Theory of Superradiant Scattering of Laser Light from Bose-Einstein Condensates

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In a recent MIT experiment, a new form of superradiant Rayleigh scattering was observed in Bose-Einstein condensates. We present a detailed theory of this phenomenon in which condensate depletion leads to mode competition, which, together with the directional dependence of the scattering rate, is ultimately responsible for the observed phenomena. The nonlinear response of the system is shown to be highly sensitive to initial quantum fluctuations which cause large run to run variations in the observed superradiant pulses.

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With the recent advent of Bose-Einstein condensation (BEC) in low-density alkali vapors [1,2], a laserlike source of coherent monochromatic atomic matter waves is now readily available. As the electromagnetic vacuum itself provides a nonlinear medium for atomic fields, an atomic BEC is thus an ideal system to study nonlinear wave mixing and related phenomena. Indeed, nonlinear atom optics [3–5] is now an experimental reality with the recent observation of atomic four-wave mixing in condensate systems [6,7]. In addition to wave mixing between atomic matter waves, the ability to generate laserlike atomic fields also raises the possibility to observe direct wave mixing between atomic and optical fields.

In the case where the atoms interact only with far off-resonant optical fields, the dominant atom-photon interaction is two-photon Rayleigh scattering. When the atoms are described as matter waves, Rayleigh scattering is formally equivalent to a cubic nonlinearity, and therefore leads to four-wave mixing between atomic and optical fields. In recent work [8–10] along these lines, the scattering of light by a condensate from a strong pump laser into a weak quantized optical cavity mode was considered. This work was an extension of the collective atomic recoil laser (CARL) [11,12] into the regime of BEC, and focused on exploiting an instability in the light-matter interaction to parametrically amplify atomic and optical waves as well as to optically manipulate matter-wave coherence properties and generate entanglement between atomic and optical fields.

Recent experiments by Ketterle and co-workers at MIT [13], however, have demonstrated that this instability can play an important role also in the case in which laser light is scattered into the vacuum modes of the electromagnetic field. In these experiments, a variation of Dicke superradiance [14] was observed in which the role of electronic coherence, which stores the memory of previous scattering events, is replaced by coherence between center-of-mass momentum states, i.e., interference fringes in the atomic density. In this paper we present a multimode theory of condensate superradiance. Beginning with the elimination of the radiated light field as in the Wigner-Weiskopf theory of spontaneous emission, we derive a linearized model which describes amplification of quantum fluctuations. This is then coupled to a "classical" nonlinear model in which mode competition quenches scattering in all but the direction(s) of maximum gain. The initial quantum fluctuations are shown to strongly influence superradiant pulse formation, and lead to large fluctuations between runs with identical experimental parameters. In the MIT experiments this is clearly demonstrated by the presence of random spots in the angular distribution of the scattered photons.

Our model consists of a Schrödinger field of two-level atoms coupled via the electric-dipole interaction to a far-off resonant pump laser field, as well as to the vacuum modes of the electromagnetic field. The pump laser has frequency ω_0, wave vector $\mathbf{k}_0 = (\omega_0/c)\hat{\mathbf{y}}$, and its polarization is taken along the $\hat{\mathbf{x}}$ axis. Because of the large detuning between the pump frequency and the atomic transition frequency ω_a, we can eliminate the excited state field, and describe the atoms as a scalar field of ground state atoms. This atomic field is self-interacting, due to ground-ground collisions, however, we note that collisions which transfer populations are generally nonresonant and should make only a small contribution to the dynamics. The remaining collisions then simply give a mean field shift to the resonance frequency for quasiparticle excitations. As the effect of these shifts on our model are negligible, at present we include collisions only implicitly in the determination of the condensate wave function.

The effective Hamiltonian which describes the coupling of the atomic and electromagnetic fields is given by

$$\hat{H} = \int d^3\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r}) H_0(\mathbf{r}) \hat{\Psi}(\mathbf{r}) + \int d^3\mathbf{k}\, \hbar\omega(\mathbf{k}) \hat{b}^\dagger(\mathbf{k}) \hat{b}(\mathbf{k}) + \int d^3\mathbf{k}\, d^3\mathbf{r} [\hbar g(\mathbf{k}) \hat{\Psi}^\dagger(\mathbf{r}) \hat{b}^\dagger(\mathbf{k}) e^{i(\mathbf{k}_0 - \mathbf{k})\cdot\mathbf{r}} \hat{\Psi}(\mathbf{r}) + \text{H.c.}], \quad (1)$$

where $\hat{\Psi}(\mathbf{r})$ is the atomic field operator, and $\hat{b}(\mathbf{k})$ is the annihilation operator for a photon in mode $\mathbf{k}$ in the frame rotating at the pump frequency ω_0. The photon energy in

