Phase Defects in Self-Focusing of Ultrashort Pulses

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We report the numerical observation of edge-type phase defects which occur during the self-focusing of ultrashort pulses. We identified two distinct kinds of defects, formed at the temporal or spatial edges of the pulse. We show the effect of phase defect creation on the dynamics and the eventual arrest of the self-focusing process. Phase defects often lead to the inversion of the phase fronts curvature, transforming contracting pulses into expanding pulses.

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Self-focusing of light beams inside a nonlinear Kerr medium was investigated extensively in the early days of nonlinear optics [1]. A high power light beam may focus by creating for itself a lenslike index profile via the Kerr effect. When light is free to diffract only along one transverse direction, as in a planar waveguide, self-focusing can lead to the formation of a stable self-trapped light beam—a spatial soliton [2]. However, stable self-trapping does not occur in the case of two transverse dimensions. When the beam has a power above a critical value, it collapses to a very small size, usually resulting in material damage [3]. Mathematically, the solution of the standard paraxial wave equation in this case predicts collapse into a singular point. This collapse is avoided when the nonlinear index saturates [4], in the presence of nonlinear absorption [5], or when nonparaxial corrections were taken into account [6].

In contrast to a continuous light beam, the self-focusing dynamics of short light pulses has been investigated only lately. For such pulses, dispersion plays a role very similar to diffraction [2,7]. A short pulse broadens in space by diffraction and in time by dispersion. The roles of dispersion and diffraction are even more similar in the case of anomalous dispersion. The evolution of optical pulses in a planar waveguide with anomalous dispersion is equivalent to self-focusing of a continuous beam in three dimensions [8]. According to standard analysis, both cases are described by the same $1 + 2$ nonlinear Schrödinger equation (NLSE), which predicts a collapse to a singular point. A high intensity short pulse propagating in an anomalous dispersive medium is therefore expected to collapse not only in space, but also in time. It has been shown that in the normal dispersion regime, temporal pulse broadening tends to counteract the spatial self-focusing, leading often to pulse splitting [9,10].

The modeling of self-focusing as described above is usually performed using the NLSE, which is derived using the slowly varying amplitude approximation [11]. This approximation is questionable in both cases: when analyzing the collapse of ultrashort pulses which contain just a few optical cycles and when modeling propagation of beams of a wavelength scale. Using this approximation is even more doubtful in the case of spatiotemporal focusing where the two limits are reached. Recently, several groups studied numerically the propagation of light pulses in nonlinear media by solving directly Maxwell's equations [12–16]. These methods do not use the standard approximations. As the electric field equations are solved directly, complete information on the instantaneous electric field is retained. High harmonics and waves propagating backwards, as well as in all other directions, are included in this model, unlike in the common beam propagation schemes [17]. In this paper, we use such a model in order to show that phase evolution, and in particular the appearance of phase singularities, are crucial for understanding the stability and dynamics of self-focusing of ultrashort pulses.

Using this scheme, it has been shown [12,13] that spatiotemporal symmetric pulses close to the critical power appear to be stable for a significant distance [18]. These pulses resemble light bullets—pulses that maintain their spatial and temporal structure [8]. Pulses that have slightly higher powers start to contract, but then they stop and disperse. Even stronger pulses often split into two weaker pulses. We examined these solutions and found that these effects are connected with the appearance of phase singularities, which explains this dynamic behavior.

The simulations use a finite difference scheme in the time domain, propagating a given field by small steps in time, as described by Goorjian *et al.* [12]. The time convolution integrals involved in the linear dispersion are translated into differential equations, hence, information has to be kept only one step backward. The medium is modeled as a two-level system; all orders of dispersion are included and there is no need to add them artificially. In order to induce anomalous dispersion in this simplified model, the resonance is assumed to be below the carrier frequency, and ε_∞ is nonzero. We assumed optical period of 5.15 fs, which corresponds to a wavelength of 1.55 μm in vacuum. We ran the simulations on a grid of 2000×300 cells of the size 8×50 nm each. The longer side was along the propagation direction and the other transverse direction was where diffraction took place. Since all the simulations involved the propagation

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of pulses, we continuously shifted the fields to keep the pulse centered in the grid. This is possible as long as dispersion effects do not broaden the pulses beyond the length of the grid. The parameters used in the modeling of the dispersion are the same as used in Ref. [18]. A run of 25 000 steps of 2.7×10^{-17} s on a Pentium 200-MHz processor took about 10 hours. On some of the simulations, absorption was added to the edges of the grid in order to avoid edge reflections. It did not change the nature of the phenomena reported here.

