Violation of Time Reversal Invariance in the Decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^- e^+e^-$

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The origin of the large CP-odd and T-odd asymmetry observed in the decay $K_L \to \pi^+\pi^- e^+e^-$ is traced to the polarization properties of the photon in the decay $K_L \to \pi^+\pi^-\gamma$. The Stokes vector of the photon $\vec{S} = (S_1, S_2, S_3)$ is studied as a function of the photon energy and found to possess CP-violating components S_1 and S_2 which are large, despite being proportional to the ϵ parameter of the K_L wave function. The component S_2 is T-even and manifests itself as a circular polarization of the photon, while S_1 is T-odd and gives rise to the asymmetry observed in $K_L \to \pi^+\pi^- e^+e^-$. The latter survives in the limit in which all unitarity phases are absent, and represents a genuine example of time reversal symmetry breaking in a CPT invariant theory.

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The KTeV experiment has reported the observation of a large CP-violating, T-odd asymmetry in the decay $K_L \to \pi^+\pi^- e^+e^-$ [1], in agreement with a theoretical prediction made some years ago [2,3]. In this Letter, we trace the origin of this effect to a large violation of CP invariance and T invariance in the decay $K_L \to \pi^+\pi^-\gamma$, which is hidden in the polarization state of the photon. We explain why the effect is large, despite the fact that it stems entirely from the ϵ impurity of the K_L wave function. Our analysis demonstrates that the T-odd asymmetry does not vanish in the limit in which unitarity phases, expressing the non-Hermiticity of the effective Hamiltonian, are switched off, and thus represents a genuine example of time reversal noninvariance.

The decay $K_L \to \pi^+\pi^-\gamma$ is known empirically [4] to contain a bremsstrahlung component (IB) as well as a direct emission component (DE), with a relative strength DE/(DE + IB) = 0.68 for photons above 20 MeV. By contrast, the decay $K_S \to \pi^+\pi^-\gamma$ is well reproduced by pure bremsstrahlung. The simplest matrix element consistent with these features is [2]

$$\mathcal{M}(K_S \to \pi^+\pi^-\gamma) = e f_S\left[\frac{\epsilon\cdot p_+}{k\cdot p_+} - \frac{\epsilon\cdot p_-}{k\cdot p_-}\right],$$
$$\mathcal{M}(K_L \to \pi^+\pi^-\gamma) = e f_L\left[\frac{\epsilon\cdot p_+}{k\cdot p_+} - \frac{\epsilon\cdot p_-}{k\cdot p_-}\right] + e\,\frac{f_{\rm DE}}{M_K^4}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma\,, \tag{1}$$

where

$$f_L \equiv |f_S| g_{\rm br}\,, \qquad g_{\rm br} = \eta_{+-} e^{i\delta_0(s=M_K^2)},$$
$$f_{\rm DE} \equiv |f_S| g_{M1}\,, \qquad g_{M1} = i(0.76) e^{i\delta_1(s)}. \tag{2}$$

Here the direct emission has been represented by a CP-conserving magnetic dipole coupling g_{M1}, whose magnitude $|g_{M1}| = 0.76$ is fixed by the empirical ratio DE/IB. The phase factors appearing in $g_{\rm br}$ and g_{M1} are dictated by the Low theorem for bremsstrahlung, and the Watson theorem for final state interactions. The factor i in g_{M1} is a consequence of CPT invariance [5]. The matrix element for $K_L \to \pi^+\pi^-\gamma$ contains simultaneously electric multipoles associated with bremsstrahlung $(E1, E3, E5, \ldots)$, which have $CP = +1$, and a magnetic $M1$ multipole with $CP = -1$. It follows that interference of the electric and magnetic emissions should give rise to CP violation.

