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Quantum Tunneling across Spin Domains in a Bose-Einstein Condensate

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Quantum tunneling was observed in the decay of metastable spin domains in gaseous Bose-Einstein condensates. A mean-field description of the tunneling was developed and compared with measurement. The tunneling rates are a sensitive probe of the boundary between spin domains, and indicate a spin structure which is prohibited in the bulk fluid. These experiments were performed with optically trapped $F = 1$ spinor Bose-Einstein condensates of sodium.

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A metastable system trapped in a local minimum of the free energy can decay to lower energy states in two ways. Classically, the system may decay by acquiring thermal energy greater than the depth of the local energy well (the activation energy). However, even in the absence of thermal fluctuations, the system may still decay by tunneling through the classically forbidden energy barrier. The theory of quantum tunneling describes a variety of physical and chemical phenomena [1,2] and finds common applications in, for example, scanning tunneling microscopy. In those systems, tunneling dominates over thermal activation because the energy barriers are much larger than the thermal energy.

Bose-Einstein condensates of dilute atomic gases [3] offer a new system to study quantum phenomena. Recently, metastable multicomponent Bose-Einstein condensates were observed in which the spatial structure of phase-separated domains persisted in spite of an external force which favored their rearrangement [4]. The metastability was due to the restriction of motion to one dimension by the narrow trapping potential and to the repulsive interaction between the domains. Even at temperatures ($\sim$100 nK) much larger than the energy barriers responsible for metastability ($\sim$5 nK), thermal relaxation was found to be extremely slow due to the scarcity of noncondensed atoms, to which the thermal energy is available.

In this Letter, we examine the decay of metastable spin domains in an $F = 1$ spinor condensate by quantum tunneling. The tunneling rates provide a sensitive probe of the boundary between spin domains and of the penetration of the condensate wave function into the classically forbidden region. Tunneling barriers are formed not by an external potential, but rather by the intrinsic repulsion between two immiscible components of a quantum fluid. These energy barriers are naturally of nanokelvin-scale height and of micron-scale width, and are thus a promising tool for future studies of quantum tunneling and Josephson oscillations [5–7].

We begin by considering the one-dimensional motion of a Bose-Einstein condensate comprised of atoms of mass m in two different internal states, $|A\rangle$ and $|B\rangle$. The condensate is held in a harmonic trapping potential which has the same strength for each component. In a mean-field description, the condensate wave function $\psi_i(z)$ is determined by two coupled Gross-Pitaevskii equations [8,9]

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V_i(z) + g_i n_i(z) + g_{A,B} n_j(z) - \mu_i \right)\psi_i(z) = 0 ,$$

where $V_i(z)$ is the trapping potential, $n_i(z)$ the density, and μ_i the chemical potential of each component ($i, j = \{A, B\}, i \neq j$). The constants g_A, g_B, and $g_{A,B}$ (all assumed positive) are given by $g = 4\pi\hbar^2 a/m$ where a is the s-wave scattering length which describes collisions

between atoms in the same (a_A and a_B) or different ($a_{A,B}$) internal states. Bulk properties of the condensate are well described by neglecting the kinetic energy (Thomas-Fermi approximation). Under the condition $g_{A,B} > \sqrt{g_A g_B}$, the two components tend to phase separate (as observed in [4,10]). The ground state configuration consists of one domain of each component on opposite sides of the trap (Fig. 1a). The chemical potentials are determined by the densities at the boundary n_i^b as $\mu_i = g_i n_i^b$, and are related to one another by the condition of equal pressure, $\mu_A^2/2g_A = \mu_B^2/2g_B$.

