Exactly Soluble Model for Umklapp Scattering at Quantum Hall Edges

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We consider the low-energy, long-wavelength excitations of a reconstructed quantum Hall edge where three branches of chiral one-dimensional edge excitations exist. We find that, in addition to forward scattering between the three edge-excitation branches, Coulomb interaction gives rise to an Umklapp-type scattering process that cannot be accounted for within a generalized Tomonaga-Luttinger model. We solve the theory including Umklapp processes exactly in the long-wavelength limit and calculate electronic correlation functions.

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Two-dimensional (2D) electron systems subject to perpendicular magnetic fields have low-lying excitations that are localized at the sample boundary [1]. At certain values of the filling factor ν when the quantum Hall (QH) effect occurs, the bulk of the 2D system turns out to be incompressible [2], and the edge excitations comprise the *only* low-lying excitations present.

The detailed electronic structure at the edge of QH systems depends sensitively on the interplay between the external potential confining the electrons to the finite sample, electrostatic repulsion, as well as exchange and correlation effects. For an ultimately sharp [3] edge, a single branch of chiral one-dimensional (1D) excitations is predicted to exist when the filling factor $\nu = 1/m$, where m is an odd integer [4]. In that case, the dynamics of edge excitations can be described [5] using a Tomonaga-Luttinger (TL) model [6] with only the right-moving [7] degrees of freedom present. However, for a confining potential that is just not sharp enough to stabilize a single-branch edge, a different configuration is realized where a lump of electron charge is separated from the bulk of the QH sample [8,9]. Such a *reconstructed* edge [10–12] supports three branches of chiral 1D edge excitations, two right-moving and one left-moving. For even weaker confining potential, further reconstructions occur, leading to a proliferation of edge-excitation modes [13]. The microscopic structure of a very smooth edge is dominantly determined by electrostatics, which favors a phase separation of the 2D electron system at the edge into a series of alternating compressible and incompressible strips [14].

Effective TL theories describing single-branch and multibranch QH edges predict Luttinger-liquid behavior [5], i.e., power laws governing the energy dependence of electronic correlation functions. The characteristic exponents of these power laws depend, in general, on details of the microscopic edge structure. However, in the absence of coupling between different chiral edge branches or, in some cases, due to disorder effects [15], power-law exponents turn out to be universally dependent on the bulk filling factor. At present, microscopic details of the edge structure that is realized in experiment [16] are not fully known. To facilitate a realistic comparison between theory and experiment, it is necessary to study the low-lying edge excitations of reconstructed and smooth edges and investigate the effect interactions have on the Luttinger-liquid power-law exponents when more than one branch of edge excitations is present. Most importantly, it turns out that edges having at least three chiral branches of 1D edge excitations can support a new kind of scattering process which does not conserve particle number in each branch separately. This new interaction process cannot be accounted for within an effective TL-model description. Here we study an exactly soluble model for the new scattering process and determine its effect on the Luttinger-liquid behavior of QH edges.

We focus on the edge of a QH sample at $\nu = 1$ that has undergone reconstruction such that three branches of edge excitations are present. To be specific, we choose the Landau gauge where lowest-Landau-level (LLL) basis states $\chi_k(x,y) = \Phi_k(y)\exp\{ikx\}/\sqrt{L}$ are labeled by a 1D wave vector k. Here, $\ell = \sqrt{\hbar c/|eB|}$ denotes the magnetic length, L is the edge perimeter, and $\Phi_k(y) = \exp\{-(y - k\ell^2)^2/(2\ell^2)\}/\sqrt{\pi^{1/2}\ell}$. In the absence of interactions between different edge branches, the ground state will be a generalized Fermi-sea state that is a Slater determinant of LLL basis states whose wave-vector label satisfies $k \le k_{\mathrm{F}}^{(R)}$ or $k_{\mathrm{F}}^{(W)} \le k \le k_{\mathrm{F}}^{(B)}$. The Fermi "surface" consists of three (Fermi) points $k_{\mathrm{F}}^{(R)} < k_{\mathrm{F}}^{(W)} < k_{\mathrm{F}}^{(B)}$. As in Tomonaga's approach to interacting 1D electron systems [6], long-wavelength electronic excitations at the reconstructed edge can be identified according to which Fermi point they belong to. This makes it possible to rewrite the long-wavelength part of the electron operator as follows:

$$\begin{aligned}\psi(x,y) = {} & \Phi_{k_{\mathrm{F}}^{(R)}}(y)e^{ik_{\mathrm{F}}^{(R)}x}\psi^{(R)}(x) \\ & + \Phi_{k_{\mathrm{F}}^{(W)}}(y)e^{ik_{\mathrm{F}}^{(W)}x}\psi^{(W)}(x) \\ & + \Phi_{k_{\mathrm{F}}^{(B)}}(y)e^{ik_{\mathrm{F}}^{(B)}x}\psi^{(B)}(x)\,. \end{aligned} \tag{1}$$