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this frame is given by $\omega(\mathbf{k}) = c|\mathbf{k}| - \omega_0$. The single-atom Hamiltonian is given by $H_0(\mathbf{r}) = -(\hbar^2/2m)\nabla^2 + V(r) + \hbar|\Omega_0|^2/2\Delta$, with $V(\mathbf{r})$ being the trap potential, Ω_0 the pump Rabi frequency, and $\Delta = \omega_0 - \omega_a$ the pump detuning. The Hamiltonian (1) includes only scattering of pump photons, i.e., multiple scatterings between vacuum modes are neglected. The coupling coefficient for Rayleigh scattering between the pump and vacuum modes is

$$g(\mathbf{k}) = \frac{|\Omega_0|}{2|\Delta|}\sqrt{\frac{c|\mathbf{k}|d^2}{2\hbar\epsilon_0(2\pi)^3}}\,|\hat{\mathbf{k}} \times \hat{\mathbf{x}}|, \tag{2}$$

where d is the magnitude of the atomic dipole moment for the transition involved.

The atomic field is initially taken to be a number state in which N atoms occupy the trap ground state $\varphi_0(\mathbf{r})$, which satisfies $[H_0(\mathbf{r}) - \hbar\mu]\varphi_0(\mathbf{r}) = 0$, with μ being the energy of the trap ground state. The effect of atomic recoil during Rayleigh scattering between the pump and the vacuum mode $\mathbf{k}$ is therefore to transfer atoms into the state $\varphi_0(\mathbf{r})\exp[i(\mathbf{k}_0 - \mathbf{k})\cdot\mathbf{r}]$. This suggests to expand the atomic field operator onto quasimodes according to

$$\hat{\Psi}(\mathbf{r},t) = \sum_{\mathbf{q}}\langle\mathbf{r}|\mathbf{q}\rangle e^{-i(\omega_{\mathbf{q}}+\mu)t}\hat{c}_{\mathbf{q}}(t), \tag{3}$$

where $\langle\mathbf{r}\,|\,\mathbf{q}\rangle = \varphi_0(\mathbf{r})\exp(i\mathbf{q}\cdot\mathbf{r})$, and $\omega_{\mathbf{q}} = \hbar|\mathbf{q}|^2/2m$. This is similar to the slowly varying envelope approximation from optical physics, the envelope being given here by $\varphi_0(\mathbf{r})$.

A discrete quantization of the $\mathbf{q}$ values follows from the requirement that the operators $\{\hat{c}_{\mathbf{q}}\}$ obey boson commutation relations $[\hat{c}_{\mathbf{q}}, \hat{c}_{\mathbf{q}'}^\dagger] = \langle\mathbf{q}\,|\,\mathbf{q}'\rangle \approx \delta_{\mathbf{q},\mathbf{q}'}$. Because of the finite size of the ground state wave function $\varphi_0(\mathbf{r})$, this means that $\mathbf{q}$ and $\mathbf{q}'$ must be separated in $\mathbf{k}$ space. Hence, the summation in Eq. (3) is taken to include the condensate mode $\mathbf{q} = 0$ as well as a grid of $\mathbf{q}$ values as closely spaced as is consistent with orthogonality. Clearly, this expansion is not rigorously orthogonal and complete, however, it is sufficient to account for the quantum statistical effects which occur above the critical phase-space density.

An important aspect of BEC superradiance is the generation of families of higher-order side modes due to the scattering of pump photons by the first-order side modes. For the scope of this paper, however, we consider a simplified model containing only the primary Rayleigh scattering process whereby a condensate atom is transferred to a first-order side mode by scattering a pump photon. With this simplification, we insert the expansion (3) into Eq. (1) and arrive at the effective Hamiltonian

$$\begin{aligned}\hat{H} = &\int d^3\mathbf{k}\,\hbar\omega(\mathbf{k})\hat{b}^\dagger(\mathbf{k})\hat{b}(\mathbf{k}) \\ &+ \sum_{\mathbf{q}\neq 0}\int d^3\mathbf{k}\,[\hbar g(\mathbf{k})\rho_{\mathbf{q}}(\mathbf{k})e^{i\omega_{\mathbf{q}}t}\hat{c}_{\mathbf{q}}^\dagger\hat{b}^\dagger(\mathbf{k})\hat{c}_0 + \text{H.c.}],\end{aligned} \tag{4}$$