Within the Thomas-Fermi approximation, the domain boundary is sharp and the two components do not overlap. Yet, the kinetic energy allows each component to penetrate within the domain of the other. The energy barrier for component A (similar for B) is $\Delta E_A(z) = V_A(z) + g_{A,B} n_B(z) - \mu_A$. Neglecting slow variations in V_A and n_B gives the barrier height

$$\Delta E_A = \mu_A \left(\frac{g_{A,B}}{\sqrt{g_A g_B}} - 1 \right). \tag{1}$$

In this work we consider a condensate of atomic sodium in the two hyperfine states $|A\rangle = |F = 1, m_F = 0\rangle$ and $|B\rangle = |F = 1, m_F = 1\rangle$, with scattering lengths of $a_1 = a_{0,1} = 2.75$ nm [11] and $(a_1 - a_0) = 0.10$ nm [12]. The barrier height for atoms in the $|m_F = 0\rangle$ state is then $0.018\mu_0$, a small fraction of the chemical potential.

A state-selective force $-F\hat{z}$ is then applied (in this case by a magnetic field gradient) and displaces the trapping potential $V_B(z)$ from $V_A(z)$ (Fig. 1b). Classically, the atoms cannot move to the other end of the trap due to the energy barrier discussed above, and thus the condensate is left in a high-energy configuration. This configuration can decay by tunneling. At a domain boundary at the center at the condensate, $dV_A/dz = g_B dn_B/dz = -F/2$, and so $\Delta E_A(z) = \Delta E_A - Fz$ [13] (Fig. 1c). The width of the barrier becomes $z_b = \Delta E_A/F$. Tunneling from the metastable spin domains is analogous to the field emission of electrons from cold metals [2], where the energy barrier height corresponds to the work function of the metal and the force arises from an applied electric field. The tunneling rate dN_A/dt of atoms in state $|A\rangle$ from the metastable spin domain is then given by the Fowler-Nordheim (WKB) relation [2]

$$\frac{dN_A}{dt} = \gamma \exp\left(-\frac{4}{3} \sqrt{\frac{2m}{\hbar^2}} \frac{\Delta E_A^{3/2}}{F} \right), \tag{2}$$

where γ is the total attempt rate for tunneling, and the exponential is the tunneling probability.

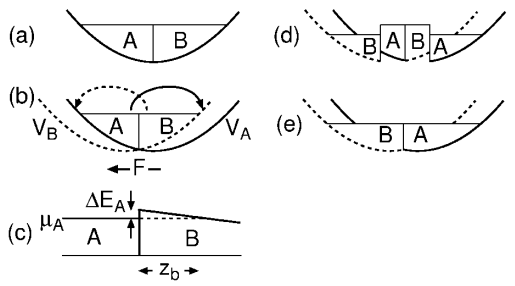

FIG. 1 Metastable spin domains and the energy barrier for decay. (a) The ground state of a two-component condensate consists of two phase-separated domains. (b) A state-selective force F displaces the trap potential V_B from V_A, creating metastable spin domains. Atoms tunnel from the metastable spin domains (direction of arrows) through an energy barrier (c) of maximum height ΔE_A and width $z_b \simeq \Delta E_A/F$ (shown for component A). (d) Tunneling proceeds from the metastable domains (inner) to the ground state domains (outer) until (e) the condensate has completely relaxed to the ground state.

The rate of quantum tunneling was studied experimentally in three steps. First, condensates of sodium in the $|F = 1, m_F = -1\rangle$ hyperfine state were created in a magnetic trap [14] and transferred to a single-beam infrared optical trap [15] with a $1/e^2$ beam radius of 12 μm, an aspect ratio (axial/radial length) of about 60, and a depth of 1–2 μK. Chirped radio-frequency pulses were used to create two-component condensates with nearly equal populations in the $|m_F = 0\rangle$ and $|m_F = 1\rangle$ states [4,10]. Shortly afterwards, the two components were separated into two domains by the application of a strong (several G/cm) magnetic field gradient along the axis of the trap in a 15 G bias field. The spin domains were typically 100–200 μm long.

