Uniform Statistical Description of the Transition between Near and Far Field Turbulence in a Wake Flow

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The transition to fully developed turbulence of a wake flow is investigated on different length scales r. It is shown that the r dependence of the velocity increments can be taken as Markov processes even in the case of not fully developed turbulence. The estimation of the deterministic part of these Markov processes shows a pronounced structural change as one passes away from the cylinder, which can be parametrized by a kind of order parameter. We propose that this effect is connected to the presence of large scale coherent structures close to the cylinder.

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In recent years considerable progress has been made in understanding the statistical features of fully developed, local isotropic turbulence [1]. Special interest has been shown in understanding intermittency effects of small scale velocity fluctuations characterized by the velocity increments $u_r(x) := u(x + r) - u(x)$ at a scale r. For most real flows these results are applicable only for small well defined regions of the flow, which may be regarded as locally isotropic. A remaining challenge is to find out how these concepts can help one to understand real flows which are not fully developed or not homogeneous and isotropic [2].

The common method to characterize the disorder of fully developed local isotropic turbulence is to investigate the scale evolution of the probability density functions (pdf), $P_r(u_r)$, either directly or by means of their moments $\langle u_r^n \rangle = \int u_r^n P(u_r)\, du_r$, which are called structure functions. For better estimation of the scaling exponents of these structure functions the so-called extended self-similarity (ESS) method was suggested [3]. Recently, it was found that the r evolution of the pdf can be related to a Markov process [4]. The Markovian properties can be evaluated thoroughly by investigating conditional probabilities. Furthermore the stochastic for the r evolution of P_r and the Langevin equation for u_r can be extracted from measured data [5,6]. This method provides a statistically more complete description of turbulence with no need of additional assumptions, like scaling [4,7].

Here we present measurements of a turbulent flow behind a circular cylinder. The main result is that the Markovian analysis is valid for all distances to the cylinder. One aspect is the deterministic part, for which we present the finding of a phase transition-like behavior to the state of fully developed turbulence. This characterizes the disappearance of the Karman vortices with respect to two parameters: the distance to the cylinder and the scale r.

In the following we describe first the experimental setup. The measurements of longitudinal and transversal velocities are analyzed with respect to the r dependent statistics of the velocity increments. Subsequently, a test of Markov properties is presented. From the conditional pdf the moments $M^{(k)}$ ($k = 1, 2, 4$) are evaluated. Then some consequences are shown and physical interpretations are finally discussed.

Our work is based on hot-wire velocity measurements performed in a wake flow generated behind a circular cylinder (diameters d of 2 and 5 cm) inserted in a wind tunnel. The used wind tunnel [8] has the following parameters: cross section 1.6 m $\times$ 1.8 m, length of the measuring section 2 m, velocity 25 m/s, and residual turbulence level below 0.1%. To measure longitudinal and transversal components of the local velocity we used x-wire probes (Dantec 55P71), placed at several distances, D, between 8 and 100 diameters of the cylinder. The spatial resolution of the probes is about 1.5 mm.

From the measurements the following characteristic lengths were evaluated: the integral length, defined by the autocorrelation function, varied between 10 and 30 cm depending on the cylinder used and the location of the probe; the Kolmogorov length was about 0.1 mm; and the Taylor length scale was about 2.0 mm. Thus we see that our measurement resolved at least the turbulent structures down to the Taylor length scales. (Note that these lengths can be calculated precisely only for distances above 40 cylinder diameters.) The Reynolds numbers of these two flow situations are $R_\lambda = 280$ and 650. Each time series consists of 10^7 data points, and was sampled with a frequency corresponding to about one Kolmogorov length. To obtain the spatial variation the Taylor hypothesis of frozen turbulence was used.

To investigate the disorder of the turbulent field the velocity increments for different scales r and at different measuring points D were calculated. First, we looked at the scaling properties of the structure functions using the above mentioned ESS method. The estimated scaling exponents vary only very slightly for the different flow

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situations, suggesting universality. These variations are close to the errors of estimation. Next, we evaluated the pdfs, which are exemplarily shown in Fig. 1 for the transversal velocity component. In Fig. 1(a) the well known intermittency effect of isotropic turbulence is shown by nearly Gaussian distributions at large scales and more intermittent distributions as r gets smaller. Coming closer to the cylinder a structural change is found. Most remarkably a double hump pdf emerges for large r. This structure reflects the fact that two finite values of the velocity increment are most probable. We interpret this as the result of counterrotating vortices passing over the detector. This is consistent with the geometric features of vortices elongated parallel to the cylinder axis (Karman vortices). For small scales the humps vanish and the pdfs become similar to the isotropic ones.

