Systematic Analytical Approach to Correlation Functions of Resonances in Quantum Chaotic Scattering

Yan V. Fyodorov[1] and Boris A. Khoruzhenko[2]
[1]*Fachbereich Physik, Universität-GH Essen, D-45117 Essen, Germany*
[2]*School of Mathematical Sciences, Queen Mary & Westfield College, University of London, London E1 4NS, United Kingdom*
(Received 8 February 1999)

We solve the problem of resonance statistics in systems with broken time-reversal invariance by deriving the joint probability density of all resonances in the framework of a random matrix approach and calculating explicitly all n-point correlation functions in the complex plane. As a by-product, we establish the Ginibre-like statistics of resonances for many open channels. Our method is a combination of Itzykson-Zuber integration and a variant of nonlinear σ model and can be applied when the use of orthogonal polynomials is problematic.

PACS numbers: 05.45.Mt, 03.65.Nk

As is well known, universal statistical properties of bound states in the regime of quantum chaos can be described in the framework of the random matrix approach [1]. The relevant methods adjusting random matrix description to the case of resonance scattering in open quantum systems are very well known since the pioneering work by the Heidelberg group [2]; see the review [3] for a thorough discussion of recent developments.

One of the basic concepts in chaotic quantum scattering is the notion of resonances. Resonances are long-lived intermediate states to which bound states of a "closed" system are converted due to coupling to continua. On the formal level the resonances show up as poles of the $M \times M$ scattering matrix $S_{ab}(E)$. The dimension of this matrix, M, equals the number of open channels in a given interval of energies. The poles of $S_{ab}(E)$ occur at complex energies $\mathcal{E}_k = E_k - \frac{i}{2}\Gamma_k$, where E_k is called the position and Γ_k the width of the corresponding resonance. Recent advances in computational techniques made available resonance patterns with high accuracy for realistic models of atomic and molecular chaotic systems [4], as well as for quantum billiards and other models related to chaotic scattering [5].

In the framework of the random matrix approach the S-matrix poles (resonances) are just the complex eigenvalues of an effective random matrix Hamiltonian $\mathcal{H}_{\text{eff}} = H - i\Gamma$. Here H is a random self-adjoint matrix of a large dimension N describing the statistical properties of the *closed* counterpart of the scattering system under consideration. Depending on the presence or absence of the time-reversal invariance H has to be chosen as a real symmetric or complex Hermitian one, respectively [1,6]. The $N \times N$ matrix Γ serves the purpose of describing transitions from the states described by H to the outer world via M open channels. It is related to the $N \times M$ matrix W of transition amplitudes in the following way: $\Gamma = \pi W W^{\dagger}$. Such a form of Γ is actually dictated by the requirement of the S-matrix unitarity and ensures that all S-matrix poles lie in the lower half-plane of complex energies, as required by causality. It is evident that the rank of Γ is M. In practice, the most interesting case is that of few open channels: $N \gg M \sim 1$. In this case the width Γ_k of a typical resonance is comparable with the mean *separation* Δ between neighboring resonances along the real axis, and statistical properties of resonances are expected to be universal [3].

Despite quite substantial efforts [3,7–9] our actual knowledge of S-matrix poles statistics for few-channel scattering is still quite restricted. Among the facts established analytically beyond perturbation theory one can mention (i) the density of joint distribution of all resonances for the system with a single open channel and Gaussian-distributed transition amplitudes W [7] and (ii) the mean density of S-matrix poles for arbitrary $M \ll N$ [3,8], as well as for $M \sim N$ [10].

At the same time, the most interesting and difficult question of correlations between resonances in the complex plane resisted systematic analytical investigations. As an attempt to get an insight into the problem, a nontrivial integral relation satisfied by the lowest (two-point) correlation function of complex eigenvalues for $\Gamma \geq 0$ was derived recently in [9]. Using that relation it turned out to be possible to put forward a conjecture on the analytic structure of the correlation functions for systems with broken time-reversal invariance. Unfortunately, the above mentioned relation neither fixed the lowest correlation function in a unique way nor provided direct information on higher correlation functions.