Second, the condensates were placed in a metastable state by applying a magnetic field gradient B' in the opposite direction of that used to initially separate the components [4]. This metastable state corresponds to that shown in Fig. 1b, where we identify the states $|A\rangle = |m_F = 0\rangle$ and $|B\rangle = |m_F = 1\rangle$. The field gradient exerted a state-selective force $F = g\mu_B m_F B'$ where $g = 1/2$ is the Landé g factor and μ_B the Bohr magneton. The condensate was then allowed to evolve freely at the gradient B' and a bias field B_0 for a variable time τ of up to 12 s.

Finally, the condensate was probed by time-of-flight absorption imaging combined with a Stern-Gerlach spin separation [4,10]. The radial expansion of the condensate in time of flight allowed for independent measurement of the chemical potentials μ_0 and μ_1 [14], while the axial distribution allowed for measurement of the number of atoms in the metastable and ground state domains of each spin state.

The mean-field description of tunneling from the metastable spin domains was tested by measuring the tunneling rate across energy barriers of constant height and variable width. Condensates in a 15 G bias field at a constant density (corresponding to $\mu_0/k_B = 300$ nK) were probed after 2 s of tunneling at a variable field gradient B' (Fig. 2). Thus, the energy barrier for tunneling had a constant height of 5 nK and a width between

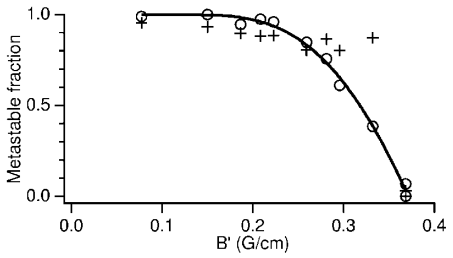

FIG. 2. Tunneling across a barrier of variable width which was adjusted by varying the field gradient B'. Condensates at constant density were probed after 2 s of tunneling. The fraction of atoms of each spin state in their metastable domain is shown. Circles represent the $m_F = 0$ atoms, and plusses the $m_F = 1$ atoms. A fit to the $m_F = 0$ data (solid line) determines the barrier attempt rate and tunneling probability. The data indicate that the tunneling rate for $m_F = 0$ atoms is larger than that for $m_F = 1$ atoms.

4 and 20 μm. As the barrier width was shortened, the tunneling rate increased, and the fraction of atoms in the $m_F = 0$ metastable spin domains decreased. As expressed in Eq. (2), the number of atoms which tunnel from the metastable to the ground state domains in a time τ should vary as $\gamma\tau e^{-\alpha/B'}$ where γ and α were determined by fits to the data as $\gamma = 1.5(5) \times 10^7$ s^{-1} and $\alpha = 1.5(2)$ G/cm . This value of α gives a tunneling probability of about e^{-4} for $B' = 370$ mG/cm, at which the metastable domains were fully depleted in 2 s.

The tunneling attempt rate γ can be estimated as the product of two factors. First, a bulk flux can be estimated by considering the pressure $g_0 n_0^2/2$ to arise from an incoming atomic flux $n_0 v/2$ which collides elastically at the boundary, imparting an impulse $2mv$ per particle. This gives $\gamma_{\text{bulk}} = \langle n_0 v_s \rangle_{\text{rad}}/2^{3/2}$ where $v_s = (g_0 n_0/m)^{1/2}$ is the Bogoliubov speed of sound and $\langle \cdots \rangle_{\text{rad}}$ denotes an integral over the radial dimension of the condensate. This bulk flux is reduced by an extinction factor f which accounts for the interpolation of the condensate wave function between the bulk spin domain and the classically forbidden region. Applying the treatment of Dalfovo *et al.* [7], we find $f \simeq 1/10$. Using $\mu_0/k_B = 300$ nK and a radial trap frequency of 500 Hz gives an estimate of $\gamma_{\text{bulk}} \simeq 5 \times 10^7$ s^{-1} and $\gamma \simeq 5 \times 10^6$ s^{-1}, in moderate agreement with the value extracted from our measurements.