Some basic features of nonlinear short-pulse propagation can be observed in Fig. 1, where the pulse propagates from the bottom to the top. Electric field values are shown as gray levels, gray represents zero value, and positive (negative) values are shown as bright (dark) shades. Also shown are lines of zero field, which highlight the phase fronts of the propagating pulses. Only a small part of the simulation grid around the center of the pulse is actually shown in all figures. At low energy, the nonlinearity could not hold the pulse, the phase front curvature becomes positive (Fig. 1a), and the pulse eventually diffracts and disperses. At higher pulse energies, this process occurs more slowly. Above a certain critical power, the nonlinearity causes the phase front curvature to become negative and the pulse starts to shrink in size (Fig. 1b). Other features are also observed, including the formation of a third harmonic companion pulse that is generated on the entrance of the main pulse into the nonlinear medium [16]. This pulse is observed to propagate faster than the main pulse due to anomalous dispersion (Fig. 1c).

At these high intensities, the phase fronts exhibited interesting behavior, which was linked to the dynamics of the evolving pulses. Since the pulse center experiences a higher refractive index due to its higher intensity, it propagates with a smaller group velocity than the low-intensity wings, resulting in a negative curvature of the phase fronts. This is the basic process that leads to the deformation of the pulse phase fronts and to self-focusing. Similarly, the difference between the velocity of the slow pulse center and its faster head and tail leads to deformation of the carrier wave and to the creation of new frequencies. We found that the transverse and longitudinal deformations of the phase fronts often led to creation of edge defects in the phase fronts [19].

We identified two distinct kinds of defect creation during the self-focusing process. In the first kind, the difference in the group velocity between the pulse center and its head and tail stretches and squeezes the leading and trailing phase fronts, respectively. This deformation is clearly seen in Fig. 1b. Eventually, a stretched phase front may split locally into two, or a squeezed phase front can be broken. Consider the simulation results shown in Fig. 2 [20]. A time-space symmetric pulse (a pulse that diffracts and disperses at the same rate) is launched close to the critical power. The phase fronts acquire a small negative curvature, leading to a slight contraction. More dramatically, the phase fronts in the leading edge are stretched, while those in the trailing edge are compressed. Fig. 2a shows that an edge dislocation pair was created in the tail of the pulse due to the squeezing out of one crest of the wave. The two defects can be seen in Figs. 2b–2d

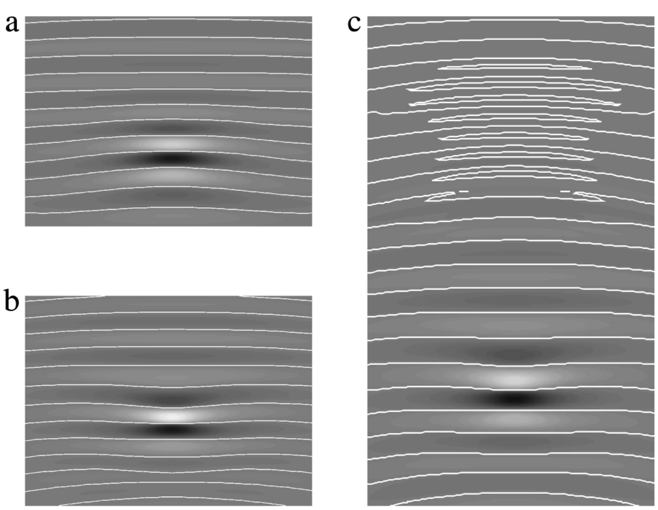

FIG. 1. The pulse phase fronts contain information about its dynamics. (a) A diverging pulse with a positive phase front curvature. (b) A converging pulse during self-focusing with a negative phase front curvature. (c) A propagating pulse with its companion third harmonic pulse that was generated when the pulse entered the nonlinear medium. This frame was taken at $t = 160$ fs after the entrance of the main pulse into the nonlinear medium.