The goal of the present paper is to develop a regular analytical approach to the statistical properties of resonances for systems with broken time-reversal invariance. To this end, we first derive the joint probability density of all resonances for an arbitrary number of open channels M. Then we reduce the problem of extracting the n-point correlation functions in the limit n, M-fixed, $N \to \infty$ to averaging a certain product of $2n$ determinants. Finally, the latter is evaluated with a method combining a mapping to a fermionic version of a nonlinear σ model with

0031-9007/99/83(1)/65(4)$15.00

the Itzykson-Zuber integration. As a result, we prove the validity of the conjecture put forward in [9].

We consider an ensemble of random $N \times N$ complex matrices $J = H + i\Gamma$ [11], where H is $N \times N$ matrix taken from a Gaussian unitary ensemble (GUE) of *Hermitian* matrices with the probability density $\mathcal{P}(H) \propto \exp(-\frac{N}{2}\,\mathrm{tr}H^2)$ and Γ is a *fixed* non-negative one: $\Gamma \geq 0$. The probability density function in our ensemble can be written in the form

$$\mathcal{P}(J) \propto \exp\left[-\frac{N}{2}\,\mathrm{tr}\left(\frac{J + J^\dagger}{2}\right)^2\right]\delta\left(\Gamma - \frac{J - J^\dagger}{2i}\right). \tag{1}$$

Here and henceforth we do not specify the multiplicative constants when dealing with probability densities and correlation functions since they can always be found from the normalization condition.

Equation (1) can be used to obtain the density of joint distribution of eigenvalues by integrating $\mathcal{P}(J)$ over the degrees of freedom that are complementary to the eigenvalues of J. This can be done following Dyson's method (see [6,12]). Neglecting matrices with repeated eigenvalues, one transforms J to triangular form: $J = U(Z + R)U^\dagger$, where U is a unitary matrix, R is strictly upper triangular, and $Z = \mathrm{diag}\{z_1, \ldots, z_N\}$ is the diagonal matrix of complex eigenvalues. The Jacobian of the transformation from J to (Z, U, R) is $|\Delta(Z)|^2$ where $\Delta(Z) = \prod_{1\leq j<k\leq N}(z_j - z_k)$ is the Vandermonde determinant. To perform the integration over R it is technically convenient to use a Fourier-integral representation for the δ function in Eq. (1). This reduces the corresponding integral to a Gaussian one and after algebraic manipulations the resulting expression is

$$P_{N,\Gamma}(Z) \propto e^{-(N/2)\,\mathrm{Re}\,\mathrm{tr}Z^2 - (N/2)\,\mathrm{tr}\Gamma^2}|\Delta(Z)|^2 Q(\mathrm{Im}Z), \tag{2}$$

where $Q(\mathrm{Im}Z)$ is the remaining integral

$$Q(\mathrm{Im}Z) = \int [dU] \prod_{l=1}^{N} \delta[\mathrm{Im}z_l - (U^\dagger \Gamma U)_{ll}], \tag{3}$$

over the unitary group $U(N)$, $[dU]$ being the Haar measure. Because of the specific structure of the matrices J their eigenvalues lie in the upper part of the complex plane and in all formulas below $\mathrm{Im}z_j \geq 0$ for all j [11].

To proceed further we need to integrate over U. Again it is convenient to use the Fourier-integral representation for the δ functions in Eq. (3),

$$Q(\mathrm{Im}Z) = \int \frac{dK}{(2\pi)^N}\, e^{i\,\mathrm{Im}\,\mathrm{tr}KZ} \int [dU] e^{-i\,\mathrm{tr}KU^\dagger \Gamma U}, \tag{4}$$

where the first integration is over all real diagonal matrices K of dimension N, dK being $dk_1 \cdots dk_N$.