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The operator $\psi^{(R,W,B)}(x)$ creates an electron belonging to the chiral 1D edge branch labeled R, W, B, respectively. The original 2D interacting Hamiltonian specializes, in the low-energy limit, to an effective 1D Hamiltonian involving scattering processes of electrons close to the three Fermi points. We can classify these interaction processes in terms of a generalized g-ology scheme that is familiar from the treatment of conventional interacting 1D electron systems [17] (see Fig. 1). Forward scattering and backscattering [18] can be accounted for within a generalized TL model [9]:

$$H_{\mathrm{TL}} = \frac{1}{2L} \sum_{q \neq 0} [\vec{\varrho}_q]^\dagger (2\pi\hbar \mathcal{K} + \mathcal{V}_q) \vec{\varrho}_{-q} . \qquad (2)$$

Here we defined the vector $\vec{\varrho}_q = (\varrho_q^{(B)}, \varrho_q^{(R)}, \varrho_q^{(W)})$ of density fluctuations at the three Fermi points, and matrices $\mathcal{K} = \mathrm{diag}(v_{\mathrm{F}}^{(B)}, v_{\mathrm{F}}^{(R)}, v_{\mathrm{F}}^{(W)})$ and

$$\mathcal{V}_q = \begin{pmatrix} g_4^{(BB)}(q) & g_4^{(BR)}(q) & g_2^{(BW)}(q) \\ g_4^{(BR)}(q) & g_4^{(RR)}(q) & g_2^{(RW)}(q) \\ g_2^{(BW)}(q) & g_2^{(RW)}(q) & g_4^{(WW)}(q) \end{pmatrix} . \qquad (3)$$

It is straightforward to diagonalize H_{TL} by a transformation $\vec{\varrho}_q = \mathcal{M}_q \vec{\rho}_q$. The long range of Coulomb interaction renders the matrix $\mathcal{M}_q$ universal in the limit $q \to 0$ [9], and we find as normal modes (a) the classical [1] edge-magnetoplasmon mode, $\rho^{(\mathrm{emp})} = \varrho^{(B)} + \varrho^{(R)} + \varrho^{(W)}$, which is right-moving, and (b) two linearly dispersing neutral modes, a right-moving one given by $\rho^{(\mathrm{rn})} = (\varrho^{(B)} - \varrho^{(R)})/\sqrt{2}$, and the left-moving neutral mode $\rho^{(\mathrm{ln})} = (\varrho^{(B)} + \varrho^{(R)} + 2\varrho^{(W)})/\sqrt{2}$.

In addition to forward scattering and backscattering, another interaction process exists which has not been noticed previously (see Fig. 1). The full effective 1D Hamiltonian describing edge excitations at a reconstructed edge is actually given by $H = H_{\mathrm{TL}} + H_U$ with

$$H_U = \iint dx\, dx'\, V_U(x - x') \times \{[\psi^{(R)}(x)]^\dagger [\psi^{(B)}(x')]^\dagger \psi^{(W)}(x') \psi^{(W)}(x) \times e^{iD[(x-x')/2] - i\delta[(x+x')/2]} + \mathrm{H.c.}\} . \qquad (4)$$

Here we have used the abbreviations $D = k_{\mathrm{F}}^{(B)} - k_{\mathrm{F}}^{(R)}$ and $\delta = k_{\mathrm{F}}^{(B)} + k_{\mathrm{F}}^{(R)} - 2k_{\mathrm{F}}^{(W)}$. As the scattering process represented by H_U converts two electrons from the left-moving W branch into electrons belonging to the right-moving R, B branches (and vice versa), it is reminiscent of Umklapp scattering which is important in lattice models for conventional 1D electron systems near half filling [19]. That this analogy reaches quite far can be seen from the fact that there is a commensuration issue for the novel Umklapp process at QH edges. It is most relevant if $k_{\mathrm{F}}^{(B)} - k_{\mathrm{F}}^{(W)} = k_{\mathrm{F}}^{(W)} - k_{\mathrm{F}}^{(R)}$, and the parameter δ arises naturally as a measure for the deviation from perfect commensuration [20] (see below.)

