Opposite Thermodynamic Arrows of Time

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A model in which two weakly coupled systems maintain opposite running thermodynamic arrows of time is exhibited. Each experiences its own retarded electromagnetic interaction and can be seen by the other. The possibility of opposite-arrow systems at stellar distances is explored and a relation to dark matter suggested.

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The possibility of simultaneous opposite running thermodynamic arrows of time has been raised on several occasions, for didactic purposes [1], for general interest [2], and to confound by "obvious" counterexample [2]. A difficulty in these considerations is the absence of a well-defined framework. For example, one might argue against opposing arrows as follows. Let the systems be A and B. An observer in A will see a succession of small miracles in B as eggs uncrack, etc. It would seem that the tiniest interference by A, the smallest cry of amazement—transmitted to B—would destroy the monumental coordination needed for B's reversed arrow. That this argument is flawed is apparent when one realizes that it is phrased from A's perspective, and takes as natural that the images from B do not destroy the coordination that B would attribute to A. But whether the flaw is correctable or whether the conclusion is that *both* arrows would be destroyed is less clear.

In [3–5] a framework for these questions was proposed. Here I use that framework to show that small interaction does *not* destroy the arrows. The question of whether the systems can communicate will be touched on. Signals are of interest because of causal paradoxes. One aspect of communication is electromagnetic radiation, and I will extend the Wheeler-Feynman absorber theory [6] to show that each system has its own retarded interactions, which appear advanced to the other.

Our usual thermodynamic arrow can be phrased as the fact that when macroscopic (coarse grained) information is given it can be used, by averaging over the evolution of all microstates consistent with the macrostate, to estimate the future, but not (in that way) the past. As argued in [3–5], an unbiased treatment of thermodynamic arrow questions can be had by giving macroscopic information at two times (typically, cosmologically remote). It was found that despite the nonstandard conditioning, arrows emerge, consistent with a thesis correlating the thermodynamic arrow with the expansion of the Universe [7] (or at least with low entropy states at the remote eras [5]).

In [5], p. 179, it was suggested that the 2-time formulation could be used to study opposing arrows, but the inquiry was dismissed as "science fiction." However, in a time-symmetric universe this possibility should be considered (in fact this was a complaint in [2], so defense of the arrow-correlation thesis requires this). Moreover, as proposed below there is also the possibility of physical relevance in our present cosmological era.

Given systems A and B (for simplicity taken identical) that interact slightly, conflicting arrows are established through the following boundary conditions. In each system there is a concept of macrostate, defined by coarse grains in phase space. At time 0, A and B are, respectively, in Δ_{Ai} and Δ_{Bi} ($\subset \Gamma \equiv$ phase space energy surface). At time T they are in Δ_{Af} and Δ_{Bf}. The entropy, S, of a grain is the logarithm of its volume. The conflict is imposed by starting A in a small grain, and putting little or no constraint on its final state. The opposite is done for B. ("Start" refers to "t," not to a thermodynamic arrow.) Thus, $S(\Delta_{Ai}) = S(\Delta_{Bf}) \ll S(\Delta_{Af}) = S(\Delta_{Bi})$. For convenience we set $\Delta_{Bi} = \Delta_{Af} = \Gamma$. The relaxation time for Δ_{Ai} to spread within Γ is denoted τ.

The equation of motion of a particle $\alpha \in A$ is schematically

$$\ddot{x}_\alpha = \sum_{\gamma \in A} F_\gamma(x_\alpha) + \sum_{\gamma \in B} F_\gamma(x_\alpha) = F^{(A)} + F^{(B)}.$$

By hypothesis $F^{(B)}$ is small, but not *ultrasmall*. Thus $F^{(B)}$ is not *so* weak that it would not destroy an entropy lowering process (such as the time reverse of a breaking egg) of a macroscopic system [8]. Now if this were a normal physical problem one would expect the effect of B on A to shorten the relaxation time: $F^{(B)}$ would be noise on top of the independent motion of A. But from B's perspective we might expect extremely rapid relaxation, because B's interaction destroys A's ability to shrink entropy (in the direction of B's arrow). Alternatively, one might expect that there simply would be *no* solution to the boundary value problem. If indeed shrinking is instantaneous or solutions do not exist, then what was wrong with the argument that suggested a small reduction in τ? Presumably correlations in the "noise" would allow the small $F^{(B)}$ to have large coherent effects.