Based on the findings that the evolution of the pdfs with r for the case of fully developed turbulence can be described by a Fokker-Planck equation [4,9], we apply the Markov analysis to the not fully developed states close to the cylinder. The basic quantity to be evaluated is the conditional pdf $P(u_{r_2}, r_2 | u_{r_1}, r_1)$, where $r_2 < r_1$ and u_{r_2} is nested into u_{r_1} by a midpoint construction. To verify the Markovian property, we evaluate the Chapman-Kolmogorov equation, cf. [6]

$$P(u_{r_2}, r_2 | u_{r_1}, r_1) = \int_{-\infty}^{\infty} P(u_{r_2}, r_2 | u_{r'}, r') \times P(u_{r'}, r' | u_{r_1}, r_1)\, du_{r'}, \quad (1)$$

where $r_2 < r' < r_1$. The validity of this equation was examined for many different pairs of (r_1, r_2). As a new result, we found that Eq. (1) also holds in the vicinity of the cylinder, i.e., in the undeveloped case of turbulence. For illustration see Fig. 2; in 2(a) the integrated conditional pdf [right-hand side of Eq. (1)] and the directly evaluated pdf [left-hand side of Eq. (1)] are shown by superimposed contour plots. In Fig. 2(b) three exemplary cuts through the contour plot are shown. The quality of the validity of Eq. (1) can be seen from the proximity of the contour lines, or by the agreements of the conditional pdfs, represented by open and bold symbols [10]. Based on this result we treat the evolution of the statistics with the scale r as a Markov process in r. Thus the evolution of the pdf $P_r(u_r)$ is described by the partial differential equation called Kramers-Moyal expansion [6],

$$-\frac{d}{dr} P(u_r, r) = \sum_{k=1}^{\infty} \left[-\frac{\partial}{\partial u_r} \right]^k D^{(k)}(u_r, r) P(u_r, r), \quad (2)$$

with the coefficients

$$M^{(k)}(u_r, r, \delta) := \frac{1}{k!} \frac{1}{\delta} \int du_{r'} (u_{r'} - u_r)^k P(u_{r'}, r' | u_r, r), \quad (3)$$

$$D^{(k)}(u_r, r) := \lim_{\delta \to 0} M^{(k)}(u_r, r, \delta), \quad (4)$$

where $\delta = r - r'$. Notice, having once evaluated the conditional pdfs, these so-called Kramers-Moyal (KM) coefficients can be estimated directly from the data without any additional assumption. For our purpose it is sufficient to consider the $M^{(k)}$ for a small length of $\delta \approx 2\eta$.

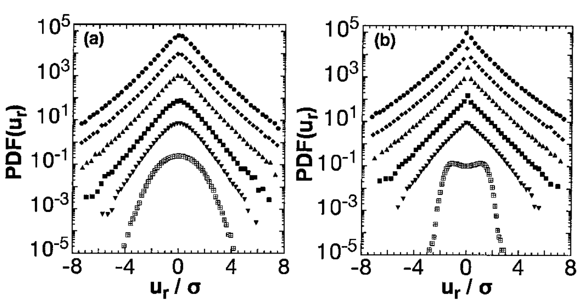

FIG. 1. Probability density functions for $r = 0.1$ mm to $r = 120$ mm (from top to bottom) obtained from two data sets of transversal velocities, cylinder diameter $d = 5$ cm. (a) Fully developed turbulence (40 d); (b) transition region close to the cylinder (8 d). pdfs are shifted along the y direction for clearness of the presentation.

