

Magnetic Field Induced Localization in a Two-Dimensional Superconducting Wire Network

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We report transport measurements on superconducting wire networks which provide the first experimental evidence of a new localization phenomenon induced by magnetic field on a 2D periodic structure. In the case of a superconducting wave function this phenomenon manifests itself as a depression of the network critical current and of the superconducting transition temperature at a half magnetic flux quantum per tile. In addition, the strong broadening of the resistive transition observed at this field is consistent with enhanced phase fluctuations due to this localization mechanism.

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In a recent paper [1] a novel case of extreme localization induced by a transverse magnetic field was predicted for noninteracting electrons in a two-dimensional (2D) periodic structure. This new phenomenon, due to a subtle interplay between lattice geometry and the magnetic field, differs from Anderson localization on two essential points: it occurs in a pure system, without disorder, and the system eigenstates are nondispersive states. In a tight-binding (TB) approach, it can be simply understood in terms of the Aharonov-Bohm effect which, at half a flux quantum per unit tile (half flux), leads to fully destructive quantum interference. For this flux, the set of sites visited by an initially localized wave packet will be bounded in Aharonov-Bohm cages [1]. This effect is absent on other regular periodic lattices at half flux, such as the square and the triangular lattices.

Superconducting wire networks are suitable to address phase interference phenomena driven by a magnetic field [2]. These systems are extremely sensitive to phase coherence of the superconducting order parameter over the network sites which is exclusively determined by the competition between the external field and the network geometry [3]. Besides, the quantum regime is accessible even in low T_c diffusive superconductors: since all Cooper pairs condense in a quantum state, the relevant wavelength is associated with the macroscopic superfluid velocity and can be much larger than the lattice elementary cell [4]. Also, the magnetic field corresponding to one superconducting flux quantum $\Phi_0 = hc/2e$ is easily accessible: it is about 1 mT for a network cell of $1 \mu\text{m}^2$, in contrast to the unattainable 10^3 T for an atomic lattice. In addition, some features of the TB spectrum, namely, the Hofstadter butterfly [5], are experimentally accessible

in the model system of a superconducting wire network [3,6]. As shown by de Gennes and by Alexander [7,8], the linearized Ginzburg-Landau (GL) equations for a superconducting wire network can be mapped onto the eigenvalues equation of a TB Hamiltonian for the same geometry. This mapping is of particular relevance since one of the remarkable findings of Ref. [1] is the total absence of dispersion in the TB spectrum at half flux. In the context of a superconducting network, the localization effect is expressed by the inability of the superconducting wave function to carry phase information throughout the network and, therefore, transport anomalies are expected.

In this Letter we present transport measurements on 2D superconducting networks with the so-called T_3 geometry (see Fig. 1 inset). Our results allow us to confirm some of the exotic features of the T_3 energy spectrum related to the localization mechanism. The field-temperature (H, T) superconducting transition line is determined and related to the ground state of the T_3 spectrum. We also compare the critical current as a function of the magnetic field with calculations of the group velocity. The striking behavior found at half flux is discussed as a possible signature of localization effects. The strong broadening of the normal to superconductor transition supports this interpretation. Very few experiments were reported so far on localization phenomena in superconducting networks and only the issues of irrational magnetic flux [9] or disorder [10] have been addressed.

The networks pattern was defined on a 600 nm thick layer of positive UV3 resist using an *e-beam* writer Leica VB6-HR. A 100 nm thick layer of pure aluminum was *e-beam* evaporated in a ultrahigh vacuum chamber, followed by the resist *lift-off* [11]. We designed two

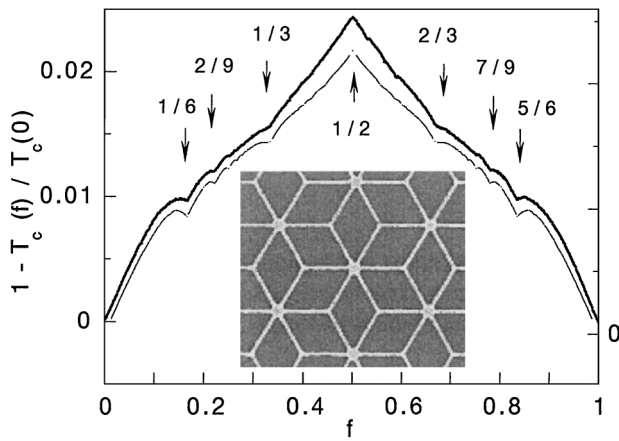


FIG. 1. Reduced critical temperature vs frustration for Star 600 (heavy line, left axis). The theoretical curve has been offset by -0.001 for clarity (small dots, right axis). Inset: Electron micrograph of the T_3 network ($a = 1.0 \mu\text{m}$). Measuring current is applied along the horizontal wires direction.

series with a large patterned area: Star 600 defined on a $0.6 \times 1 \text{ mm}^2$ surface and Star 20 defined on a surface of $0.02 \times 1 \text{ mm}^2$, which required stitching of $200 \times 200 \mu\text{m}^2$ writing fields. The elementary tile side length is $a = 1 \mu\text{m}$, the wires having 100 nm width and 100 nm thickness.

