Vortex Fluctuations in High-T_c Films: Flux Noise Spectrum and Complex Impedance

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The flux noise spectrum and complex impedance for a 500 Å thick YBCO film are measured and compared with predictions for two-dimensional vortex fluctuations. It is verified that the complex impedance and the flux noise spectra are proportional to each other, that the logarithm of the flux noise spectra for different temperatures has a common tangent with slope ≈ -1, and that the amplitude of the noise decreases as d^{-3}, where d is the height above the film at which the magnetic flux is measured. A crossover from normal to anomalous vortex diffusion is indicated by the measurements and is discussed in terms of a two-dimensional decoupling.

Spontaneously created two-dimensional (2D) vortices drive the phase transition of Kosterlitz-Thouless (KT) type between the superconducting and the normal state for thin film superconductors [1]. This means that the physics of the vortices is responsible for the features in a region around the transition [1]. More recently it has been found that high-T_c superconductors also show behavior characteristic of 2D vortex fluctuations [1–5]. This is expected for films thin enough to be in the "quasi" 2D regime, while the phase transition for thicker films is related to the 3D bulk transition. However, also for such thick films 2D vortex fluctuations have been observed just above the transition [4,5], which has been ascribed to a decoupling of the superconducting planes associated with the CuO$_2$ layers [6–8]. The evidence in case of BSCCO, which has a fairly broad resistive transition, is better established than for YBCO which has a very narrow transition [4,5]. One may argue that for BSCCO, since it has a much larger anisotropy than YBCO, the interlayer coupling should be much weaker, and as a consequence the quasi 2D character much stronger [1]. From this perspective the possibility and the significance of 2D vortex fluctuations remains to be clarified for YBCO. In the present Letter, we address this question by focusing on the dynamical features of the vortices reflected in the flux noise spectrum and the complex impedance for a 500 Å thick YBCO film; a strategy which was also used in Ref. [3] for a quasi-2D BSCCO film. To resolve the narrow resistive region for our YBCO sample, we use a high resolution temperature control in the experimental setup.

The sample is a square shaped (5 × 5 mm^2) 500 Å thick YBCO-123 film grown epitaxially on a LaAlO$_3$ substrate by pulsed laser deposition. According to x-ray θ-2θ and ϕ scans the film is highly c-axis oriented and only [100] YBCO‖[100] LaAlO$_3$ in-plane orientation is ob-

served. The experimental setup is a SQUID based system designed for complex impedance and flux noise measurements. The sample space is magnetically shielded by μ-metal and niobium cans, resulting in a residual dc field of approximately 1 mOe. The pickup coil having a diameter of 1.2 mm is a first order gradiometer, each section containing 2 layers (8 + 7 turns) of 0.05 mm NbTi wire. The amplitude of the field in the complex impedance measurements is 0.4 mOe at the center of the drive coil, which is low enough to ensure a linear sample response. The construction of the system allows a very high precision temperature control of the sample. The long time stability of the sample temperature is of the order 0.1 mK [9].

There exist predictions for the flux noise spectrum associated with 2D vortex fluctuations [10–14]. In the present analysis we focus on the relation between the flux noise spectrum and the conductance: The flux noise spectrum $S(\omega)$ caused by 2D vortex fluctuations should be proportional to the conductance $G(\omega)$ in the limit where the distance d between the sample and the pickup coil is larger than the perpendicular penetration depth Λ [15]:

$$S(\omega) = A\,\text{Re}[G(\omega)] = \frac{A\rho_0(T)}{\omega}\left|\text{Im}\left[\frac{1}{\epsilon(\omega)}\right]\right| . \quad (1)$$

Here $1/\epsilon(\omega)$, which is the quantity often studied theoretically, is the vortex dielectric function [1], and ρ_0 is the bare superfluid density which within a Ginzburg-Landau description decreases to zero as $\rho_0(T) \propto 1 - T/T_{c0}$ [1]. The proportionality constant A, apart from trivial constants, contains a factor $Tf(d/R)/R^2$, where R is the linear dimension of the pickup coil. The dissipation is proportional to $|\text{Im}[1/\epsilon(\omega)]|$ and has a maximum as a function of ω at a fixed T [1]. The frequency ω_0 at this maximum defines the characteristic frequency scale which vanishes

as the transition is approached from above due to critical slowing down [11,16].