The measured value of α can be compared with the prediction of the Fowler-Nordheim equation [Eq. (2)]. Using the scattering lengths above gives $\alpha = 1.5(2)$ G/cm, in agreement with our measurement (the error reflects a 10% systematic uncertainty in μ_0).

In addition, $g_1 > g_0$ implies $\mu_1 > \mu_0$ and thus the tunneling rate of $m_F = 1$ atoms across the $m_F = 0$ domain should be *slower* than that of the $m_F = 0$ atoms across the $m_F = 1$ domain. The data in Fig. 2 show evidence of this behavior.

The dependence of the tunneling rate on the energy barrier height was probed by varying the condensate density. For this, the number of trapped atoms was varied between about 10^5 and 10^6 by allowing for a variable duration of trap loss [15] before creating the metastable state. Figure 3 shows data collected at two different settings of the optical trap depth U and tunneling time τ (see caption). For each data series, at a given field gradient B', there was a threshold value of the chemical potential μ_0 below which the condensates had relaxed completely to the ground state, and above which they had not. Since the total condensate number and the attempt rate γ should both scale as $\mu_0^{5/2}$ [14], one expects the chemical potential threshold to vary as $\mu_0 \propto B'^{2/3}$. The data shown in Fig. 3 suggest a slightly steeper dependence.

The chemical potential thresholds were approximately the same for both settings of the optical trap depth. Varying the optical trap depth U also changed the temperature [$T \simeq (1/10)U$ [15]], and trap frequencies ($\omega \propto U^{1/2}$). That the threshold is independent of temperature confirms that the decay proceeds by quantum tunneling rather than thermal activation. That the threshold is independent of the trap frequencies confirms that the decay occurs by quantum tunneling of one spin component through the other, rather than by radial motion of one component around the other.

Thus, we have shown the decay of the metastable spin domains at high magnetic fields (15 G) to be due to quantum tunneling in a two-component condensate. At lower magnetic fields, a dramatic change in the tunneling behavior was observed. Metastable spin domains of initial chemical potential $\mu_0/k_B = 600$ nK were prepared at a constant field gradient of $B' = 130$ mG/cm and a bias field B_0 between 0.4 and 2 G. During a variable tunneling time τ of up to 12 s, the condensate density dropped due

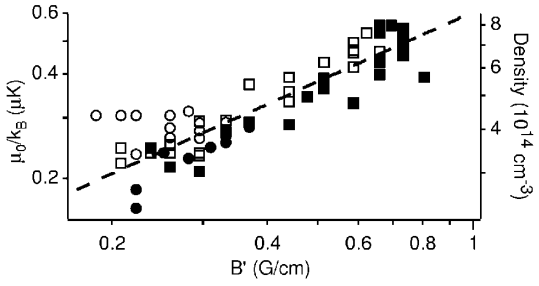

FIG. 3. Threshold behavior for tunneling. The chemical potential μ_0 and gradient B' are shown on a logarithmic scale. Closed symbols represent condensates which had fully decayed to the ground state, and open symbols those which had not. At higher condensate densities, the barrier is higher, and larger gradients B' are required to ensure tunneling. Data were taken at two different settings of the optical trap depth U and tunneling time τ: $U = 1.0$ μK and $\tau = 2$ s (circles), and $U = 2.0$ μK and $\tau = 1$ s (squares). The dashed line shows a $\mu_0 \propto B'^{2/3}$ dependence for the $U = 2.0$ μK threshold.

to the loss of atoms from the trap. At fields below about 1 G, the tunneling rates dramatically increased, leading to relaxation of the ground state at earlier times (Fig. 4a), and thus at higher chemical potentials (Fig. 4b).

The increase in the tunneling rates at low magnetic fields is inconsistent with the dynamics of a two-component condensate. Our measurements thus serve as a probe of the spin domain boundary and reveal the presence of the third $F = 1$ spin component ($m_F = -1$).