Dynamic resistance measurements were performed using a 33 Hz ac four terminal resistance bridge with an ac measuring current of 20 nA . Sample probes were connected to the cryostat terminals by ultrasonic bonding of $25 \mu\text{m}$ gold wires. Noninvasive voltage probes were placed at 0.2 mm from the current pads. The zero field transition temperatures $T_c(0)$ were 1.234 K for Star 600 and 1.240 K for Star 20, using a resistance criterion of half the normal state resistance R_n at 1.25 K , which are 4.20Ω and 63.56Ω , respectively. The resistive transition width in zero field is 3 mK (10% – 90%) for both samples indicating a good homogeneity of the networks.

The field dependent transition temperature $T_c(H)$ was monitored by locking the temperature controller to keep the sample resistance at $0.5 R_n$ as the magnetic field is varied. The experimental data are to be compared with the lowest energy solution of the network linear GL equations, which is expressed in terms of the ground state eigenvalue $\epsilon_g(f)$ of the TB spectrum [8] by

$$1 - \frac{T_c(f)}{T_c(0)} = \frac{\xi(0)^2}{a^2} \arccos^2\left(\frac{\epsilon_g(f)}{\sqrt{18}}\right), \quad (1)$$

where $\xi(0)$ is the superconducting coherence length at zero temperature and f is the frustration. Neglecting field screening effects, $f = \Phi/\Phi_0$, where $\Phi = Ha^2\sqrt{3}/2$ is the magnetic flux through a rhombus tile.

The transition line of Star 600 is plotted in Fig. 1 in reduced units $1 - T_c(f)/T_c(0)$ as a function of frustration. Since the transition line is periodic on Φ_0 we displayed it only in the field range $0 \leq f \leq 1$ [3]. A small para-

bolic background due to field penetration in the wires was subtracted from the experimental $T_c(f)$. We also display the theoretical $T_c(f)$ obtained using Eq. (1) and $\xi(0) = 157 \text{ nm}$, the only adjustable parameter. The fine field structure of the experimental data is very well described by the theoretical curve. Distinct downward cusps are visible at low order rationals $f = 1/q$, for $q = 3, 4, 6$, and $f = 2/9$. They reflect the long range phase ordering of the order parameter among network sites, established at fields commensurate to the underlying lattice. These features were discussed previously [3,9]. The novel feature of the transition line occurs at $f = 1/2$, where the maximum of $T_c(f)$ depression (30 mK) is achieved, associated with an inversion of the field modulation concavity. This anomalous cusp persists distinctly at all criteria used on $T_c(f)$ determination, from $0.06 R_n$ to $0.87 R_n$, though the downward cusps at other rationals fade out with increasing temperature. This cusp is similar to the T_c variation in a *single* loop geometry close to $f = 1/2$ [12] and is characteristic of quantum effects determined on a finite length scale. It indicates that at half flux the network transition is determined by fluxoid quantization at independent tiles.

The maximum depression of $T_c(f)$ at half flux shows the strong incommensurability at this field. To our knowledge, these results are the first experimental observation of such an effect on an extended periodic network. Besides, they indicate that 2D periodicity is not a sufficient condition for a commensurate state to exist at rational f .

We also observed a strong broadening of the resistive transition ΔT_{width} , at half flux, as displayed in Fig. 2 for Star 600. ΔT_{width} is obtained as the difference between the $T_c(f)$ curves taken for criteria $0.6 R_n$ and $0.1 R_n$, respectively. The anomalous enhancement (up to 12 mK) at half flux, twice the average width over most of the field range, confirms the singular behavior found at this field. At the strong commensurate fields $f = 0, \frac{1}{6}, \frac{1}{3}$, ΔT_{width} is sharply reduced to a few mK as expected for a phase ordered system. Close to these fields, the phase of the order parameter at the network sites is able to “lock” in