To decide between these alternatives I have done computer simulations using variations on dynamical systems used to study ergodicity. As will be seen, the effect of one system on the other is not at all traumatic. There is simply a moderate shortening of relaxation times.

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Each system, A and B, is an ideal gas of particles evolving under the cat map [9]. This is a measure preserving map of the unit square: $\phi(x, y) = (x + y, x + 2y) \bmod 1$. A single such system has been used to illustrate conceptual issues and analytic results are available [4,5]. We also use the map $\psi_\alpha(u, v) \equiv (u + \alpha v, v) \bmod 1$. Each point (x_A, y_A) in A has a corresponding one in B, (x_B, y_B). A time step consists of three maps: (1) $\psi_{\alpha/2}$ applied to (x_A, y_B) and (x_B, y_A) separately; (2) ϕ applied to (x_A, y_A) and (x_B, y_B) separately; (3) repeat (1) [10].

In Fig. 1 results are shown for a simulation of 500 pairs of points in which the initial state of A was confinement in a particular 0.1×0.1 box with the same final state for B [11]. Entropies (S) of A and B are shown separately, where $S = -\sum_k \rho_k \log\rho_k$, k labels coarse grains, N = number of points, and $N\rho_k$ = number of points in grain k. Figure 1a is the 0-coupling result. As expected, the boundary condition gives opposing arrows. Relaxation times are both about 5. In Fig. 1b a coupling (α) of 0.2 is used. This conveys the main result of the simulation, the observation that the two arrows *do* persist. What A feels from B is noise, and the effect is to hasten relaxation. For this moderate coupling, all that happens is that relaxation takes about 4 time steps rather than 5. Finally, in Fig. 1c $\alpha = 0.5$, for which the ability of each system to maintain its arrow is clearly compromised.

We next explore whether the entropy changes yield another property of arrows, macroscopic causality. By this I mean that effect follows cause, to be distinguished from microscopic causality, stated, e.g., in terms of field commutators. Defining a test of (macro) causality requires caution. Thus with initial conditions only the effect of a perturbation is *by definition* subsequent. In [5] a consistent test is given by providing macroscopic data (coarse grains) at two times. The system is evolved microscopically from initial to final grains with a particular evolution law, and then again (for the same boundary data) with the same law on all but one time step, at which time some other law is used. With low entropy at both ends there are relatively few phase space points satisfying the boundary conditions. Solution points for perturbed and unperturbed evolutions are in general different. The test of macroscopic causality is whether the *macroscopic* behavior is different before the perturbation, after it, or perhaps both [12]. For our elaborated cat map, perturbation means that on a particular time step, instead of applying ϕ and ψ, another rule is used.

In Fig. 2a an entropic history is shown for uncoupled systems. The perturbation is a faster cat (higher Lyapunov exponent) at time-4 [generated by the matrix [3,2;4,3] (in MATLAB notation)]. The entropy, $S(t)$, in the figure is calculated between t and $t + 1$. To better see the effects, in Fig. 2b we show only the entropy *change* due to the perturbation. For A the major difference occurs at 4, while for B it is at 3, consistent with causality. For uncoupled systems this result is trivial and shows only that our method works. In Fig. 2c, coupling (0.2) is turned on and the same comparison made. Qualitatively causality persists, although the coupling reduces all deviations.