In Fig. 1 we plot $\omega S(\omega)$ and $\text{Re}[\omega G(\omega)]$ for three temperatures above T_c: They agree very well in all three cases which shows that $S(\omega) \propto \text{Re}[G(\omega)]$ as expected for vortex fluctuations in the large d limit. The inset shows the corresponding characteristic frequency ω_0 (filled and open circles correspond to the maximum of $\omega S(\omega)$ and the dissipation peak of $\text{Re}[\omega G(\omega)]$ vs T [compare Fig. 3(b)], respectively). One notes the excellent agreement between the characteristic frequencies for $S(\omega)$ and $G(\omega)$ measured independently. A striking feature in the inset is the dramatic decrease of ω_0 [$O(10^4)$ over a 0.1 K interval] and a linear extrapolation gives 10^{-4} Hz at 84.7 K (which we take as a rough estimate of T_c). This rapid decrease of ω_0 is indeed expected from 2D vortex fluctuations because these imply that ω_0 is proportional to the resistance $\mathcal{R}$ [17], which has a similar rapid decrease, as is also seen in the inset of Fig. 1.

The proportionality between the flux noise spectrum and the conductance is by no means a general property. For example, the flux noise spectra discussed in Refs. [11,12] are not proportional to the conductance [15], because they correspond to the case when the distance d from the sample to the pickup coil is zero. Our experiment corresponds to d larger than Λ and only in this limit is the proportionality expected for 2D vortex fluctuations [15].

A further test of the prediction for the flux noise spectra when $d > \Lambda$ is the dependence of the flux noise amplitude on d. For a fixed temperature this dependence has the form $f(d/R) \propto 1/d^3$ for $R \gg d$ and $1/d^6$ for $R \ll d$ [see Eq. (1)] [15]. Figure 2 shows simulation results in

the region $R \approx d$ (open circles) and demonstrates that the $1/d^3$ behavior is valid also in this parameter range. We have obtained experimental data in the same range (filled circles), which are in reasonable agreement with the expected $1/d^3$ behavior [18].

Since $\omega S(\omega)$ is proportional to $|\text{Im}[1/\epsilon(\omega)]|$, properties of the vortex dielectric function $1/\epsilon(\omega)$ should be reflected in the flux noise. The connection between the vortex dynamics and $1/\epsilon(\omega)$ was worked out in Ref. [19]. In the high-temperature phase $1/\epsilon(\omega)$ for 2D vortex fluctuations close to T_{c0} is of normal Drude type describing the response of "free" vortices:

$$\left| \text{Im}\left[\frac{1}{\epsilon(\omega)} \right] \right| = \frac{1}{\tilde{\epsilon}} \frac{\omega \omega_0}{\omega^2 + \omega_0^2}, \qquad (2)$$

where $\tilde{\epsilon}$ describes an effective static polarization of vortex-antivortex pairs. The maximum value is $1/2\tilde{\epsilon}$ and the peak ratio defined as $\text{Im}[1/\epsilon(\omega)]/|\text{Re}[1/\epsilon(\omega)]|$ at this maximum is 1. Close to T_c the response is expected to be dominated by bound vortex pairs and has been found to be well parametrized by the Minnhagen phenomenology (MP) form [1,3,16]:

$$\left| \text{Im}\left[\frac{1}{\epsilon(\omega)} \right] \right| = \frac{1}{\tilde{\epsilon}} \frac{2}{\pi} \frac{\omega \omega_0 \ln \omega / \omega_0}{\omega^2 - \omega_0^2}, \qquad (3)$$