To determine the nature of this interference, we write the $K_L \to \pi^+\pi^-\gamma$ amplitude more generally as

$$\mathcal{M}(K_L \to \pi^+\pi^-\gamma) = \frac{1}{M_K^3}\{E(\omega,\cos\theta)[\epsilon\cdot p_+ k\cdot p_- - \epsilon\cdot p_- k\cdot p_+] + M(\omega,\cos\theta)\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma\}\,, \tag{3}$$

where ω is the photon energy in the K_L rest frame, and θ is the angle between π^+ and γ in the $\pi^+\pi^-$ rest frame. In the model represented by Eqs. (1) and (2), the electric and magnetic amplitudes are (omitting a common factor $e|f_S|/M_K$)

$$E = \left(\frac{2M_K}{\omega}\right)^2 \frac{g_{\rm br}}{1 - \beta^2\cos^2\theta}\,,$$
$$M = g_{M1}\,, \tag{4}$$

where $\beta = (1 - 4m_\pi^2/s)^{1/2}$, with $\sqrt{s}$ being the $\pi^+\pi^-$ invariant mass. The Dalitz plot density, summed over photon polarizations, is

$$\frac{d\Gamma}{d\omega\, d\cos\theta} = \frac{1}{512\pi^3}\left(\frac{\omega}{M_K}\right)^3 \beta^3\left(1 - \frac{2\omega}{M_K}\right) \times \sin^2\theta[|E|^2 + |M|^2]\,. \tag{5}$$

Clearly, there is no interference between the electric and magnetic multipoles if the photon polarization is unobserved. Therefore, any CP violation involving the interference of $g_{\rm br}$ and g_{M1} is encoded in the polarization state of the photon.

The photon polarization can be defined in terms of the density matrix

$$\rho = \begin{pmatrix} |E|^2 & E^*M \\ EM^* & |M|^2 \end{pmatrix} = \frac{1}{2}(|E|^2 + |M|^2)[\mathbb{1} + \vec{S} \cdot \vec{\tau}], \tag{6}$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ denotes the Pauli matrices, and $\vec{S}$ is the Stokes vector of the photon with components

$$\begin{aligned} S_1 &= 2\,\mathrm{Re}(E^*M)/(|E|^2 + |M|^2), \\ S_2 &= 2\,\mathrm{Im}(E^*M)/(|E|^2 + |M|^2), \\ S_3 &= (|E|^2 - |M|^2)/(|E|^2 + |M|^2). \end{aligned} \tag{7}$$

The component S_3 measures the relative strength of the electric and magnetic radiation at a given point in the Dalitz plot. The effects of CP violation reside in the components S_1 and S_2, which are proportional to $\mathrm{Re}(g^*_{\mathrm{br}} g_{M1})$ and $\mathrm{Im}(g^*_{\mathrm{br}} g_{M1})$, respectively. Physically, S_2 is the net circular polarization of the photon: it is proportional to the difference of $|E - iM|^2$ and $|E + iM|^2$, which are the probabilities for left-handed and right-handed radiation. Such a polarization is a CP-odd, T-even effect, which is known to be possible in decays like $K_L \rightarrow \pi^+\pi^-\gamma$ or $K_{L,S} \rightarrow \gamma\gamma$ whenever there is CP violation accompanied by unitarity phases [5,6]. To understand the significance of S_1, we examine the dependence of the $K_L \rightarrow \pi^+\pi^-\gamma$ decay on the angle ϕ between the polarization vector $\vec{\epsilon}$ and the unit vector $\vec{n}_\pi$ normal to the decay plane [we choose coordinates such that $\vec{k} = (0,0,k)$, $\vec{n}_\pi = (1,0,0)$, $\vec{p}_+ = (0, p\sin\theta, p\cos\theta)$, and $\vec{\epsilon} = (\cos\phi, \sin\phi, 0)$]:

$$\begin{aligned} \frac{d\Gamma}{d\omega\, d\cos\theta\, d\phi} &\sim |E\sin\phi - M\cos\phi|^2, \\ &\sim 1 - [S_3\cos 2\phi + S_1 \sin 2\phi]. \end{aligned} \tag{8}$$