When the eigenvalues of Γ are all *distinct*, the integration over U can be performed using the famous Itzykson-Zuber-Harish-Chandra (IZHC) formula [13]. We, however, are mostly interested in the case when Γ has a small rank $M \ll N$; i.e., it has only M nonzero eigenvalues which we denote by $\gamma_1, \ldots, \gamma_M$. This limit of highly degenerate eigenvalues is difficult to perform in the original IZHC formula. Nevertheless, the difficulty can be circumvented and the result is as follows:

$$Q(\mathrm{Im}Z) = \frac{\det^{M-N}\boldsymbol{\gamma}}{\Delta(\boldsymbol{\gamma})} \int_{R^M} d\Lambda\, \det[e^{-i\gamma_l \lambda_m}]_{l,m=1}^{M} \times \Delta(\Lambda) \prod_{j=1}^{N} \sum_{m=1}^{M} \frac{e^{i\lambda_m \mathrm{Im}z_j}}{\prod_{s\neq m}(\lambda_m - \lambda_s)}, \tag{5}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_M)$ and $\boldsymbol{\gamma} = \mathrm{diag}(\gamma_1, \ldots, \gamma_M)$.

Equations (2) and (5) give an explicit representation for the joint probability density of N resonances z_i in the complex plane. As such, they constitute one of the main results of the present paper and provide the basis for calculating the n-eigenvalue correlation functions

$$R_n(\mathsf{z}) = \frac{N!}{(N-n)!} \int d\mathsf{w}\, P_{N,\Gamma}(\mathsf{z}, \mathsf{w}), \tag{6}$$

where, for the sake of brevity, we decompose $Z = \mathrm{diag}(\mathsf{z}, \mathsf{w})$ with $\mathsf{z} = (z_1, \ldots, z_n)$ and $\mathsf{w} = (w_1, \ldots, w_{N-n})$, $d\mathsf{w} = \prod_{j=1}^{N-n} d\,\mathrm{Re}w_j\, d\,\mathrm{Im}w_j$, identifying $w_k \equiv z_{n+k}$.

In what follows we will calculate $R_n(\mathsf{z})$ for arbitrary fixed n and M in the limit $N \to \infty$. On the first stage we will replace the integration over w in (6) by averaging over the ensemble of non-Hermitian random matrices $J_{N-n}(\boldsymbol{\gamma}) = H_{N-n} + i\Gamma$, with H_{N-n} being a GUE matrix of the reduced size $(N-n) \times (N-n)$. This step involves cumbersome algebraic manipulations and will be presented in full details elsewhere. Here we outline the ideas on the simplest, still nontrivial example of one-channel systems ($M = 1$). In this case Γ has only one nonzero eigenvalue γ and the λ integration in Eq. (5) can be explicitly performed yielding

$$P_{N,\gamma}(Z) \propto \frac{|\Delta(Z)|^2}{\gamma^{N-1}}\, e^{-(N/2)[\mathrm{Re}\,\mathrm{tr}Z^2 + \gamma^2]}\delta\left(\gamma - \sum_{j=1}^{N} \mathrm{Im}z_j\right). \tag{7}$$

Introducing the notation $\tilde{\gamma} = \gamma - \sum_{j=1}^{n} \mathrm{Im}z_j$, we write the δ function in Eq. (7) as $\delta(\tilde{\gamma} - \sum_{l=1}^{N-n} \mathrm{Im}w_l)$. Now we replace N in the exponent of Eq. (7) by $N - n$ (this act is justified by the limit $N \to \infty$). With the resulting relation in hand one readily obtains that