which has the maximum value $1/\pi\tilde{\epsilon}$ and the peak ratio $2/\pi$ [16,20]. In reality the response crosses over from the anomalous MP form to the normal Drude form as the temperature is increased from T_c to T_{c0} [16,20]. This is because at T_{c0} there are only "free" vortices and no bound vortex-antivortex pairs as a consequence of the thermal unbinding of vortex pairs. However, as the temperature is lowered the proportion of bound pairs increases and for the pure 2D case these bound pairs dominate the response in a region above the 2D transition [1]. This knowledge of
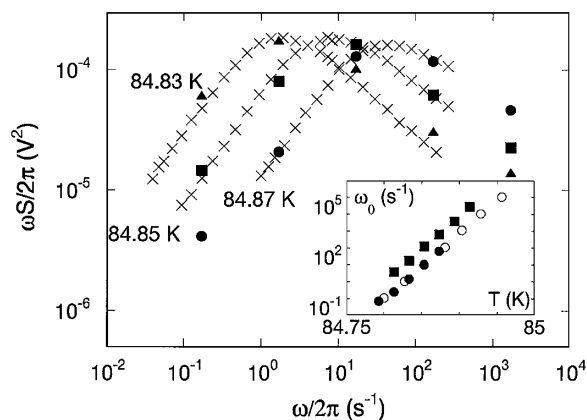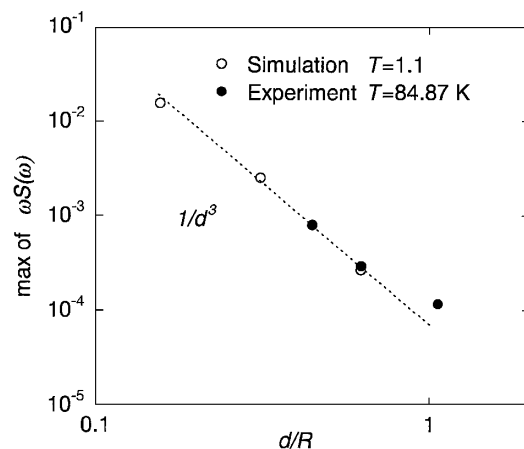

FIG. 1.   Flux noise $\omega S(\omega)$ (crosses) and ac conductance $\text{Re}[\omega G(\omega)]$ (filled circles, squares, and triangles) for three different T plotted in a log-log scale. The ac conductance results have been adjusted in the vertical direction. The flux noise results are given in units of the SQUID voltage. The inset shows the characteristic frequency ω_0 obtained from the maxima of $\omega S(\omega)$ (filled circles) and from the maxima in $\text{Re}[\omega G(\omega)]$ vs T for fixed ω [open circles, compare Fig. 3(b)]. Filled squares correspond to the resistance $\mathcal{R}$ calculated as the inverse of the magnitude of the small frequency plateaus of the noise spectra.

FIG. 2.   Maximum of $\omega S(\omega)$ as a function of distance d to the pickup coil in a log-log plot. The horizontal axis is d/R, where R is the linear dimension of the pickup coil. The open circles give results from simulations (see Ref. [15]) slightly above T_c. The filled circles give the corresponding experimental results (for convenience shifted in the vertical direction).

$1/\epsilon(\omega)$ together with the proportionality to $\omega S(\omega)$ leads to the prediction of a common tangent for the flux noise spectra for different temperatures [16]:

$$\omega S(\omega)|_{\omega_0} \propto \frac{T \rho_0(T) h(T)}{\tilde{\epsilon}(T)}. \qquad (4)$$

If the right-hand side is independent of T then $S(\omega_0)$ for different T lie on a common curve $\propto 1/\omega$ such that all spectra $\ln S(\omega)$ as a function of $\ln \omega$ has a common tangent with slope -1. The point is that, although the right hand side does have a weak T dependence [$\rho_0 \propto 1 - T/T_{c0}$, the height $h(T)$ of $\tilde{\epsilon} \, \text{Im}(1/\epsilon)$ goes from $1/\pi$ to $1/2$ as T goes from T_c to T_{c0}, and $\tilde{\epsilon}$ is of order unity and goes from a value >1 at T_c to 1 at T_{c0} [1]], the dramatic T dependence of $\omega_0(T)$ dominates, leading to the common tangent with the slope ≈ -1. It should be noted that the common tangent hinges on only two things, the proportionality between $\omega S(\omega)$ and $|\text{Im}[1/\epsilon(\omega)]|$ in the large d limit and the weak T dependence of the proportionality factor in comparison with the large temperature dependence of the characteristic frequency ω_0, but it does not hinge on the explicit frequency dependence of $|\text{Im}[1/\epsilon(\omega)]|$. One may note that the flux noise spectra for $d = 0$ (discussed, e.g., in Refs. [11,12]) are not proportional to the conductance and, as a consequence, do not have a common tangent. Figure 3(a) tests this prediction for the flux noise spectra for six different T: A common tangent is indeed found and has the slope -1.05 very close to -1. As also seen in Fig. 3(a) each individual noise spectrum has a slope $-3/2$ for frequencies $\omega > \omega_0$. Because of the proportionality shown in Fig. 1, this also means a corresponding slope $-3/2$ for $\text{Re}[G(\omega)]$. But $\text{Re}[G(\omega)]$ for a superconductor must eventually have a crossover to an ω^{-2} dependence for high enough frequencies. However, this crossover frequency is outside our present experimental range. By comparison the simulations in Ref. [15] for the flux noise spectrum from 2D vortex fluctuations and, correspondingly, the conductance show a slope close to $-3/2$ only over a limited frequency region before the crossover to -2 sets in.