where $\rho_{\mathbf{q}}(\mathbf{k}) = \int d^3\mathbf{r}|\varphi_0(\mathbf{r})|^2\exp[-i(\mathbf{k} - \mathbf{k}_0 + \mathbf{q})\cdot\mathbf{r}]$ is the Fourier transform of the ground state density distribution centered at $\mathbf{k} = \mathbf{k}_0 - \mathbf{q}$.

From the Hamiltonian (4) it is straightforward to derive the equation of motion for $\hat{b}(\mathbf{k})$, which upon formal integration yields

$$\begin{aligned}\hat{b}(\mathbf{k},t) = &\hat{b}(\mathbf{k},0)e^{-i\omega(\mathbf{k})t} - i\sum_{\mathbf{q}\neq 0} g(\mathbf{k})\rho_{\mathbf{q}}(\mathbf{k})e^{i\omega_{\mathbf{q}}t} \\ &\times\int_0^t d\tau\, e^{-i[\omega(\mathbf{k})+\omega_{\mathbf{q}}]\tau}\hat{c}_{\mathbf{q}}^\dagger(t-\tau)\hat{c}_0(t-\tau),\end{aligned} \tag{5}$$

where the first term gives the free electromagnetic field, i.e., vacuum fluctuations, and the second term is the radiation field due to Rayleigh scattering. A nonzero expectation value of the coherence operator $\hat{c}_{\mathbf{q}}^\dagger\hat{c}_0$ indicates the presence of interference fringes, hence the radiated field increases as fringes build up. *This term therefore leads to an instability where the memory of previous scattering events, stored in the matter-wave interference fringes, enhances the present rate of Rayleigh scattering.*

Equation (5) is then substituted into the equation of motion for $\hat{c}_{\mathbf{q}}$. In the Markov approximation this yields

$$\begin{aligned}\frac{d}{dt}\hat{c}_{\mathbf{q}} = &-i\int d^3\mathbf{k}\,g(\mathbf{k})\rho_{\mathbf{q}}(\mathbf{k})\hat{b}^\dagger(\mathbf{k},0)e^{i[\omega(\mathbf{k})+\omega_{\mathbf{q}}]t}\hat{c}_0 \\ &+\frac{G_{\mathbf{q}}}{2}\hat{c}_0^\dagger\hat{c}_0\hat{c}_{\mathbf{q}},\end{aligned} \tag{6}$$

where

$$G_{\mathbf{q}} = 2\pi\int d^3\mathbf{k}|g(\mathbf{k})|^2|\rho_{\mathbf{q}}(\mathbf{k})|^2\delta[\omega(\mathbf{k}) + \omega_{\mathbf{q}}] \tag{7}$$

is the single-atom gain. In deriving Eq. (6) we have used the approximation $\rho_{\mathbf{q}}^*(\mathbf{k})\rho_{\mathbf{q}'}(\mathbf{k}) \approx |\rho_{\mathbf{q}}(\mathbf{k})|^2\delta_{\mathbf{q},\mathbf{q}'}$, and neglected the principal part which accompanies the δ function.

For a closed atomic system, the total number of atoms is conserved, hence $\hat{c}_0^\dagger\hat{c}_0 = N - \sum_{\mathbf{q}\neq 0}\hat{c}_{\mathbf{q}}^\dagger\hat{c}_{\mathbf{q}}$. For very short times we can therefore take $\hat{c}_0^\dagger\hat{c}_0 \approx N$. In this case Eq. (6) reduces to

$$\frac{d}{dt}\hat{c}_{\mathbf{q}} = \frac{G_{\mathbf{q}}}{2}N\hat{c}_{\mathbf{q}} + \hat{f}_{\mathbf{q}}^\dagger(t). \tag{8}$$

where $\hat{f}_{\mathbf{q}}(t)$ is a noise operator whose correlation functions are given in the Markoff approximation by

$$\begin{aligned}\langle\hat{f}_{\mathbf{q}}^\dagger(t)\hat{f}_{\mathbf{q}}(t')\rangle &= 0, \\ \langle\hat{f}_{\mathbf{q}}(t)\hat{f}_{\mathbf{q}}^\dagger(t')\rangle &= G_{\mathbf{q}}N\delta(t-t').\end{aligned} \tag{9}$$

These noise operators allow the system to be triggered by quantum fluctuations, and hence describe "spontaneous" scattering which occurs in the absence of any side mode population.