Atoms in the $|m_F = -1\rangle$ state can be produced by spin relaxation, wherein two $m_F = 0$ atoms collide to produce an atom in the $|m_F = 1\rangle$ and the $|m_F = -1\rangle$ state. Their production is influenced by two competing effects: a quadratic Zeeman energy shift suppresses spin relaxation by lowering the energy of two $m_F = 0$ atoms below that of their spin-relaxation product, while a spin-dependent interaction energy favors a mixture of atoms in the $|m_F = \pm 1\rangle$ states [10].

Atoms in the $|m_F = -1\rangle$ state are energetically excluded from the bulk spin domains at fields $B_0 \gtrsim 250$ mG for typical densities of $n \sim 3 \times 10^{14}$ cm^{-3}. However, in the boundary between spin domains, where atoms in the $|m_F = 0\rangle$ and $|m_F = 1\rangle$ states overlap, the production of $m_F = -1$ atoms is always energetically favored [16]. The population of atoms in the domain boundary in the $|m_F = -1\rangle$ state scales roughly as $1/B_0^2$; thus, as the magnetic field is lowered, their population in the boundary increases. At a field of 1 G, the fraction of atoms in the domain boundary in the $|m_F = -1\rangle$ state is at most $\sim 2\%$ (about 300 atoms).

The presence of the $m_F = -1$ atoms in the barrier weakens the effective repulsion between the spin domains. Consider a two-component system as before where $|A\rangle = |m_F = 0\rangle$ and $|B\rangle = \cos\theta|m_F = 1\rangle - \sin\theta|m_F = -1\rangle$ where $0 \le \theta \le \pi/2$. Evaluating the spin-dependent interaction energy [10,17] one finds $g_B = g_0 + \Delta g \cos^2 2\theta$ and $g_{A,B} = g_0 + \Delta g(1 - \sin 2\theta)$, where $\Delta g = g_1 - g_0$.

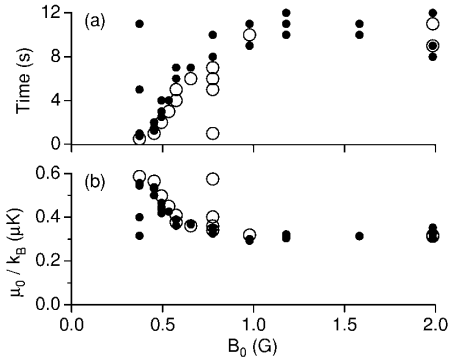

FIG. 4. Variation of tunneling threshold with magnetic bias field B_0. Condensates probed after a variable tunneling time are represented by a closed symbol if total relaxation to the ground state was observed, and with an open symbol if not. As the field was lowered, condensates (a) relaxed in shorter times and thus (b) at higher chemical potentials μ_0.

Thus, as the fraction of atoms in the $|m_F = -1\rangle$ state rises, the repulsion of the $m_F = 0$ atoms at the domain walls is weakened, and the tunneling rate is increased.

Future studies using metastable spin domains as tunneling barriers may focus on the roles of coherence and damping in quantum tunneling. In the current setup, rapid Josephson oscillations might be expected at frequencies (~ 1 kHz) given by the energy difference between the metastable and ground state spin domains. Over long time scales such oscillations are presumably damped. While no evidence for oscillatory behavior was found in the present work, the use of smaller spin domains and better time resolution is warranted.

In conclusion, we have identified and studied quantum tunneling across phase-separated spin domains in a Bose-Einstein condensate. The energy barriers due to the interatomic repulsion are a small fraction of the chemical potential, and their width is varied simply by the application of a weak force. The tunneling rates at high field ($B_0 > 1$ G) were described by a two-component mean-field model, while the tunneling at lower fields revealed changes in the spin-state composition of the domain boundaries.

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