

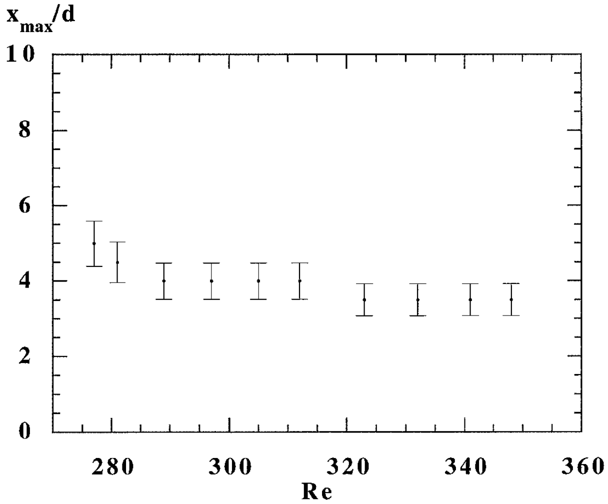

FIG. 7. Variation of the streamwise location of the maximum of energy with the Reynolds number.

number. There is a small change from [4.5d, 5.5d] near the threshold to [4d, 4.5d] at the upper Reynolds value of the periodic range [280, 360]. This tendency is similar to the measurement by Wesfreid *et al.* [14] of a downstream evolution of the maximum of amplitude in the wake of a cylinder, near the threshold leading to a scaling of x_{max}/d as a function of $(\mathrm{Re} - \mathrm{Re}_c)^{-0.5}$.

Measurements of the velocity amplitude and spectral analysis as well as visualizations give a more precise description of the different regimes occurring when one increases the Reynolds number from subcritical value. Though an extrinsic velocity gradient controls the spatial organization of the wake, the shedding of horseshoe vortices appear as an intrinsic phenomenon common to different axisymmetrical configurations (cone, disk, or sphere).

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