Umklapp processes do not conserve particle number in each edge branch separately. Therefore, H_U cannot be written in terms of a TL model. However, using the bosonization identity [21,22] for 1D fermionic operators,

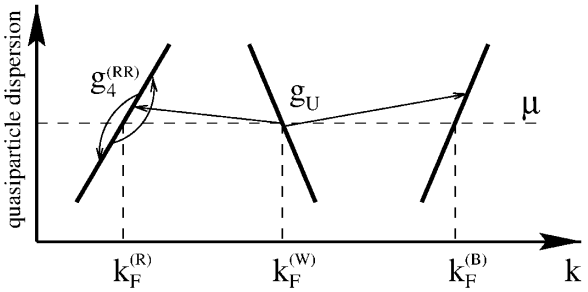

FIG. 1. Schematic depiction of quasiparticle dispersion and interaction processes at a reconstructed QH edge. At low energies, only interaction processes involving electrons close to the Fermi points $k_{\mathrm{F}}^{(R)}$, $k_{\mathrm{F}}^{(W)}$, and $k_{\mathrm{F}}^{(B)}$ are important. As examples, we show forward scattering within the R branch (coupling constant $g_4^{(RR)}$) and the Umklapp process (g_U).

$$\psi^{(R)}(x) = 1/\sqrt{L} \; : \exp[i\phi^{(R)}(x)] : , \qquad (5a)$$

$$\psi^{(W)}(x) = 1/\sqrt{L} \; : \exp[-i\phi^{(W)}(x)] : , \qquad (5b)$$

$$\psi^{(B)}(x) = 1/\sqrt{L} \; : \exp[i\phi^{(B)}(x)] : , \qquad (5c)$$

where $: \cdots :$ symbolizes normal ordering, and

$$\phi^{(\alpha)}(x) = i \frac{2\pi}{L} \sum_{q \neq 0} \frac{e^{-iqx}}{q} \varrho_q^{(\alpha)} , \qquad (6)$$

with $\alpha \in \{R, W, B\}$, it is possible to rewrite H_U entirely in terms of bosonic degrees of freedom:

$$H_U = 2\Lambda^2 g_U \int dx \cos[\phi^{(R)}(x) + \phi^{(B)}(x) + 2\phi^{(W)}(x) + \delta x] . \qquad (7)$$

Both the coupling constant g_U and the incommensuration parameter δ can be determined from a microscopic calculation [23], and $\Lambda \lesssim (D\ell^2)^{-1}$ is a physical ultraviolet cutoff. From Eq. (7), it is immediately obvious that H_U couples only to one of the three normal modes of H_{TL}, namely, the left-moving neutral mode $\rho^{(\mathrm{ln})}$. Hence, the full Hamiltonian of low-energy excitations at a reconstructed edge is the sum of three terms, $H = H_{\mathrm{emp}} + H_{\mathrm{rn}} + H_{\mathrm{ln}}$, where H_{emp} and H_{rn} describe the dynamics of free chiral bosons that are associated with the edge-magnetoplasmon and right-moving neutral modes, respectively, and

$$H_{\mathrm{ln}} = \int dx \left\{ \frac{\hbar v_{\mathrm{ln}}}{4\pi} [\partial_x \phi^{(\mathrm{ln})}(x)]^2 + 2\Lambda^2 g_U \cos[\sqrt{2}\, \phi^{(\mathrm{ln})}(x) + \delta x] \right\} . \qquad (8)$$

Here, $\phi^{(\mathrm{ln})}(x)$ is defined in terms of $\rho^{(\mathrm{ln})}$ as in Eq. (6), and v_{ln} is the velocity of the left-moving neutral mode.