Notice that the Stokes parameter S_1 appears as a coefficient of a term $\sin 2\phi$ which changes sign under CP as well as T. Thus S_1 is a measure of a CP-odd, T-odd correlation [7]. The essential idea of Refs. [2,3] is to use in place of $\vec{\epsilon}$ the vector $\vec{n}_l$ normal to the plane of the Dalitz pair in the reaction $K_L \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-$. This motivates the study of the distribution $d\Gamma/d\phi$ in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$, where ϕ is the angle between the $\pi^+\pi^-$ and e^+e^- planes.

To obtain a quantitative idea of the magnitude of CP violation in $K_L \rightarrow \pi^+\pi^-\gamma$, we show in Fig. 1a the three components of the Stokes vector as a function of the photon energy. These are calculated from the amplitudes (4) using weighted averages of $|E|^2$, $|M|^2$, E^*M, and EM^* over $\cos\theta$ [8]. The values of S_1 and S_2 are remarkably large, considering that the only assumed source of CP violation is the ϵ impurity in the K_L wave function ($\epsilon = \eta_{+-}$). Clearly the factor $(2M_K/\omega)^2$ in E enhances it to a level that makes it comparable to the CP-conserving amplitude M. This is evident from the behavior of the parameter S_3, which swings from a dominant electric behavior at low ω ($S_3 \approx 1$) to a dominant magnetic behavior at large ω

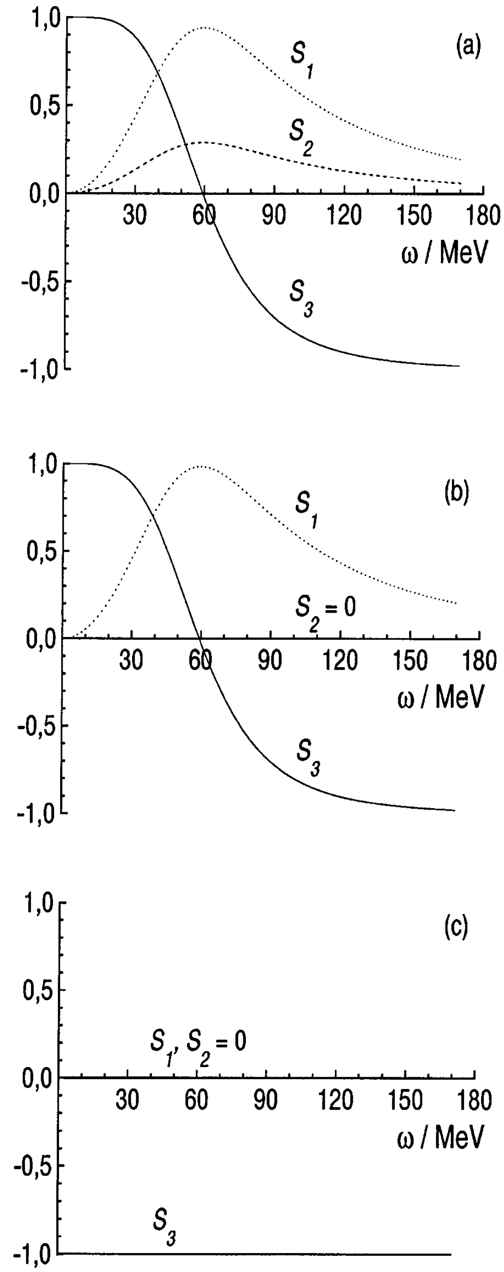

FIG. 1. (a) Stokes parameters of photon in $K_L \rightarrow \pi^+\pi^-\gamma$; (b) Hermitian limit $\delta_0 = \delta_1 = 0$, $\arg\epsilon = \pi/2$; (c) CP-invariant limit $\epsilon \rightarrow 0$.