Understanding radiation with opposing arrows is no less in need of a defining framework than our considerations up to now. The language to be used is time-symmetric electrodynamics and the Wheeler-Feynman absorber theory [6]. Classically there is no loss of generality, since

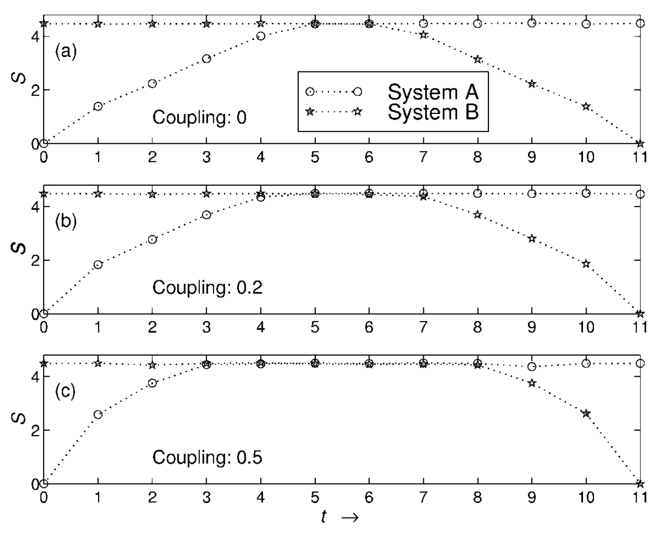

FIG. 1. Entropy, S (log to base e), as a function of time for systems A and B, with opposite thermodynamic arrows. There are 100 course grains in the unit square and each simulation uses 500 points. In (a), (b), and (c) the coupling is 0, 0.2, and 0.5, respectively.

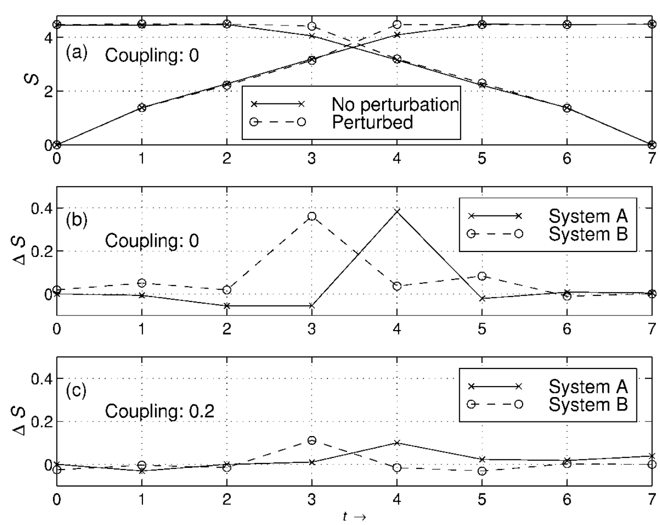

FIG. 2. Entropy (S) and entropy difference (ΔS) due to perturbation. Coarse grains, etc., are as in Fig. 1. The solid lines in (a) are (up to statistics) the same as Fig. 1a. For the dashed lines the system is perturbed at $t = 4$. Entropy is calculated *between* time steps, so for A ($S\uparrow$ for $t\uparrow$) the fact that entropy is nearly unchanged for $t \leq 4$ means that cause follows effect. For B causality implies that changes should be at $t < 4$, which is confirmed in the figure. For clarity, in (b) only the difference is shown. Part (c) shows differences for *coupled* systems ($\alpha = 0.2$). Again causality is evident, but because of coupling-induced relaxation the perturbation does not have so marked an effect.

differences from the standard representation can be eliminated using free fields. Again consider systems A and B, and write the force on a particle in, say, A in terms of the advanced and retarded fields of all particles: $\ddot{x}_i = \sum_{k \neq i}[F_a^{(k)}(x_i) + F_r^{(k)}(x_i)]/2$, where a and r refer to advanced and retarded, respectively, $k \in A \cup B$, and $i \in A$. As before, a low entropy macrostate is given for A at small t, high entropy for large t, and contrarily for B. As in the fourth derivation in [6], we rearrange the sum for $\ddot{x}_i$, but in a new way:

$$\begin{aligned} \ddot{x}_i = & \sum_{k \in A'} F_r^{(k)} + \frac{1}{2} \sum_{k \in A} [F_a^{(k)} - F_r^{(k)}] \\ & + \sum_{k \in B} F_a^{(k)} + \frac{1}{2} \sum_{k \in B} [F_r^{(k)} - F_a^{(k)}] \\ & - \frac{1}{2} [F_a^{(i)} - F_r^{(i)}], \end{aligned} \tag{1}$$

where the prime on A' means $k \neq i$. The term $[F_r^{(i)} - F_a^{(i)}]/2$ was found by Dirac to give radiation reaction. We rewrite Eq. (1) in obvious notation,

$$\ddot{x}_i = F_r^{(A')} + F_a^{(B)} + f_{\text{rad react}} + E_h , \tag{2}$$

where $E_h \equiv \frac{1}{2}\sum_k \sigma_k(F_a^{(k)} - F_r^{(k)})$, $\sigma_k = 1\ (-1)$ for $k \in A\ (B)$, and is homogeneous (sourceless). These manipulations reduce to the Wheeler-Feynman calculation when B is empty. They argued that their E_h was zero, based on the randomness of the particles (this is the absorber theory). Their explanation of why one should not reverse the development (to get advanced interactions, etc.) is statistical. In particular, they suppose that the source (i) suffers an acceleration. When only retarded fields are used, they "had no particular effect on the acceleration of the source" ([6], p. 170). On the other hand, with a time reversed representation there is coherence in the source, leading to unlikely behavior. In their words, "As the result of chaotic motion going on in the absorber, we see each one of the particles receiving at the proper moment just the right impulse to generate a disturbance which converges upon the source at the precise instant when it is accelerated." As to choosing a representation, they say "Small *a priori* probability of the given initial conditions provides our only basis on which to exclude such phenomena."

In our case, for A the unlikely states come at the beginning; for B at the end. Therefore there should be a different expansion for each. That is just Eq. (2). The key point is that *this is still consistent with electrodynamics*. The field, E_h, apparently more complicated (because of σ_k) than the one vanishing in [6], is nevertheless *sourceless*.

So it is mathematically consistent for E_h to vanish. Can arguments like those of Wheeler and Feynman be applied showing that it does in fact vanish? Since A and B are only weakly coupled this is reasonable. But the argument could fail if the weak-in-magnitude forces managed peculiar coherences. It is the point of the numerical simulations reported above that such correlations do not occur. Those simulations dealt with the conceptual issues of opposing arrows and although we now have more complex interactions the conceptual statistical mechanics issues are the same.

Assuming then that E_h vanishes, what would A see when looking at B? A's images arise from the advanced field coming from his future. Successive images present earlier times, as measured by the causal-entropic arrow of B. Indeed eggs uncrack.

Can this yield causal paradoxes? Can B close the windows and avoid getting the carpet wet [13] because A tells him it's raining in? In principle such signals could be exchanged and paradoxes avoided as discussed in [14]. It is also possible that such an interaction would violate the small coupling assumption. At this stage I draw no conclusion.

Focusing on situations where the small coupling assumption *is* valid, we arrive at the real possibility that at some distance from us there are regions of opposite running thermodynamic arrows. The extended absorber theory indicates that we would see them at an era later than our own, later by the time for light travel to them. How could those regions have arisen? One possibility is that our Universe has a big crunch in the (our) future and that the other-arrow regions are survivors coming the other way. If the bang-to-crunch time is long, they would be further away from their start, hence less likely to have luminous matter. As such, we would pretty much not see them electromagnetically (but not for the reasons in [1]). On the other hand, there would be no suppression of gravity. According to this description, this material has all the properties now attributed to dark matter.

Based on what was learned from the simulations above, there is no bar to such objects being within our galaxy [15]. Specifically, the radiation from them could be noticeable, but sufficiently weak as not to overwhelm our normal thermodynamics. A dead star at 50 pc should satisfy this [16]. However, this conclusion is not firm, since with a signal (which gravitational lensing may be) there arises the issue of whether the small coupling assumption is satisfied (cf. the causal paradoxes). I remark further that these relics might be distinguished from other dark matter candidates if some hint of composition could be gleaned (e.g., higher metallicity).