Note that Eq. (8) does *not* correspond to the Hamiltonian of the familiar sine-Gordon model; but rather to

a chiral version of it. To be able to evaluate electronic correlation functions, we employ a refermionization technique [24] that has been used before to study the effect of disorder on transport at hierarchical QH edges [25]. We introduce an auxiliary *ghost* field $\eta(x)$ that has the same chirality as the real bosonic field $\phi^{(\mathrm{ln})}(x)$ and whose dynamics is given by the first term of H_{ln} in Eq. (8):

$$H_\eta = \frac{\hbar v_{\mathrm{ln}}}{4\pi} \int dx [\partial_x \eta(x)]^2. \tag{9}$$

The Hamiltonian $H' = H_{\mathrm{ln}} + H_\eta$ is then equivalent to the bosonized representation of a model of chiral 1D spin-1/2 fermions $\Psi = (\Psi_+, \Psi_-)$ subject to a magnetic field $\vec{h}(x)$ that couples to the (pseudo)spin [26] degrees of freedom,

$$H' = \int dx\, \Psi^\dagger(x) \{ i\hbar v_{\mathrm{ln}} \partial_x 1 + \vec{h}(x) \cdot \vec{\sigma} \} \Psi(x), \tag{10a}$$

provided we define $\vec{h}(x) = \Lambda g_U (\cos[\delta x], \sin[\delta x], 0)$ and $\Psi_\pm(x) = 1/\sqrt{L} : \exp\{ i[\eta(x) \pm \phi^{(\mathrm{ln})}(x)]/\sqrt{2} \} :$. (We denoted the vector of Pauli matrices by $\vec{\sigma}$.) The introduction of the ghost field $\eta(x)$ turns out to be favorable because it is possible to calculate physical observables more easily in the refermionized representation of H' than in the original bosonic theory with H_{ln}. Note that real physical observables do not depend on the auxiliary field $\eta(x)$. In Fourier space, the Hamiltonian H' reads

$$H' = -\hbar v_{\mathrm{ln}} \sum_{ks} k c^\dagger_{ks} c_{ks} + \Lambda g_U \sum_k [c^\dagger_{k-\delta,+} c_{k,-} + \mathrm{H.c.}], \tag{10b}$$

and is easily diagonalized, yielding

$$H' = \sum_{ks} \{ -\hbar v_{\mathrm{ln}} k + s\Delta/2 \} \varphi^\dagger_{ks} \varphi_{ks}. \tag{10c}$$

The "Zeeman splitting" induced by the fictitious magnetic field $\vec{h}(x)$ is $\Delta = 2\Lambda g_U [\sqrt{1+\xi^2} - \xi]$, where $\xi = \hbar v_{\mathrm{ln}} \delta / (2\Lambda g_U)$ is a measure of the incommensuration. In the ground state, the free fictitious fermions φ_{ks} form two Fermi seas, one for each spin direction, having different Fermi wave vectors due to the Zeeman splitting.

Having diagonalized H', we are now in the position to calculate dynamic correlation functions for adding electrons at a reconstructed edge. We consider the real-time correlation functions

$$\mathcal{G}^{(\alpha)}(x,t) = \langle \psi^{(\alpha)}(x,t) [\psi^{(\alpha)}]^\dagger(0,0) \rangle, \tag{11}$$

where $\alpha \in \{R, W, B\}$. Within the bosonized representation [Eqs. (5)] of fermionic operators, the correlation functions $\mathcal{G}^{(\alpha)}(x,t)$ factorize into a product of correlation functions in each of the three normal modes of H_{TL}:

$$\mathcal{G}^{(\alpha)}(x,t) = \Lambda \mathcal{G}^{(\alpha)}_{\mathrm{emp}}(x,t) \mathcal{G}^{(\alpha)}_{\mathrm{rn}}(x,t) \mathcal{G}^{(\alpha)}_{\mathrm{ln}}(x,t), \tag{12a}$$

$$\mathcal{G}^{(\alpha)}_\beta(x,t) = \langle e^{-i\lambda^{(\alpha)}_\beta \phi^{(\beta)}(x,t)} e^{i\lambda^{(\alpha)}_\beta \phi^{(\beta)}(0,0)} \rangle, \tag{12b}$$

with $\beta \in \{\mathrm{emp}, \mathrm{rn}, \mathrm{ln}\}$. The coefficients $\lambda^{(\alpha)}_\beta$ can be read off from the matrix $\mathcal{M}_q$ that relates the density fluctuations at the R, W, B branches to the normal modes of H_{TL}. As the edge-magnetoplasmon mode and the right-moving neutral mode are free bosons, the calculation of $\mathcal{G}^{(\alpha)}_{\mathrm{emp}}(x,t)$ and $\mathcal{G}^{(\alpha)}_{\mathrm{rn}}(x,t)$ is standard [22], yielding

$$\mathcal{G}^{(\alpha)}_\beta(x,t) = [\Lambda(x - t v_\beta)]^{-[\lambda^{(\alpha)}_\beta]^2}, \tag{13}$$