($S_3 \approx -1$), with a zero in the region $\omega \approx 60$ MeV. The essential difference between the T-odd parameter S_1 and the T-even parameter S_2 comes to light when we compare their behavior in the "Hermitian" limit: this is the limit in which the T matrix or effective Hamiltonian governing the decay $K_L \rightarrow \pi^+\pi^-\gamma$ is taken to be Hermitian, all unitarity phases related to real intermediate states being dropped. This limit is realized by taking δ_0, $\delta_1 \rightarrow 0$, and $\arg\epsilon \rightarrow \pi/2$. The last of these follows from the fact that ϵ may be written as

$$\epsilon = \frac{\Gamma_{12} - \Gamma_{21} + i(M_{12} - M_{21})}{\gamma_S - \gamma_L - 2i(m_L - m_S)}, \tag{9}$$

where $H_{\rm eff} = M - i\Gamma$ is the mass matrix of the K^0-$\overline{K}^0$ system. The Hermitian limit obtains when $\Gamma_{12} = \Gamma_{21} = \gamma_S = \gamma_L = 0$. As seen from Fig. 1b, S_2 vanishes in this limit, but S_1 survives, as befits a CP-odd, T-odd observable. This difference in behavior is obvious from the fact that in the Hermitian limit

$$\begin{aligned} S_1 &\sim \mathrm{Re}(g^*_{\rm br} g_{M1}) \sim \sin(\phi_{+-} + \delta_0 - \delta_1) \to 1\,, \\ S_2 &\sim \mathrm{Im}(g^*_{\rm br} g_{M1}) \sim \cos(\phi_{+-} + \delta_0 - \delta_1) \to 0\,. \end{aligned} \tag{10}$$

Figure 1c shows what happens in the CP-invariant limit $\epsilon \to 0$: the parameters S_1, S_2 collapse to zero, while S_3 attains the uniform value -1. It is clear that we are dealing here with an exceptional situation in which a CP impurity of a few parts in a thousand in the K_L wave function is magnified into a huge CP-odd, T-odd effect in the photon polarization.

We can now examine how these large CP-violating effects are transported to the decay $K_L \to \pi^+\pi^-e^+e^-$. The matrix element for $K_L \to \pi^+\pi^-e^+e^-$ can be written as [2,3]

$$\begin{aligned} \mathcal{M}(K_L \to \pi^+\pi^-e^+e^-) = {} & \mathcal{M}_{\rm br} + \mathcal{M}_{\rm mag} \\ & + \mathcal{M}_{\rm CR} + \mathcal{M}_{\rm SD}\,. \end{aligned} \tag{11}$$

Here $\mathcal{M}_{\rm br}$ and $\mathcal{M}_{\rm mag}$ are the conversion amplitudes associated with the bremsstrahlung and $M1$ parts of the $K_L \to \pi^+\pi^-\gamma$ amplitude. In addition, we have introduced an amplitude $\mathcal{M}_{\rm CR}$ denoting $\pi^+\pi^-$ production in a $J = 0$ state (not possible in a real radiative decay), as well as an amplitude $\mathcal{M}_{\rm SD}$ associated with the short-distance interaction $s \to de^+e^-$. The last of these turns out to be numerically negligible because of the smallness of the Cabibbo-Kobayashi-Maskawa factor $V_{ts}V^*_{td}$. The s-wave amplitude $\mathcal{M}_{\rm CR}$, if approximated by the K^0 charge radius diagram, makes a small ($\sim$1%) contribution to the decay rate. Thus the dominant features of the decay are due to the conversion amplitude $\mathcal{M}_{\rm br} + \mathcal{M}_{\rm mag}$.