Although I have refrained from claiming definite answers to some of the important questions it is, nevertheless, clear that at the conceptual level further progress is possible. In particular, the question of whether signaling is consistent with weak coupling can be approached by simulations analogous to but more complicated than what I report above.

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Theoretical Physics and the Petrie Fund of the Technion and to the NSF (PHY 97 21459) for support.

[1] N. Wiener, *Cybernetics* (M.I.T. Press, Cambridge, MA, 1961), 2nd ed., p. 34.
[2] *The Nature of Time,* edited by R. Flood and M. Lockwood (Blackwell, Oxford, 1986). M. Dummet (p. 153) considers conflicting arrows a stepping stone to “incoherence,” while R. Penrose (p. 41) uses causal paradoxes to argue against the proposal that the thermodynamic arrow is related to the cosmological one (cf. [7]).
[3] L. S. Schulman, Phys. Rev. D **7**, 2868 (1973).
[4] L. S. Schulman, J. Stat. Phys. **16**, 217 (1977).
[5] L. S. Schulman, *Time’s Arrows and Quantum Measurement* (Cambridge University Press, Cambridge, 1997).
[6] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945); **21**, 425 (1949).
[7] T. Gold, Am. J. Phys. **30**, 403 (1962).
[8] The coupling in the model below has a significant effect on the motion. It is not “ultrasmall.”
[9] V. I. Arnold and A. Avez, *Ergodic Problems of Classical Mechanics* (Benjamin, New York, 1968).
[10] The alternation enhances apparent time reversal symmetry. ϕ itself is *not* time symmetric, but for our purposes this is unimportant. What matters is that Lyapunov exponents for ϕ and ϕ^{-1} are equal (making Fig. 1a symmetric). Thus [5] microscopic T violation alone does not provide a thermodynamic arrow.
[11] As in [5], solution points were found by giving $\sim N \times$ (number of grains) points the correct *initial* conditions and discarding all but the N points that found their way into the desired final grain. This method relies on the lack of interactions *within* A or B. It yields a random sample of the time-symmetric true solution set ($\Delta_f \cap \Phi^T \Delta_i$, with Φ the combined A-B map, $\Delta_i = \Delta_{Ai} \otimes \Delta_{Bi}$, etc.).
[12] Reader response suggests that this definition [5] is novel. To me it seems a take off on P. A. M. Dirac, Proc. R. Soc. London A **167**, 148 (1938), where future boundary conditions suppress runaway solutions and establish *near*-causality. (Dirac calls this “the most beautiful feature of the theory.”) Future conditioning allows comparison of perturbed and unperturbed motion without making causality circular.
[13] A sees rain coming in B’s window (at, say, 8 a.m. B time). With an hour’s delay A informs B. B receives the signal at 7 and closes the window before the rain starts (say at 7:30).
[14] L. S. Schulman, Am. J. Phys. **39**, 481 (1971); A. Peres and L. S. Schulman, Phys. Rev. D **5**, 2654 (1972). In these articles a paradox is resolved by finding a solution despite seemingly contradictory initial conditions. The present case is more problematic. The conflict must be set up as a 2-time boundary condition and then a solution found.
[15] This proposal does not depend on the cosmological scenario given above. “They” could be there for the same unknown reason we are here. With a temporal cosmological principle, preferring neither arrow, there is no reason to rule this out.
[16] A further possibility is regions where opposite-arrow systems interact strongly and the arrows mutually destroyed. Perhaps this is *usually* the case, so that most matter in the Universe resembles Fig. 1c. For gravitational reasons I expect this matter to be part of galaxies, but there may need to be more segregation than the 50 pc mentioned in the text in order to prevent galactic-dynamical stirring from bringing different-arrow subsystems into excessive contact.