We can also use the knowledge of $1/\epsilon(\omega)$ when analyzing the complex conductance $G(\omega)$. Figure 3(b) shows $\text{Re}[\omega G(\omega)]$, which is proportional to $|\text{Im}[1/\epsilon(\omega)]|$, as a function of T for several frequencies. If the proportionality factor $\rho_0(T)/\tilde{\epsilon}(T)$ was independent of T, then the frequency at the maximum of $\text{Re}[\omega G(\omega)]$ in Fig. 3(b) for each T would give the corresponding characteristic frequency $\omega_0(T)$ of $|\text{Im}[1/\epsilon(\omega)]|$. Since the T dependence of $\omega_0(T)$ completely dominates over the T dependence of the proportionality factor this gives a very good estimate (this is the estimate used in the inset of Fig. 1). In case of $G(\omega)$ we can push the analysis one step further by using the relation between the peak ratio and the height $h(T)$ for $\tilde{\epsilon} \, \text{Im}[1/\epsilon(\omega)]$ inferred from simulations of 2D vortex fluctuations [20]. This relation is shown in the inset of Fig. 4(a), as obtained from the parametrization given in Ref. [20]. Figure 4(a) shows $\text{Re}[\omega G(\omega)]$
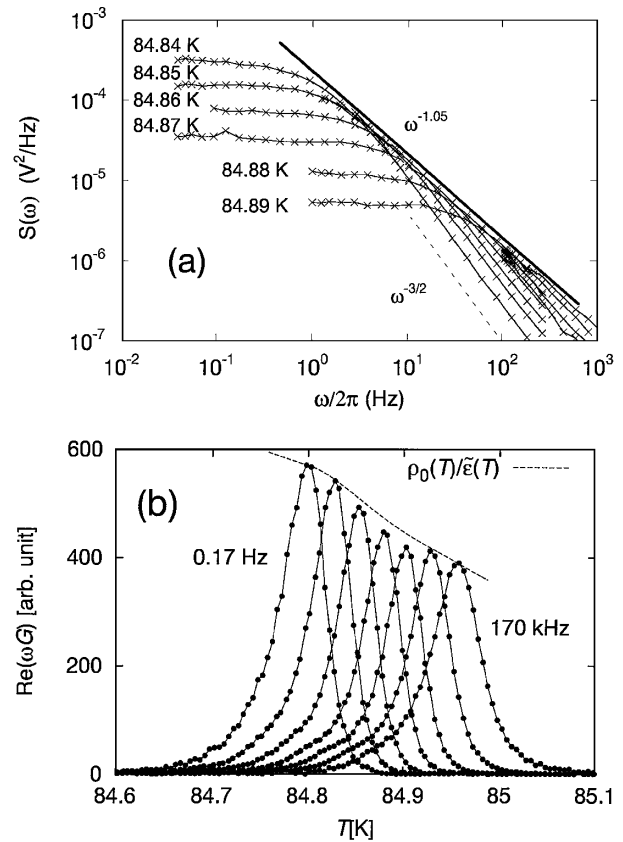