with $\beta \in \{\mathrm{emp}, \mathrm{rn}\}$. However, the left-moving neutral mode is *not* free due to Umklapp scattering, and the calculation of $\mathcal{G}^{(\alpha)}_{\mathrm{ln}}(x,t)$ is nontrivial. Now our refermionized representation turns out to be useful because we can calculate $\mathcal{G}^{(W)}_{\mathrm{ln}}(x,t)$ *exactly* using the identity

$$\mathcal{G}^{(W)}_{\mathrm{ln}}(x,t) = \Lambda^{-2} \langle \Psi^\dagger_+(x,t) \Psi_-(x,t) \Psi^\dagger_-(0,0) \Psi_+(0,0) \rangle. \tag{14}$$

An elementary calculation in the representation of the free fermions φ_{ks} yields the particle-hole correlation function for fictitious fermions shown on the right-hand side of Eq. (14). The leading term in the long-time, large-distance limit turns out to be a constant,

$$\mathcal{G}^{(W)}_{\mathrm{ln}}(x,t) = C^2_{\mathrm{ln}} + \mathcal{O}([x + t v_{\mathrm{ln}}]^{-2}), \tag{15a}$$

$$C_{\mathrm{ln}} = \frac{g_U}{2\pi\hbar v_{\mathrm{ln}}} \left(1 - \frac{\xi}{\sqrt{1+\xi^2}} \right). \tag{15b}$$

In the limit of large deviation from commensuration, i.e., $\xi \to \infty$, the constant C_{ln} vanishes and the leading behavior of $\mathcal{G}^{(W)}_{\mathrm{ln}}(x,t)$ is given by the standard result in the absence of Umklapp scattering.

Unlike for $\mathcal{G}^{(W)}_{\mathrm{ln}}(x,t)$, there is no simple representation of $\mathcal{G}^{(R)}_{\mathrm{ln}}(x,t)$ and $\mathcal{G}^{(B)}_{\mathrm{ln}}(x,t)$ in terms of fictitious fermions. This spoils the possibility to evaluate these correlation functions exactly. However, applying arguments that are familiar [27] from the study of conventional 1D electron systems, we conjecture

$$\mathcal{G}^{(B)}_{\mathrm{ln}}(x,t) \approx \mathcal{G}^{(R)}_{\mathrm{ln}}(x,t) \approx C_{\mathrm{ln}}. \tag{16}$$

The tunneling density of states $\mathcal{A}^{(\alpha)}(\varepsilon)$ for adding electrons at the branch $\alpha \in \{R, W, B\}$ can be calculated straightforwardly [28] from $\mathcal{G}^{(\alpha)}(0,t)$. We find

$$\mathcal{A}^{(\alpha)}(\varepsilon) \propto \begin{cases} \varepsilon^{[\lambda^{(\alpha)}_{\mathrm{emp}}]^2 + [\lambda^{(\alpha)}_{\mathrm{rn}}]^2 - 1} & \text{for } \varepsilon < \Delta \\ \varepsilon^{[\lambda^{(\alpha)}_{\mathrm{emp}}]^2 + [\lambda^{(\alpha)}_{\mathrm{rn}}]^2 + [\lambda^{(\alpha)}_{\mathrm{ln}}]^2 - 1} & \text{for } \varepsilon > \Delta. \end{cases} \tag{17}$$

We see that Umklapp scattering at reconstructed QH edges leads to a *suppression* of Luttinger-liquid behavior at energies smaller than Δ. This suggests a scenario for the experimental verification of Umklapp scattering. As it is the long range of Coulomb interaction that makes Umklapp scattering possible in the first place, screening by a nearby metallic gate will suppress it. The clearest indication for the presence of Umklapp scattering would be gained in the measurement of the edge-tunneling exponent [16] for various distances λ of the gate. The exponent should have a nonmonotonic dependence on λ, showing a peak for intermediate distances $\ell < \lambda < D$. In contrast, the exponent would be a monotonously

increasing function of λ in the absence of Umklapp scattering.

In conclusion, we have studied low-energy excitations at a reconstructed QH edge and identified an Umklapp process that has not been discussed previously. We solved the theory including Umklapp exactly and evaluated electronic correlation functions. It turns out that Umklapp scattering suppresses Luttinger-liquid behavior.

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Note added.—After submission of this work, I became aware of an independent recent study of chiral sine-Gordon theory [29].

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