$$R_n(\mathsf{z}) \propto \frac{C_{\tilde{\gamma}}(\mathsf{z})}{\gamma^n} |\Delta(\mathsf{z})|^2 e^{-[(N-n)/2]\sum_{j=1}^{n} \mathrm{Re}z_j^2} \times \left[\frac{\tilde{\gamma}}{\gamma}\right]^{N-n-1} e^{-[(N-n)/2](\gamma^2 - \tilde{\gamma}^2)}, \tag{8}$$

where

$$C_{\tilde{\gamma}}(\mathsf{z}) = \int d\mathsf{w}\, P_{N-n,\tilde{\gamma}}(\mathsf{w}) \prod_{l=1}^{N-n} \prod_{j=1}^{n} |z_j - w_l|^2 = \left\langle \prod_{j=1}^{n} |\det[z_j - J_{N-n}(\tilde{\boldsymbol{\gamma}})]|^2 \right\rangle_{\mathrm{GUE}}. \tag{9}$$

In the limit when $N \to \infty$ and M is finite (in particular, for the present case $M = 1$), the imaginary part of almost all eigenvalues of J is of the order $\frac{1}{N} \ll \gamma_m$ [14], hence so is $\tilde{\gamma} - \gamma$. Therefore one can reinstate γ in place of $\tilde{\gamma}$ in the determinants in Eq. (9). On the other hand, rescaling the imaginary parts $y_j = N \,\mathrm{Im} z_j$, one finds that

$$\left[\frac{\tilde{\gamma}}{\gamma}\right]^{N-n-1} e^{-[(N-n)/2](\gamma^2-\tilde{\gamma}^2)} = e^{-2\sum_{j=1}^{n} y_j g}$$

in the limit $n \ll N \to \infty$, with $g = \frac{1}{2}(\gamma + \gamma^{-1})$.

Essentially similar manipulations can be performed for an arbitrary fixed number of open channels M. In the limit $n, M \ll N \to \infty$ we arrive at the following general representation of the correlation functions:

$$R_n(\mathsf{z}) \propto \frac{C_{\tilde{\gamma}}(\mathsf{z})}{\det^n \tilde{\gamma}} |\Delta(\mathsf{z})|^2 e^{-[(N-n)/2]\sum_{j=1}^{n} \mathrm{Re} z_j^2} \times \prod_{j=1}^{n} \sum_{m=1}^{M} \frac{e^{-2y_j g_m}}{\prod_{s\neq m}(g_m - g_s)}, \quad (10)$$

where $g_m = \frac{1}{2}(\gamma_m + \gamma_m^{-1})$.

Thus, the problem amounts to evaluation of the correlation function of the determinants in Eq. (9) [15]. To proceed, we first write each of the determinants as a Gaussian integral over a set of Grassmann variables. When this is done, the GUE average becomes trivial and yields terms quartic with respect to the Grassmannians. These terms can be further traded for an auxiliary integration over a Hermitian matrix S of the size $2n \times 2n$ (the so-called Hubbard-Stratonovich transformation). Then the integration over the Grassmann fields is trivially performed and yields again a determinant. As the result, we arrive at the following expression:

$$C_{\gamma}(\mathsf{z}) \propto \int [dS] e^{-(N-n)\,\mathrm{tr}[(1/2)S^2 - \ln(\mathbb{Z}_{2n} - iS)]} \times \prod_{m=1}^{M} \det[\mathbb{1}_{2n} + i\gamma_m \mathbb{L}_{2n}(\mathbb{Z}_{2n} - iS)^{-1}], \quad (11)$$

where we have introduced the diagonal matrices $\mathbb{Z}_{2n} = \mathrm{diag}(\mathsf{z}, \mathsf{z}^{\dagger})$ and $\mathbb{L}_{2n} = \mathrm{diag}(\mathbb{1}_n, -\mathbb{1}_n)$.