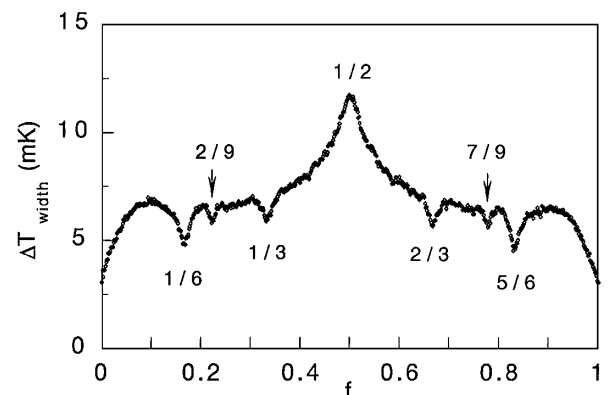


FIG. 2. Transition width vs frustration for Star 600. The large broadening close to $f = 1/2$ indicates the presence of strong phase fluctuations.

the nearest commensurate state with the creation of few mobile defects, broadening slightly the transition. Close to half flux no commensurate state is available; thus phase correlations between network sites cannot be established, leading to a strong broadening of the transition.

In fact, the T_3 tiling geometry can be viewed as an ensemble of three coupled triangular sublattices, two formed by the 3-fold sites and another by the 6-fold sites. The singular properties of the T_3 spectrum $\epsilon(f)$ at frustration f are simply revealed by the transformation

$$\epsilon^2(f) - 6 = 2 \cos(\pi f) \epsilon_T(f_T) \quad (2)$$

that relates $\epsilon(f)$ to the triangular lattice eigenvalues $\epsilon_T(f_T)$ at frustration $f_T = 3f/2$ [13]. At half flux, due to cancellation of the $\cos(\pi f)$ prefactor, all the energy levels collapse into two highly degenerate discrete levels at $\epsilon = \pm\sqrt{6}$, forming flat, nondispersive bands, in addition to the $\epsilon = 0$ flat band. Because of the mapping of the TB problem onto the linearized GL approach, the superfluid velocity can be expressed in terms of the group velocity of the band spectrum close to the ground state. In the context of a superconducting wave function, a nondispersive state cannot carry phase information through the network, contrary to a Bloch state. Therefore, critical current measurements give information on the network ability to sustain a supercurrent, i.e., both a finite order parameter and a finite superfluid velocity.

The critical current was studied as a function of field from the dynamic resistance characteristics vs increasing dc bias current at temperatures close to $T_c(0)$. The used criterion was the threshold current for which the dynamic resistance exceeds 0.2% R_n . Within the sensitivity limits of our measurements, it corresponds to the maximum current that the circuit is able to carry without dissipation. To avoid heating effects due to feeding a large current, we used sample Star 20 with 23 cells (20 μm) width. The critical current density per wire $J_c(T, f)$ is obtained from the network critical current divided by the number of parallel wires (25) and the wire cross section.

Close to T_c , we expect the critical current to follow a 3/2 power law that generalizes the depairing current of a one-dimensional superconducting wire [14] to a superconducting network [15]:

$$J_c(T, f) = J_n C(f) \left(\frac{T_c(f) - T}{T_c(0)} \right)^{3/2}, \quad (3)$$

where J_n is the zero field critical current density at $T = 0$ K. The field dependent coefficient $C(f)$ is derived from the band curvature, $\partial^2 \epsilon_g / \partial k^2$, close to the ground state, $\epsilon_g(f)$, by

$$C^2(f) = - \frac{1}{a^2 \sqrt{18 - \epsilon_g^2}} \frac{\partial^2 \epsilon_g}{\partial k^2} \arccos \frac{\epsilon_g}{\sqrt{18}}. \quad (4)$$

In Fig. 3 is displayed the field dependence of J_c , at $T = 0.96 T_c(0)$ (1.185 K). Sharp peaks are obtained for the same frustrations as the downward cusps observed in

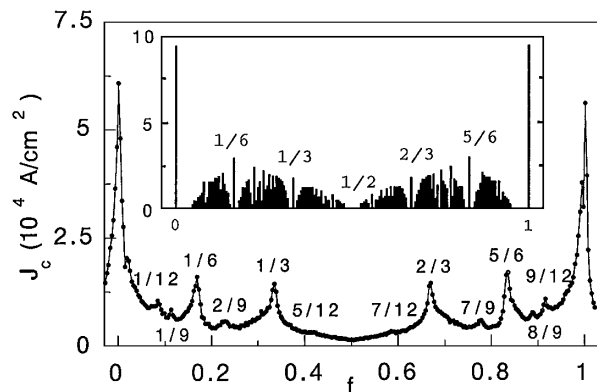


FIG. 3. Critical current density of Star 20 as a function of f at $T = 1.185$ K. The depression at half flux is the signature of the nondispersive state. Inset: Theoretical values at the same temperature.

the transition line. The remarkable finding is the total absence of a peak in the critical current at the lowest order rational $f = 1/2$, exhibiting a clear minimum at this field. For all studied temperatures the critical current was always found to exhibit the lowest values at $f = 1/2$.