FIG. 3. (a) Flux noise spectra for six temperatures in a log-log plot have a common tangent (full drawn line) with a slope -1.05. The slopes for the individual spectra are close to $-3/2$ for larger frequencies (dashed line). The flux noise is given in units of SQUID voltage. (b) $\text{Re}[\omega G(\omega)]$ as a function of T for different frequencies. The dashed curve corresponds to a self-consistent determination of $\rho_0(T)/\tilde{\epsilon}(T)$ (see discussion of Fig. 4).

and $\text{Im}[\omega G(\omega)]$ for $\omega/2\pi = 170$ Hz. The dashed line is $C \times \text{Im}[\omega G(\omega)]$, where the constant C is determined so that the dashed curve cuts $\text{Re}[\omega G(\omega)]$ precisely at the peak. This gives $C \approx 2/\pi$ consistent with bound vortex-antivortex pair response described by Eq. (3) and, consequently, with $h(T) = 1/\pi$ for the T corresponding to the maximum. This construction can be further improved by taking the T dependence of $\rho_0(T)/\tilde{\epsilon}(T)$ into account: The quantity corresponding to $1/\epsilon(\omega)$ is $\tilde{\epsilon} \omega G(\omega)/\rho_0$. We start from the T dependence for $\rho_0/\tilde{\epsilon}$ corresponding to a T-independent $h(T)$. From this, one gets the peak ratio for each frequency and hence $h(T)$ for each T corresponding to a maximum. Using this obtained $h(T)$, one gets a new estimate for $\rho_0/\tilde{\epsilon}$. In this way we can self-consistently determine the T dependence of $\rho_0/\tilde{\epsilon}$ and the peak ratio [both shown in Fig. 4(b)]. The inset in Fig. 4(b) shows that $\rho_0(T)/\tilde{\epsilon}$ goes linearly to zero at $T_{c0} \approx 85.3$ K. This is consistent with the Ginzburg-Landau prediction $\rho_0(T) \propto 1 - T/T_{c0}$ and the 2D Coulomb gas prediction $\tilde{\epsilon} \approx 1$ for T slightly above T_c [1].

The peak ratios in Fig. 4(b) suggest an increase towards the Drude value 1 as T_{c0} is approached and a decrease
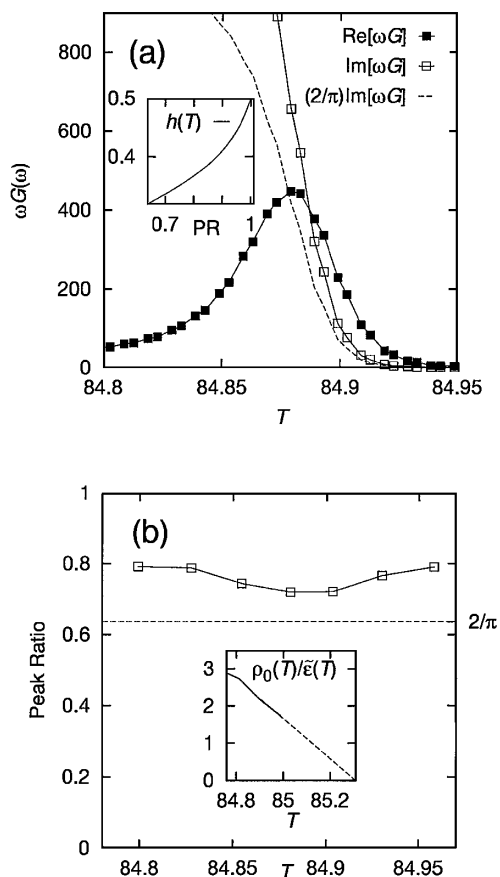
FIG. 4. (a) $\omega G(\omega)$ as a function of T for the frequency $\omega/2\pi = 170$ Hz (filled squares for real part and open squares for imaginary part). The dashed curve is $C \times \text{Im}[\omega G(\omega)]$, where $C \approx \pi/2$ is the peak ratio (see text for explanation). The inset shows the relation between the peak height $h(T)$ and the peak ratio (PR). (b) The peak ratios as a function of T obtained from the self-consistent determination described in text. The inset shows that the self-consistently determined $\rho_0(T)/\tilde{\varepsilon}(T)$ is proportional to $1 - T/T_{c0}$, which gives $T_{c0} \approx 85.3$ K.