Let us now recall that nontrivial eigenvalue correlations are expected to occur [12] on the scale when the eigenvalues are separated by distances comparable with the mean eigenvalue separation for GUE matrices H, the latter being of the order $(N - n)^{-1}$ with our choice of $\mathcal{P}(H)$. Accordingly, it is convenient to separate the "center of mass" coordinate $x = \frac{1}{n}\sum_{j=1}^{n} \mathrm{Re} z_j$ so that $z_j = x + \frac{\tilde{z}_j}{N-n}$, where both the real and imaginary parts of $\tilde{z}_j$ are of the order of 1 in the limit when $N \to \infty$ and M is fixed. In this limit $R_n(\mathsf{z})$ is effectively a function of $\tilde{\mathsf{z}}$ (x is fixed) which we are going to calculate.

To evaluate the integral in (11) let us first diagonalize S: $S = U_{2n} \Sigma U_{2n}^{-1}$, where $U_{2n} \in U(2n)$ and $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_{2n})$. Then, keeping only the leading terms in the limit $N \to \infty$, we obtain

$$C_{\gamma}(\mathsf{z}) = \int d\Sigma\, \Delta^2(\Sigma) e^{-(N-n)\sum_{k=1}^{2n}[(\sigma_k^2/2) - \ln(x - i\sigma_k)]} \times \langle C(S)\rangle_{U(2n)}, \quad (12)$$

where

$$\langle C(S)\rangle_{U(2n)} = \int [dU_{2n}] e^{-\mathrm{tr}[\tilde{\mathbb{Z}}_{2n}(x\mathbb{1}_{2n} - iS)^{-1}]} \times \prod_{m=1}^{M} \det[\mathbb{1}_{2n} + i\gamma_m \mathbb{L}_{2n}(x\mathbb{1}_{2n} - iS)^{-1}].$$

The form of the integrand in (12) suggests exploiting the saddle-point method in the integral over σ_k, $k = 1, \ldots, 2n$. Altogether there are 2^{2n} saddle points: $\sigma_k^{(s)} = -\frac{i}{2}(x + i\epsilon_k\sqrt{4 - x^2})$, where $\epsilon_k = \pm 1$. The leading order contribution comes from integration around those saddle points where exactly n parameters ϵ_k equal 1 (the rest being equal -1). All other choices can be neglected as they lead to lower order terms. This is because of the presence of the Vandermonde determinant in the integrand. At the same time, all relevant saddle points produce the same contribution and we obtain that

$$C_{\gamma}(\mathsf{z}) \propto e^{[(N-n)/2]\sum_{j=1}^{n} \mathrm{Re} z_j^2} C_{\gamma}^{s}(\tilde{\mathsf{z}}), \quad (13)$$

where $\tilde{\mathsf{z}} = (\tilde{z}_1, \ldots, \tilde{z}_n)$, $\tilde{z}_j = N(z_j - x)$, $j = 1, \ldots, n$, and

$$C_{\gamma}^{s}(\tilde{\mathsf{z}}) = \int [d\mathbb{Q}_{2n}] e^{-i\pi\nu(x)\,\mathrm{tr}\tilde{\mathbb{Z}}_{2n}\mathbb{Q}_{2n}} \times \prod_{c=1}^{M} \det\left[\mathbb{1}_{2n} + \frac{i\gamma_c x}{2}\mathbb{L}_{2n} + \pi\nu(x)\gamma_c \mathbb{L}_{2n}\mathbb{Q}_{2n}\right]. \quad (14)$$

In (14) $\mathbb{Q}_{2n} = U_{2n}^{-1}\mathbb{L}_{2n}U_{2n}$, the integration is over the coset space $U(2n)/U(n) \otimes U(n)$, and the symbol $\nu(x)$ stands for the semicircular density of real eigenvalues of the matrices H, $\nu(x) = \frac{1}{2\pi}\sqrt{4 - x^2}$.