Within such a model, one can calculate the differential decay rate in the form [3]

$$\begin{aligned} d\Gamma = {} & I(s_\pi, s_l, \cos\theta_l, \cos\theta_\pi, \phi) \\ & \times ds_\pi\, ds_l\, d\cos\theta_l\, d\cos\theta_\pi\, d\phi\,. \end{aligned} \tag{12}$$

Here s_π (s_l) is the invariant mass of the pion (lepton) pair, and θ_π (θ_l) is the angle of the π^+ (l^+) in the $\pi^+\pi^-$ (l^+l^-) rest frame, relative to the dilepton (dipion) momentum vector in that frame. The all-important variable ϕ is defined in terms of unit vectors constructed from the pion momenta $\vec{p}_\pm$ and lepton momenta $\vec{k}_\pm$ in the K_L rest frame:

$$\begin{aligned} \vec{n}_\pi &= (\vec{p}_+ \times \vec{p}_-)/|\vec{p}_+ \times \vec{p}_-|\,, \\ \vec{n}_l &= (\vec{k}_+ \times \vec{k}_-)/|\vec{k}_+ \times \vec{k}_-|\,, \\ \vec{z} &= (\vec{p}_+ + \vec{p}_-)/|\vec{p}_+ + \vec{p}_-|\,, \\ \sin\phi &= \vec{n}_\pi \times \vec{n}_l \cdot \vec{z}(CP = -, T = -)\,, \\ \cos\phi &= \vec{n}_\pi \cdot \vec{n}_l(CP = +, T = +)\,. \end{aligned} \tag{13}$$

In Ref. [2], an analytic expression was derived for the 3-dimensional distribution $d\Gamma/ds_l\, ds_\pi\, d\phi$, which has been used in the Monte Carlo simulation of this decay. In Ref. [3], a formalism was presented for obtaining the fully differential decay function $I(s_\pi, s_l, \cos\theta_l, \cos\theta_\pi, \phi)$.

The principal results of the theoretical model discussed in [2,3] are as follows:

(1) Branching ratio: This was calculated to be [2]

$$\begin{aligned} B(K_L \to \pi^+\pi^-e^+e^-) &= (1.3 \times 10^{-7})_{\rm br} \\ &\quad + (1.8 \times 10^{-7})_{M1} \\ &\quad + (0.04 \times 10^{-7})_{\rm CR} \\ &\approx 3.1 \times 10^{-7}, \end{aligned} \tag{14}$$

which agrees well with the result $(3.32 \pm 0.14 \pm 0.28) \times 10^{-7}$ measured in the KTeV experiment [1]. (A preliminary branching ratio 2.9×10^{-7} has been reported by the NA48 Collaboration [9].)

(2) Asymmetry in ϕ distribution: The model predicts a distribution of the form

$$\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3 \cos 2\phi + \Sigma_1 \sin 2\phi) \tag{15}$$

which is in complete analogy with the distribution given by Eq. (8) in the case of $K_L \to \pi^+\pi^-\gamma$. The last term is CP and T violating, and produces an asymmetry

$$\mathcal{A} = \frac{(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi})\frac{d\Gamma}{d\phi}d\phi}{(\int_0^{\pi/2} + \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} + \int_{3\pi/2}^{2\pi})\frac{d\Gamma}{d\phi}d\phi} = -\frac{2}{\pi}\Sigma_1\,. \tag{16}$$

The predicted value [2,3] is

$$|\mathcal{A}| = 15\% \sin[\phi_{+-} + \delta_0(M_K^2) - \overline{\delta}_1] \approx 14\% \tag{17}$$

to be compared with the KTeV result [1]

$$|\mathcal{A}|_{\rm KTeV} = (13.6 \pm 2.5 \pm 1.2)\%\,. \tag{18}$$

The parameters Σ_3 and Σ_1 are calculated to be $\Sigma_3 = -0.133$, $\Sigma_1 = 0.23$. The ϕ distribution measured by KTeV agrees with this expectation (after acceptance corrections made in accordance with the model). It should be noted that the sign of Σ_1 (and of the asymmetry $\mathcal{A}$) depends on whether the numerical coefficient in g_{M1} is taken to be $+0.76$ or -0.76. The data support the positive sign chosen in Eq. (2).