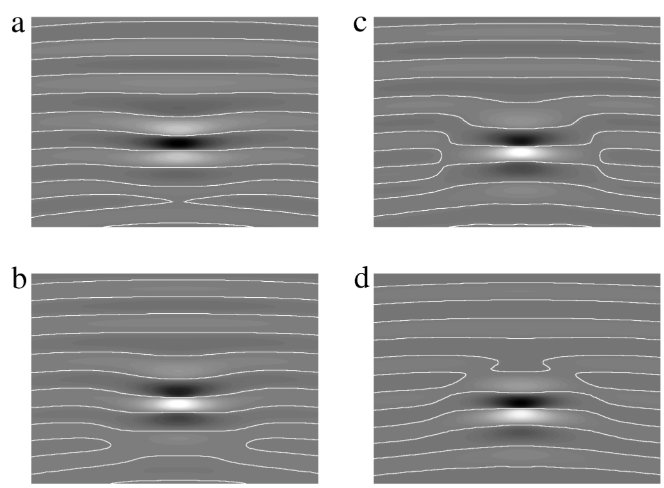

FIG. 2. Phase defects of the first kind, created behind the pulse. (a) The pulse pushes back, squeezing one crest, resulting in a pair of phase defects ($t = 192$ fs). (b),(c): The phase defect pair overtakes the pulse from both sides due to the fact that the phase velocity is larger than the pulse group velocity. While the defects are propagating, they flip the curvature of the phase fronts that they pass ($t = 216$ fs and $t = 245$ fs). (d) The two defects meet at the front of the pulse and annihilate each other ($t = 264$ fs).

as they overtake the pulse from both sides, since the phase velocity is larger than the pulse group velocity. While the two defects are sliding around the pulse, they replace each wave front going out of the pulse center with the wave front before. As a result, the negatively curved phase fronts become positively curved. Note, for example, in Fig. 2c that the phase fronts in the front of the pulse are concave, while behind the dislocation the fronts are convex. When the two defects meet in the front side of the pulse, they annihilate each other. The curvature of all wave fronts has been now inverted, the self-focusing process stops, and the pulse starts to expand.

The difference between the group velocity of the pulse center and its wings could cause defects of a second kind to be created. A row of phase fronts close to the pulse center tears apart and jumps one wavelength backwards. This jump results in two pairs of defects, one near the head and one near the tail of the pulse. The local curvature of the phase fronts changes sign and the collapse is stopped temporarily. This is exemplified in the simulation shown in Fig. 3. The pulse has the same energy as in Fig. 2, but it is now asymmetric—it is three times wider, and therefore it diffracts slower. The difference between the group velocities of the pulse center and its wings causes a strong deformation of the phase fronts (Fig. 3b) until they eventually tear along the edges of the pulse creating several pairs of defects (Fig. 3c). All these defects eventually annihilate by moving to the front of the pulse: by then, the entire pulse is practically shifted one wavelength backwards. This shift leads to an inversion of curvature, and the concave wave fronts of Fig. 3a are all inverted in Fig. 3d. Self-focusing has been arrested, and the pulse starts to broaden. If the intensity is high enough, the process could repeat itself (Figs. 3e, 3f) until the peak power of the diverging field has been reduced sufficiently.

It is worth noting that similar defects appeared in many of our simulations, for various optical power and spatiotemporal shapes. In particular, we found that defects were created also for pulses of considerably higher power, that eventually collapsed, but their dynamics was fast and abrupt. We believe that phase defects are ubiquitous in self-focusing.