Thus, the problem reduces to evaluation of an integral over a coset space. This type of integrals is known in the literature under the name of zero-dimensional nonlinear σ models and our considerations enjoy many useful insights; see, e.g., Ref. [16]. In particular, the following polar parametrization proves to be the most effective:

$$\mathbb{U}_{2n} = \begin{pmatrix} U_A & \\ & U_R \end{pmatrix} \begin{pmatrix} \cos\hat{\psi} & e^{i\hat{\phi}}\sin\hat{\psi} \\ e^{-i\hat{\phi}}\sin\hat{\psi} & -\cos\hat{\psi} \end{pmatrix} \times \begin{pmatrix} U_A^{-1} & \\ & U_R^{-1} \end{pmatrix},$$

where $U_{A,R} \in U(n)$, $\hat{\psi} = \mathrm{diag}(\psi_1, \ldots, \psi_n)$, $\hat{\phi} = \mathrm{diag}(\phi_1, \ldots, \phi_n)$, and $0 < \phi_k, \psi_k < 2\pi$. The corresponding measure $[d\mathbb{Q}_{2n}]$ is proportional to

$$[dU_A][dU_R]\prod_{j=1}^{n} d\phi_j\, d\psi_j \sin 2\psi_j \times \prod_{1\le l<k\le n} |\cos 2\psi_l - \cos 2\psi_k|^2.$$

The use of such a parametrization makes the integration especially simple because of the determinant factor being independent of the unitary matrices $U_{A,R}$. As a result, these matrices appear only in the exponential factor and the corresponding integrals can be evaluated according to the IZHC formula [13]. Passing to the variables $\lambda_k = \cos 2\psi_k$ we find

$$C^s_{\boldsymbol{\gamma}}(\tilde{z}) \propto \frac{\det^n \boldsymbol{\gamma}}{|\Delta(\tilde{z})|^2}\int_{-1}^{1}\prod_{j=1}^{n} d\lambda_j \prod_{j=1}^{n} G_M(\lambda_j) \times \det[e^{+i\pi\nu(x)\tilde{z}_j\lambda_k}]\det[e^{-i\pi\nu(x)\tilde{z}_j^*\lambda_k}] \propto \frac{n!\,\det^n \boldsymbol{\gamma}}{|\Delta(\tilde{z})|^2}\det\left[\int_{-1}^{1} d\lambda\, G_M(\lambda) e^{i\pi\nu(x)\lambda(\tilde{z}_j-\tilde{z}_k^*)}\right],$$

where

$$G_M(\lambda) = \prod_{m=1}^{M}[g_m + \pi\nu(x)\lambda].$$

Combining this with Eqs. (10) and (13) and restoring the normalization we finally see that the correlation functions, in the limit $N\to\infty$, have the following simple structure:

$$\frac{1}{N^{2n}} R_n\left(x + \frac{\tilde{z}_1}{N}, \ldots, x + \frac{\tilde{z}_n}{N}\right) = \det[K(\tilde{z}_j, \tilde{z}_k^*)]_{j,k=1}^{n},$$

where the kernel $K(\tilde{z}_j, \tilde{z}_k^*)$ is given by

$$K(\tilde{z}_1, \tilde{z}_2^*) = F^{1/2}(\tilde{z}_1)F^{1/2}(\tilde{z}_2^*) \times \int_{-1}^{1} d\lambda\, e^{-i\pi\nu(x)\lambda(\tilde{z}_1-\tilde{z}_2^*)} G_M(\lambda) \quad (15)$$

with $F(\tilde{z}) = \sum_{m=1}^{M}\frac{e^{-2|\mathrm{Im}\tilde{z}|g_m}}{\prod_{s\neq m}(g_m-g_s)}$. This is equivalent to the form conjectured in [9].