(3) Variation of $\Sigma_{1,3}$ with s_π: As shown in Fig. 2, the parameters Σ_1 and Σ_3 have a variation with s_π that is in close correspondence with the variation of S_1 and S_3. (Recall that the photon energy ω in $K_L \to \pi^+\pi^-\gamma$ can be expressed in terms of s_π: $s_\pi = M_K^2 - 2M_K\omega$.) In particular, the zero of Σ_3 and the zero of S_3 occur at almost the same value of s_π. The similarity in the shape of Σ_1 and S_1 confirms the assertion that the asymmetry seen in $K_L \to \pi^+\pi^-e^+e^-$ is related to the CP-odd, T-odd component of the Stokes vector in $K_L \to \pi^+\pi^-\gamma$. The difference in scale is a measure of the analyzing power of the Dalitz pair process, viewed as a probe of the photon polarization.

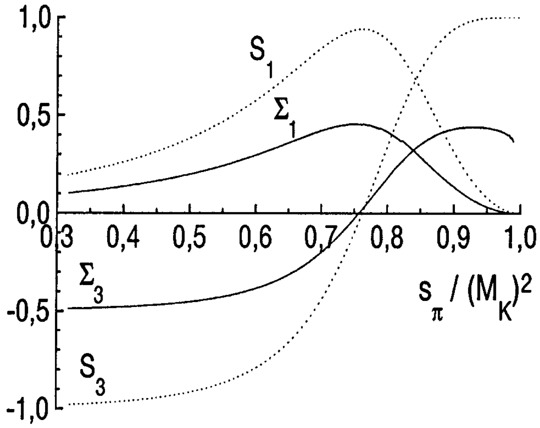

FIG. 2. Parameters Σ_1 and Σ_3 describing the ϕ distribution in $K_L \to \pi^+\pi^- e^+ e^-$, compared with the Stokes parameters S_1 and S_3 in $K_L \to \pi^+\pi^-\gamma$.

Finally, we remark that our analysis takes for granted the validity of CPT invariance in the decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^- e^+ e^-$. If the assumption of CPT invariance is relaxed, the asymmetry observed in the KTeV experiment may be interpreted as some combination of T and CPT violation [10]. From the point of view of the present paper, the effect is understandable in a CPT-invariant framework, and follows inexorably from the empirical features of the decays $K_{L,S} \to \pi^+\pi^-\gamma$ mentioned at the outset.

Some of the ideas of this paper were presented by L. M. S. at the Kaon 99 Conference in Chicago [11].

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[7] The vectors appearing in the $K_L \to \pi^+\pi^-\gamma$ amplitude transform under CP and T as $(\vec{k}, \vec{p}_\pm, \vec{\epsilon}) \xrightarrow{CP} (-\vec{k}, -\vec{p}_\mp, \vec{\epsilon})$, $(\vec{k}, \vec{p}_\pm, \vec{\epsilon}) \xrightarrow{T} (-\vec{k}, -\vec{p}_\pm, -\vec{\epsilon})$. Thus $\sin\phi = \vec{n}_\pi \times \vec{\epsilon} \cdot \hat{k}$ has $CP = +$, $T = +$, while $\cos\phi = \vec{n}_\pi \cdot \vec{\epsilon}$ has $CP = -$, $T = -$.

[8] In our numerical work we take $\phi_{+-} = 43°$, $\delta_0(M_K^2) = 40°$, and an average value $\overline{\delta}_1 = 10°$. We have also carried out a calculation which includes the s dependence of $\delta_1(s)$, as well as the measured form factor $g_{M1}(s)$ [1,4] in place of the constant value $g_{M1} = 0.76$. The curves in Fig. 1a are mildly affected, the zero in S_3 and the maximum in S_1, S_2 shifting to about 50 MeV.

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