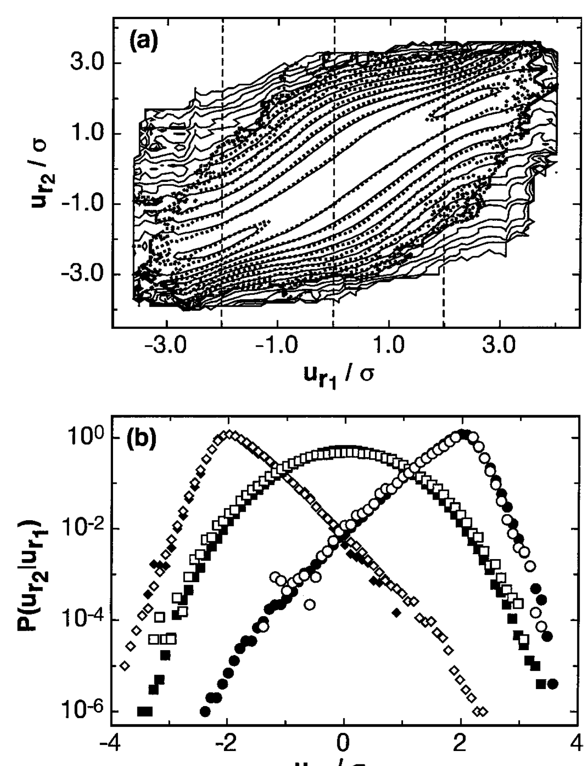

FIG. 2. Verification of the Chapman-Kolmogorov equation in the transition region ($D = 8$ d) for the cylinder with $d = 5$ cm. (a) Contour plot of the directly evaluated conditioned probability distribution, presented as dashed lines, and numerically integrated conditional pdf [right-hand side of Eq. (1)] represented by solid lines ($r_1 = 10.5$ cm, $r_2 = 12.9$ cm). (b) Corresponding cuts for selected u_{r_1} values [see marked lines in (a)]. Bold symbols stand for the directly evaluated conditional pdf and open symbols for the integrated conditional pdfs.

In Fig. 3 we show exemplarily the evaluated $M^{(1)}$ for the measurement in the near field of the cylinder estimated for several length scales. We find that close to the cylinder the form of the $M^{(1)}$ changes from a nearly linear u_r dependence at small scales to a third order polynomial behavior. The first KM coefficient (called "drift term") represents the deterministic evolution. From the corresponding Langevin equation [6] we know that the zeros of the drift term correspond to the fixed points of the deterministic dynamics. Fixed points with negative slope belong to accumulation points, having the tendency to build up local humps in the pdf. Thus the structural change of the pdfs (Fig. 1) is mainly given by $M^{(1)}$. The change of the local slope of a fixed point (Fig. 3) can be set into correspondence to a kind of phase transition, cf. [6,11].

The second KM coefficient (called "diffusion term") grasps the influence of delta correlated Gaussian noise. As shown in Figs. 3(d) and 3(e), $M^{(2)}$ also depends on r. According to the number of stable fixed points of $M^{(1)}$, $M^{(2)}$ has one or two minima. Thus the noise raises for unstable fixed points and decreases for stable fixed points.

We also checked the fourth KM coefficient $M^{(4)}$ which reflects the deviations from the Gaussian distributed noise. $M^{(4)}$ is less than 10^{-3} times $M^{(2)}$. Taking $M^{(4)}$ as negligible and using Pawula's Theorem [6], the stochastic process is driven by Gaussian white noise and Eq. (2) reduces to a Fokker-Planck equation.

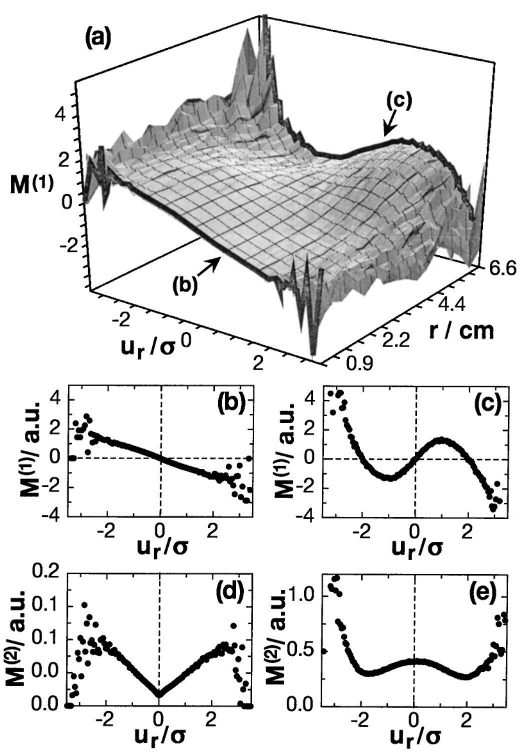

FIG. 3. Approximate KM coefficient $M^{(1)}(u_r, r, \delta)$ for $\delta =$ 0.1 mm at a distance of 8 d behind the cylinder ($d = 2$ cm). (b) and (c) Correspond to $M^{(1)}$ for $r \approx 0.5$ cm and $r \approx$ 6.5 cm. Note the change of the sign of the slope of $M^{(1)}$ at $u_r = 0$. (d) and (e) Show the corresponding $M^{(2)}$ coefficient.

An important point of our analysis is that we can determine the evolution equation in the form of the KM coefficients. This tool is much more sensitive than merely looking at the pdfs or its moments, because the pdfs reflect only the transient behavior due to the underlying evolution equation. Thus it becomes clear that we are able to work out the structural change of the stochastic behavior even in the case where the double hump structure in the pdf may not be clearly visible, and in the structure function even with ESS nothing remarkable can be seen. Note that the double hump behavior of the pdfs can be well reproduced by calculating the stationary solution of the corresponding Fokker-Planck equation, using our measured KM coefficient $M^{(1)}$ [12].