Equation (8) can be solved exactly, giving

$$\hat{c}_{\mathbf{q}}(t) = e^{(G_{\mathbf{q}}/2)Nt}\hat{c}_{\mathbf{q}}(0) + \int_0^t d\tau\, e^{(G_{\mathbf{q}}/2)N\tau}\hat{f}_{\mathbf{q}}^{\dagger}(t-\tau). \tag{10}$$

From Eq. (10) it is possible to compute the probability $P_{\mathbf{q}}(n,t)$ of having n atoms in mode $\mathbf{q}$ at time t, assuming zero population at $t=0$. To accomplish this we first compute the antinormally ordered characteristic function, $\chi_{\mathbf{q}}(\eta) = \langle \exp[-\eta^* \hat{c}_{\mathbf{q}}]\exp[\eta \hat{c}_{\mathbf{q}}^{\dagger}]\rangle$, yielding

$$\chi_{\mathbf{q}}(\eta) = e^{-|\eta|^2[\bar{n}_{\mathbf{q}}(t)+1]}, \tag{11}$$

where $\bar{n}_{\mathbf{q}}(t) = \exp(G_{\mathbf{q}}Nt) - 1$ is the mean population of mode $\mathbf{q}$ at time t. We can identify expression (11) as corresponding to a chaotic field [15]. The number distribution for a chaotic field is given by

$$P_{\mathbf{q}}(n,t) = \frac{1}{\bar{n}_{\mathbf{q}}(t)}\left(1 + \frac{1}{\bar{n}_{\mathbf{q}}(t)}\right)^{-(n+1)}, \tag{12}$$

which for $\bar{n}_{\mathbf{q}}(t) \gg 1$ is well approximated by $\exp[-n/\bar{n}_{\mathbf{q}}(t)]/\bar{n}_{\mathbf{q}}(t)$.

When the mean population of a field mode is sufficiently large, correlation functions effectively factorize to all orders, and it becomes possible to formulate a classical description of the field dynamics. In the classical theory, we can consider the side mode populations as c numbers and neglect the influence of the quantum noise operators. With the inclusion of condensate depletion, the side mode populations then obey the coupled equations

$$\frac{d}{dt} n_{\mathbf{q}} = G_{\mathbf{q}}\left(N - \sum_{\mathbf{q}' \neq 0} n_{\mathbf{q}'}\right) n_{\mathbf{q}}. \tag{13}$$

Momentum conservation tells us that for each atom scattered into the side mode $\mathbf{q}$, there is a photon scattered roughly in the direction $\mathbf{k} = \mathbf{k}_0 - \mathbf{q}$. Hence, $N \times dI_{\mathbf{q}}/dt$ is the ideal photon count rate generated by the $|0\rangle \rightarrow |\mathbf{q}\rangle$ atomic center-of-mass transition.

The classical nonlinear model is applicable when $\bar{n}_{\mathbf{q}}(t) \gg 1$, whereas the linearized quantum theory requires $\sum_{\mathbf{q}\neq 0} \bar{n}_{\mathbf{q}}(t) \ll N$. We therefore join them by choosing initial conditions for Eq. (13) from $P_{\mathbf{q}}(n, t_{\mathrm{cl}})$, where t_{cl} satisfies $1 \ll \bar{n}_{\mathbf{q}}(t_{\mathrm{cl}}) \ll N$. Because the response is still linear at time t_{cl} the resulting nonlinear evolution does not depend on the particular choice of t_{cl}.

We now analyze the geometrical dependence of the single-atom gain given by Eq. (7), and show that it is largest for radiation along the long axis of the condensate. We note that $G_{\mathbf{q}}$ depends on $g(\mathbf{k})$, which contains the dipole radiation pattern, as well as on $\rho_{\mathbf{q}}(\mathbf{k})$, which depends on the geometry of the condensate. For a cigar-shaped condensate aligned along the $\hat{\mathbf{z}}$ axis, $\rho_{\mathbf{q}}(\mathbf{k})$ is a disk which lies parallel to the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane in $\mathbf{k}$ space. The dimensions of the disk in $\mathbf{k}$ space are roughly the inverse of the condensate dimensions in $\mathbf{r}$ space. Thus for a condensate whose dimensions are large compared to an optical wavelength, the dimensions of $\rho_{\mathbf{q}}(\mathbf{k})$ are small compared to k_0.