towards the bound vortex-antivortex value $2/\pi$ as the temperature is decreased. This is consistent with the expected crossover from normal to anomalous vortex diffusion for 2D vortex fluctuations [1,16]. However, as the temperature is further decreased towards T_c the peak ratio again increases. This might be caused by a plane decoupling [1,6]: Closely above T_c the vortex fluctuations induce a decoupling of the YBCO material into essentially decoupled superconducting CuO_2 layers. The 2D vortex fluctuations associated with these planes are responsible for the peak ratios close to the bound vortex-antivortex value as well as the increase at higher T. The coupling of the planes very close to T_c suppresses the vortex-antivortex pair fluctuations and this we suggest could be the cause of the increase of the peak ratio with decreasing T since the peak ratio depends on the relative proportion of free and bound vortices [21].

In summary, we have measured the flux noise and the complex impedance for a YBCO film and shown that in a narrow temperature interval just above the resistive transition the dynamical features can be consistently interpreted in terms of 2D vortex fluctuations. To this end we explicitly verified the proportionality between flux noise and the complex conductance valid when the distance d to the pickup coil is large, as well as the $1/d^3$ dependence of the flux noise amplitude. The connection to the vortex dielectric function gives rise to a common tangent with a slope ≈ -1 for the flux noise at different temperatures which was also verified. Finally, we self-consistently determined the peak ratio associated with the vortex dielectric function and suggested an interpretation in terms of a plane decoupling just above the resistive transition. This work suggests that 2D vortex fluctuations play an identifiable role even in an YBCO thin film sample with a very narrow zero field resistive transition.

[1] For a general review, see, e.g., P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987); for connections to high-T_c superconductors, see, e.g., P. Minnhagen, in *Models and Phenomenology for Conventional and High-Temperature Superconductors*, Proceedings of the International School of Physics "Enrico Fermi," Course CXXXVI (IOS Press, Amsterdam, 1998), p. 451.

[2] D. P. Norton and D. H. Lowndes, Phys. Rev. B **48**, 6460 (1993).

[3] C. T. Rogers *et al.,* Phys. Rev. Lett. **69**, 160 (1992).

[4] L. Miu *et al.,* Phys. Rev. B **57**, 3144 (1998).

[5] V. Persico *et al.,* Physica (Amsterdam) **260C**, 41 (1996).

[6] P. Minnhagen and P. Olsson, Phys. Rev. Lett. **67**, 1039 (1991).

[7] M. Friesen, Phys. Rev. B **51**, 632 (1995).

[8] S. W. Pierson, Phys. Rev. Lett. **73**, 2496 (1994).

[9] J. Magnusson *et al.,* Rev. Sci. Instrum. **68**, 3761 (1997).

[10] J. Houlrik *et al.,* Phys. Rev. B **50**, 3953 (1994).

[11] I.-J. Hwang and D. Stroud, Phys. Rev. B **57**, 6036 (1998).

[12] P. H. E. Tiesinga *et al.,* Phys. Rev. Lett. **78**, 519 (1997).

[13] K.-H Wagenblast and R. Fazio, JETP Lett. **68**, 312 (1998).

[14] C. Timm, Phys. Rev. B **55**, 3241 (1997).

[15] B. J. Kim and P. Minnhagen, Phys. Rev. B **60**, 6834 (1999).

[16] A. Jonsson and P. Minnhagen, Phys. Rev. B **55**, 9035 (1997).

[17] We estimate $\mathcal{R}$ as $\mathcal{R} = 1/G(0)$ and note that $G(0)$ corresponds to the magnitude of the small frequency plateau of the noise spectrum.

[18] In the experimental setup the section of the pickup coil being closest to the film has a vertical extent of 0.45 mm. d is determined as the weighted average over this coil, $d = \Sigma_i d_i \times p(d_i)$ with $p(d_i) = (1/d_i^3)/\Sigma_j(1/d_j^3)$.

[19] V. Ambegaokar *et al.,* Phys. Rev. Lett. **40**, 783 (1978); Phys. Rev. B **21**, 1806 (1980).

[20] B. J. Kim, P. Minnhagen, and P. Olsson, Phys. Rev. B **59**, 11 506 (1999).

[21] A. Jonsson and P. Minnhagen, Physica (Amsterdam) **277C**, 161 (1997).