Beside the spatial scale parameter r, the second parameter of the wake experiment is the distance of the probe to the cylinder. As is well known, with increasing the distance a transition to fully developed turbulence takes place, and the double hump structure vanishes. To characterize the structural change of the stochastic behavior of the flow in this two-dimensional parameter space more completely we performed the above mentioned data analysis at several distances. The local slope at $M^{(1)}(u_r, r, z) = 0$ was determined for the different scales r and distances z from the cylinder. For the case of one fixed point we get a negative slope. It is positive for the three fixed point case, which reflects the occurrence of the Karman vortices. The magnitude of this local slope for the two parameters is shown in Fig. 4 as a contour plot. The dark colored region marks the parameter space, where 3 zeros for $M^{(1)}$ are present, or where the local slope at $u_r = 0$ is positive. The critical line of the structural change is marked by the bold black line. As displayed in Fig. 4 we see a reentrant negative slope at

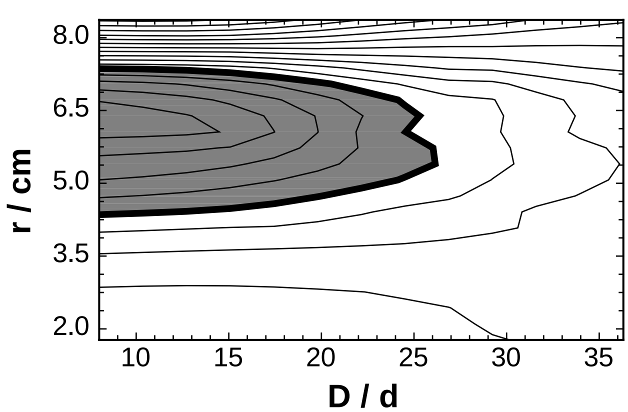

FIG. 4. Phase diagram for the transition to fully developed turbulence in a flow behind a cylinder ($d = 2$ cm), given by the value of the slope a_1 of $M^{(1)}$ at $u_r = 0$ (see Fig. 3). Shadowy region corresponds to the occurrence of a positive slope, i.e., the tendency to form the double hump shape of the pfds [see Fig. 1(b)].

very large r values (larger than 7 cm). This effect is due to the periodic structure of the Karman vortices. Taking into account that our analysis of the stochastic features of the increment u_r is closely related to a correlation analysis, it is easily seen that the periodicity of vortices is reflected in a periodic structural change of $M^{(1)}$ and $M^{(2)}$ with increasing r. For still larger r (not shown here) this periodicity vanishes as a direct consequence of the finite coherence of the Karman vortices.

We have presented that the new approach of a Markov analysis can also be used in the case of not fully developed turbulence. Thus we can describe the underlying stochastic process of the velocity increment and their changes with changing the scale r and the distance from the cylinder due to deterministic and stochastic forces. Both forces can be estimated directly from the data sets via Kramers-Moyal coefficients of the conditional probabilities. We find significant changes in the deterministic force, the drift term, as one passes from not fully developed turbulence (close to the cylinder) into fully developed turbulence (far behind the cylinder). In the far field the drift term causes a stable fixed point at $u_r = 0$. Approaching the near field at large r this fixed point becomes unstable, the slope of the drift term changes its sign at $u_r = 0$. In our one-dimensional analysis we find the appearance of two new stable (attracting) fixed points which are related to the double hump structure of the corresponding pdfs. This phenomenon may be set formally into relation with a kind of phase transition.

Finally some critical remarks are presented to show in which direction work should be done in the future. Visualizations indicate that even in the case of strong turbulence the near field still resembles time periodic structures of counterrotationg vortexlike structures detaching from the cylinder [13]. These time periodic large scale structures ask for a two-dimensional (two variable) modeling, in the sense of a noisy limit cycle. This apparent contradiction to our one variable analysis has to be seen on the background of the signal treatment. Applying to a time series the construction of increments (which represents a kind of high pass filter) the locality in time is lost. Thus also coherency in time may get lost, at least as long as one investigates small scale statistics. In this sense only a stochastic aspect of the counterrotating vortices is grasped. The challenge of a more complete characterization of the near field structures will require, in our opinion, a combination of increment analysis and real time modeling of the velocity data. At least for the latter case a higher dimensional ansatz is required. Nevertheless, we have presented in this work clear evidence of how methods and results obtained from the idealistic case of fully developed turbulence can be used to characterize also the statistics in the transition region of a wake flow.

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