In the inset in Fig. 3 is displayed the theoretical J_c obtained using (3) and (4) for $f = p/q$, $q < 50$. We used $T = 0.96 T_c(0)$ and $J_n = 7 \times 10^6 \text{ A cm}^{-2}$, estimated from a 3/2 power law fit of the zero field $J_c(T, 0)$ data. The critical current consists of successive δ functions at rational frustration $f = p/q$ except at $1/2$. The highest values of J_c were obtained for $f = 0, \frac{1}{6}, \frac{1}{3}$ and the symmetric values. For these rational frustrations the spectrum is bandlike and the group velocity is finite (Bloch states). The same applies for other regular periodic lattices such as the square lattice where a checkerboard commensurate state leads to an important peak at $f = 1/2$ [15]. However, in the T_3 case at half flux the group velocity is strictly zero due to the absence of dispersive states and hence the critical current vanishes. This situation is original for an infinite tiling and is due to the special T_3 geometry. A similar effect is found when f approaches irrational frustration (for example, at small frustration $f = 1/q$, with large q): the bandwidth then becomes exponentially small and therefore the group velocity and the critical current are suppressed.

The experimental data follow the same qualitative behavior vs field as the theoretical predictions. The 3/2 power dependence of J_c vs temperature at constant magnetic field was observed in the temperature range $T_c(f) - T < 20$ mK. Namely, the field dependent coefficient $C(f)$ at half flux is reduced to 17% of its zero field value. The $C(f)$ depression, thus of J_c , reflects the effect of the band structure on the superfluid velocity and provides strong evidence of the nondispersive character of the state at $f = 1/2$, although the measured critical current does not vanish.

One possible explanation for the incomplete suppression of J_c is the network's finite size. A current carrying state (an edge state similar to surface superconductivity in finite type-II superconductors) exists along each edge of the finite network and is expected to lead to a non-zero supercurrent. A second possible origin for finite J_c is the influence of the GL nonlinear term which was neglected in Eq. (3). Presumably, the nonlinear terms in the GL formulation are responsible for degrading the fine features of the band structure and therefore give a finite critical current. The critical current observed in Fig. 3 at small frustrations, for example, close to zero, may have the same origin. To go further, an exact solution of the nonlinear GL equations would be needed. Nevertheless, as demonstrated by Abrikosov [16], a good physical insight into the superconducting properties can be obtained from the eigenstates of the linearized GL equation.

This phenomenon suggests interesting properties of the vortex sublattice. In this context, the coupling between network sites can be expressed as a landscape of energy barriers against vortex motion. For example, at $f = 1/3$, a periodic vortex configuration can be easily constructed, matching perfectly the underlying lattice. This configuration is strongly pinned and very stable against driving currents, leading to a large critical current. The decoupling of some network sites at $f = 1/2$ suggests that, in the absence of pinning, the vortex configuration will be highly disordered. Therefore, a significant dissipation is expected for small driving currents, as revealed in our experiments by the suppression of critical current and the anomalous transition broadening. These considerations are supported by preliminary experiments on vortex decoration which indicate a highly disordered vortex distribution at $f = 1/2$ and will be addressed elsewhere.

More subtle is the commensurate state at $f = 1/6$, which corresponds to the $f_T = 1/4$ state of the triangular lattice formed by the 6-fold sites [see Eq. (2)]. As shown in Ref. [17], the uniformly frustrated XY model on a triangular lattice at $f_T = 1/4$ presents an accidental degeneracy of the ground state with zero energy domain walls which can weaken the global phase coherence. In our experiments we do observe a critical current peak at $f = 1/6$, almost as large as at $f = 1/3$. The singular behavior observed experimentally at $f = 1/2$ ($f_T = 3/4$) is completely absent at $f = 1/6$ ($f_T = 1/4$) and, therefore, cannot be simply related to the triangular lattice problem. Besides, it is not clear if the accidental degeneracy persists for a tight-binding coupling.

In summary, the anomalous transport behavior of the T_3 superconducting networks at half flux is consistent with the localization effect predicted in Ref. [1]. The transition line is in excellent agreement with the related T_3 ground state. The broad transition width at half flux indicates

a strong enhancement of phase fluctuations which we assign to destructive quantum interference at this field. The reduction of the critical current at $f = 1/2$, which was never observed so far in periodic superconducting networks, illustrates the inability of the network to sustain a transport current. This behavior is the analog, in the superconductor case, of the metal-insulator transition predicted in Ref. [1].

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