Phase defects are not new in optics. Screw dislocations have been investigated extensively. They are encountered when a two dimensional spatial phase front has a singularity, known in optics as an optical vortex [21,22]. The field at the defect location is zero due to the phase ambiguity around it. The defects are accompanied by a canonical angular momentum of their field [23]. This angular momentum is usually conserved, which means that for every positively charged vortex that is created, there will be a negative one too.

In contrast with screw dislocations, edge defects appear as a bifurcation or a sudden end of a phase front along the transverse spatial dimension, a phenomenon known as an edge dislocation [19]. Edge dislocations are also formed in pairs because of the conservation of the field canonical angular momentum. All optical defects that have been studied to date have occurred in the two dimensional spatial plane of light beams. To the best of our knowledge, this is the first numerical observation of a spontaneous creation of defects in the spatiotemporal domain.

We also found that edge defects may cause pulses to split. The reason for splitting is the zero electric field in the defects center. Defects of the first kind, that are created in front or behind the pulse, can split it in the time domain. Usually a pulse will split into two pulses, one main pulse where most of the energy remains and a smaller pulse ahead of the main pulse. After splitting, the pulses move apart due to the different frequencies they contain and the group velocity dispersion. A phase jump of the second kind, as shown in Fig. 3, may divide the pulse into three parts—one main central pulse and two weaker satellite pulses on both sides.

In conclusion, we reported numerical observation of spatiotemporal defects of the electric field, which are created during self-focusing of short optical pulses in two dimensions. Using a finite-difference scheme, we

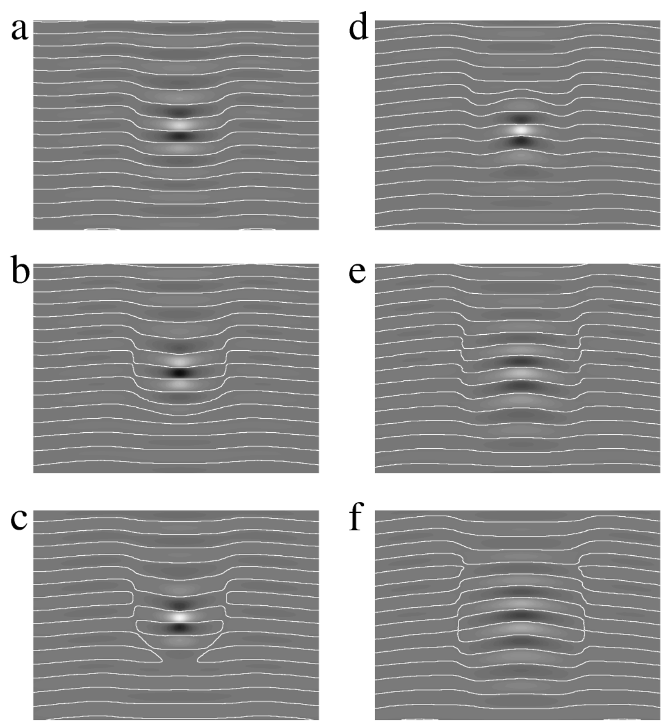

FIG. 3. Phase defects of the second kind, created at the pulse edges. (a),(b) Phase fronts are deformed due to the slower velocity of the pulse center ($t = 480$ fs and $t = 533$ fs). (c) Two pairs of defects are formed as phase fronts are torn. Another defect pair of the first kind is created behind the pulse ($t = 560$ fs). (d) All the defects advance to the pulse front due to their larger phase velocity and are annihilated there. Note the inversion of the local phase front curvature and the arrest of the collapse ($t = 640$ fs). (e),(f) Secondary defects of the second kind are created at the pulse outer regions ($t = 720$ fs and $t = 747$ fs).

integrated directly Maxwell's equations for the electric field. We found that while phase fronts are deformed during self-focusing, their continuity is not necessarily preserved. Edge defects are created when the phase fronts are strongly deformed. These defects affect the dynamics of self-focusing, leading often to its arrest. In some cases, pulse splitting in the temporal or transverse spatial directions may occur. It is conceivable that phase defects of the second kind could also play a part in continuous wave self-focusing in bulk media. This has to be verified in additional studies.

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