Since $g(\mathbf{k})$ is slowly varying compared to $\rho_{\mathbf{q}}(\mathbf{k})$, it can be removed from the integral in Eq. (7), and evaluated at the center of $\rho_{\mathbf{q}}(\mathbf{k})$. In addition we neglect the recoil shift $\omega_{\mathbf{q}}$ in the δ function as it has negligible effect. The remaining integral then defines the solid angle $\Omega_{\mathbf{q}}$ for the scattered radiation associated with the $\mathbf{q}$th mode according to

$$\Omega_{\mathbf{q}} = \frac{1}{k_0^2}\int d^3\mathbf{k}|\rho_{\mathbf{q}}(\mathbf{k})|^2\delta(|\mathbf{k}| - k_0), \tag{14}$$

which shows that only $\mathbf{q}$ values for which the center of $\rho_{\mathbf{q}}(\mathbf{k})$ lies at a distance k_0 from the origin experience gain, a consequence of energy conservation. Thus for every active quasimode $\mathbf{q}$ there is a radiation direction $\hat{\mathbf{k}}$ such that $\mathbf{q} = k_0(\hat{\mathbf{y}} - \hat{\mathbf{k}})$.

We can estimate for $\Omega_{\mathbf{q}}$ by taking $|\rho_{\mathbf{q}}(\mathbf{k})|^2$ to be an ellipsoid solid with the inverse dimensions of the condensate. This gives

$$\Omega_{\mathbf{q}} = \frac{4\pi}{k_0^2 W^2}[\cos^2\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}} + (L/W)^2\sin^2\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}}]^{-1/2}, \tag{15}$$

where L is the length of the condensate along the $\hat{\mathbf{z}}$ axis, W is the radial diameter, and $\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}}$ is the angle between the radiation direction and the long axis of the condensate. Thus $\Omega_{\mathbf{q}}$ is maximized for $\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}} = 0, \pi$, corresponding to radiation along $\hat{\mathbf{z}}$ and $-\hat{\mathbf{z}}$, where it is given by $\Omega_{\mathbf{q}} = 4\pi/k_0^2W^2$. As $\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}}$ moves away from the $\hat{\mathbf{z}}$ axis, $\Omega_{\mathbf{q}}$ is relatively flat until it reaches the geometric angle W/L, after which it drops off rapidly. It is important to note that for the isotropic case $L = W$ there is no preferred direction, and a ring of radiation is instead observed.

Taking into account all of these considerations, the expression for the single-atom gain becomes

$$G_{\mathbf{q}} = \mathcal{G}\frac{\sin^2\theta_{\hat{\mathbf{k}},\hat{\mathbf{x}}}}{\sqrt{\cos^2\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}} + (L/W)^2\sin^2\theta_{\hat{\mathbf{k}},\hat{\mathbf{z}}}}}, \tag{16}$$

where $\mathcal{G} = 3|\Omega_0|^2\Gamma/8|\Delta|^2k_0^2W^2$ is the maximum single-atom gain, $\Gamma = k_0^3d^2/3\pi\hbar\epsilon_0$ being the single-atom spontaneous decay rate, and $\theta_{\hat{\mathbf{k}},\hat{\mathbf{x}}}$ is the angle between the radiation and polarization directions. For the parameters of the MIT experiment we find $\mathcal{G} \sim 4 \times 10^{-4} \cdot I$, where I is the laser intensity in mW/cm^2, and $\mathcal{G}$ is given in Hz. A rough estimate of the duration of a superradiant pulse for the case $N = 10^6$ and $I = 100$ mW/cm^2 is $t = \ln(N)/\mathcal{G}N \sim 150\ \mu$s, in excellent agreement with experimentally observed time scales.

The interplay between the dependence of $G_{\mathbf{q}}$ on the radiation direction and the nonlinearity in Eq. (13) leads to mode competition, the outcome of which depends sensitively on the initial quantum fluctuations. When modes with different values of $G_{\mathbf{q}}$ compete, the competition is "unfair" and the mode with the largest $G_{\mathbf{q}}$ generally depletes all of the condensate atoms before the populations of the other modes build up. Modes with the same $G_{\mathbf{q}}$, such as the quasimodes corresponding to radiation along the $\hat{\mathbf{z}}$ and $-\hat{\mathbf{z}}$ directions, instead compete "fairly," with results which depend sensitively on the random initial conditions. We note that there is no "winner-takes-all" effect and multiple modes can "fire" simultaneously.