One of the important physical limits of the scattering system is the case of many equivalent open channels: $g_m = g$ for all m and $M \gg g$. Resonances in that case form a dense cloud in the complex plane characterized by a mean density $\rho(z)$ inside the cloud. This fact and expression for $\rho(z)$ were found in [3,10]. Using our formulas derived above we are able to show that the statistics of the resonances in that case is determined by a *Ginibre-like* kernel,

$$|K(z_1, z_2)| = \rho(z)\exp -\frac{1}{2}\pi\rho(z)|z_1 - z_2|^2 \quad (16)$$

with $z = (z_1 + z_2)/2$, generalizing a classical result by Ginibre [17] to the case of a nonuniform (i.e., position-dependent) mean density of complex eigenvalues $\rho(z)$. As such, it has a good chance to be universally valid for strongly non-Hermitian random matrices.

In conclusion, we considered a non-Hermitian random matrix model of chaotic quantum scattering. We started with deriving the joint probability density of all complex eigenvalues describing S-matrix poles (resonances) in chaotic systems with broken time-reversal invariance. Then we found a way to extract all correlation functions of the resonances in complex plane. As a by-product, we established the Ginibre-like statistics of resonances for many open channels.

Both authors are very much obliged to O. Zaboronsky for inviting them to the Workshop "Random Matrices and Integrable Systems" at Mathematical Institute, University of Warwick, where discussions were instrumental in developing the ideas underlying the present paper. Y. V. F. acknowledges with thanks the financial support by SFB-237 and Grant No. INTAS 97-1342.

[1] O. Bohigas, in *Chaos and Quantum Physics,* Proceedings of the Les-Houches Summer School, Session LII, edited by M. J. Giannoni *et al.* (North-Holland, Amsterdam, 1991), p. 91.
[2] J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Rep. **129**, 367 (1985).
[3] Y. V. Fyodorov and H.-J. Sommers, J. Math. Phys. (N.Y.) **38**, 1918 (1997).
[4] R. Blumel, Phys. Rev. E **54**, 5420 (1996); V. A. Mandelshtam and H. S. Taylor, J. Chem. Soc. Faraday. Trans. **93**, 847 (1997).
[5] S. Albeverio *et al.*, J. Math. Phys. (N.Y.) **37**, 4888 (1996); M. Glück, A. R. Kolovsky, and H.-J. Korsch, Phys. Rev. Lett. **82**, 1534 (1999).
[6] M. L. Mehta, *Random Matrices* (Academic Press, San Diego, CA, 1991).
[7] N. Ullah, J. Math. Phys. (N.Y.) **10**, 2099 (1969); V. V. Sokolov and V. G. Zelevinsky Phys. Lett. B **202**, 10 (1988); Nucl. Phys. **A504**, 562 (1989); H.-J.Stöckmann and P. Seba, J. Phys. A **31**, 3439 (1998).
[8] H.-J. Sommers *et al.*, J. Phys. A **32**, L77 (1999).
[9] Y. V. Fyodorov *et al.*, Phys. Rev. E **58**, 1195 (1998).
[10] F. Haake *et al.*, Z. Phys. B **88**, 359 (1992).
[11] We changed the sign in front of Γ, keeping the condition $\Gamma \geq 0$ for the sake of notational convenience.
[12] Y. V. Fyodorov *et al.*, Phys. Rev. Lett. **79**, 557 (1997).
[13] C. Itzykson and J. B. Zuber, J. Math. Phys. (N.Y.) **21**, 411 (1980); Harish-Chandra, Am. J. Math. **79**, 87 (1957).
[14] Precisely speaking, the fraction m of resonances having widths $\mathrm{Im}z \sim N^{-1}$ is known to satisfy $1 - M/N \leq m \leq 1$; see [3] pp. 1949–1950.
[15] Correlation functions of determinants for Hermitian matrices were considered by A. V. Andreev and B. Simons, Phys. Rev. Lett. **75**, 2304 (1995). Our method shares some features with theirs, but is in many respects different.
[16] M. R. Zirnbauer, J. Phys. A **29**, 7113 (1996).
[17] J. Ginibre, J. Math. Phys. (N.Y.) **6**, 440 (1965).