The angular dependence of the scattered light from a typical simulation is shown in Fig. 1, where we have plotted the photon count density as recorded by an ideal detector array located at a distance Z from the center of the condensate along the $\hat{\mathbf{z}}$ axis. The dark regions correspond to maximum light intensity, while the white regions indicate negligible intensity. The center of the figure lies along the symmetry axis of the BEC, and the half-width of the box, given by ZW/L, corresponds to the geometric radiation angle. The simulation was performed for a condensate with $N = 10^6$ atoms in a BEC with a width of 10 μm and a length of 100 μm, in rough agreement with the MIT experiment. From the figure, we see typical results of the interplay between quantum fluctuations and mode competition. The pattern of dark spots indicates multimode superradiance, and exhibits variation on the scale of Z/k_0W, corresponding to the solid angle of radiation for an endfire mode. The pattern, which arises from the amplification of quantum fluctuations, varies randomly from run to run, an effect which has been directly observed experimentally.

In conclusion, we note that the quasimode populations will experience losses as the recoiling atoms eventually propagate out of the condensate volume. The lifetime of the quasimode, however, is on the order of $T_{\mathbf{q}} \equiv mL_{\mathbf{q}}/\hbar|\mathbf{q}|$, where $L_{\mathbf{q}}$ is the length of the condensate along $\mathbf{q}$. These losses tend to destroy the coherence between the condensate and the quasimode, which accounts for the observation of a threshold for superradiance in the MIT experiment: for insufficient laser power, the growth of matter-wave coherence cannot overcome the losses. As this threshold is very small, we have simply neglected it here.

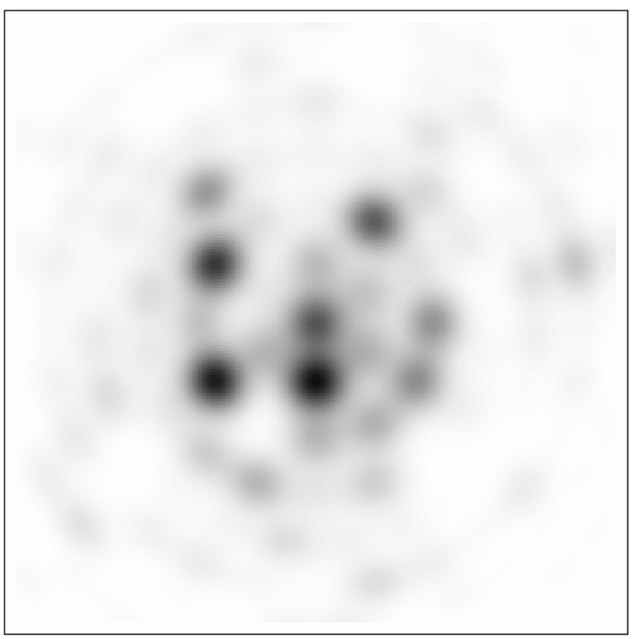

FIG. 1. A typical numerical simulation of condensate superradiance. The scattered photon intensity is plotted as seen by a detector array located at a distance Z along the symmetry axis of the cigar-shaped BEC. The black regions correspond to maximum and the white regions to negligible intensity, respectively. The width of the figure is twice the geometric angle of the condensate.

In general, the rate of matter-wave decoherence in superradiance or CARL-type experiments is given by the ratio between the recoil velocity and the matter-wave coherence length. As the coherence length of a BEC is significantly larger than that of a noncondensed atomic cloud, the threshold for superradiance above T_c is much larger than below T_c, which explains why superradiance was never observed above T_c in the MIT experiment. We remark that in the case of the CARL, the presence of an optical cavity provides additional feedback, which can compensate for the lack of atomic coherence.

Lastly, we remark that the Hamiltonian (1) describes the creation of correlated atom-photon pairs, and is therefore analogous to the optical parametric amplifier, which generates entangled two-photon states for a variety of applications, e.g., tests of Bells inequality, quantum cryptography, and quantum teleportation. It should therefore be possible to perform analogous experiments using entangled atom-photon states generated in a BEC superradiance experiment.

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