Modulus Stabilization with Bulk Fields

Walter D. Goldberger* and Mark B. Wise†
California Institute of Technology, Pasadena, California 91125
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We propose a mechanism for stabilizing the size of the extra dimension in the Randall-Sundrum scenario. The potential for the modulus field that sets the size of the fifth dimension is generated by a bulk scalar with quartic interactions localized on the two 3-branes. The minimum of this potential yields a compactification scale that solves the hierarchy problem without fine-tuning of parameters.

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The standard model for strong, weak, and electromagnetic interactions based on the gauge group SU(3) × SU(2) × U(1) has been extremely successful in accounting for experimental observations. However, it has several unattractive features that suggest new physics beyond that incorporated in this model. One of these is the gauge hierarchy problem, which refers to the vast disparity between the weak scale and the Planck scale. In the context of the minimal standard model, this hierarchy of scales is unnatural since it requires a fine-tuning order by order in perturbation theory. A number of extensions have been proposed to solve the hierarchy problem, notably technicolor [1] (or dynamical symmetry breaking) and low energy supersymmetry [2].

Recently, it has been suggested that large compactified extra dimensions may provide an alternative solution to the hierarchy problem [3]. In these models, the observed Planck mass $M_{\rm Pl}$ is related to M, the fundamental mass scale of the theory, by $M_{\rm Pl}^2 = M^{n+2}V_n$, where V_n is the volume of the additional compactified dimensions. If V_n is large enough, M can be of the order of the weak scale. Unfortunately, unless there are several large extra dimensions, a new hierarchy is introduced between the compactification scale, $\mu_c = V_n^{-1/n}$, and M.

Randall and Sundrum [4] have proposed a higher dimensional scenario to solve the hierarchy problem that does not require large extra dimensions. This model consists of a spacetime with a single S^1/Z_2 orbifold extra dimension. Three-branes with opposite tensions reside at the orbifold fixed points and together with a finely tuned cosmological constant serve as sources for five-dimensional gravity. The resulting spacetime metric contains a redshift factor which depends exponentially on the radius r_c of the compactified dimension:

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \qquad (1)$$

where k is a parameter which is assumed to be of order M, x^μ are Lorentz coordinates on the four-dimensional surfaces of constant ϕ, and $-\pi \le \phi \le \pi$ with (x,ϕ) and $(x,-\phi)$ identified. The two 3-branes are located at $\phi = 0$ and $\phi = \pi$. A similar scenario to the one described in Ref. [4] is that of Horava and Witten [5], which arises within the context of M theory. Supergravity solutions similar to Eq. (1) are presented in Ref. [6]. In Ref. [7], it is shown how this model may be obtained from string theory compactifications.

The nonfactorizable geometry of Eq. (1) has several important consequences. For instance, the four-dimensional Planck mass is given in terms of the fundamental scale M by

$$M_{\rm Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}], \qquad (2)$$

so that, even for large kr_c, $M_{\rm Pl}$ is of order M. Because of the exponential factor in the spacetime metric, a field confined to the 3-brane at $\phi = \pi$ with mass parameter m_0 will have physical mass $m_0 e^{-kr_c\pi}$ and for kr_c around 12, the weak scale is dynamically generated from a fundamental scale M which is on the order of the Planck mass. Furthermore, Kaluza-Klein gravitational modes have TeV scale mass splittings and couplings [8]. Similarly, a bulk field with mass on the order of M has low-lying Kaluza-Klein excitations that reside primarily near $\phi = \pi$ and hence, from a four-dimensional perspective, have masses on the order of the weak scale [9].

In the scenario presented in Ref. [4], r_c is associated with the vacuum expectation value of a massless four-dimensional scalar field. This modulus field has zero potential and consequently r_c is not determined by the dynamics of the model. For this scenario to be relevant, it is necessary to find a mechanism for generating a potential to stabilize the value of r_c. Here we show that such a potential can arise classically from the presence of a bulk scalar with interaction terms that are localized to the two 3-branes [10]. The minimum of this potential can be arranged to yield a value of $kr_c \sim 10$ without fine-tuning of parameters.

Imagine adding to the model a scalar field Φ with the following bulk action:

$$S_b = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi\, \sqrt{G}\,(G^{AB}\partial_A\Phi\,\partial_B\Phi - m^2\Phi^2), \qquad (3)$$

where G_{AB} with $A, B = \mu, \phi$ is given by Eq. (1). We also include interaction terms on the hidden and visible

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branes (at $\phi = 0$ and $\phi = \pi$, respectively) given by

$$S_h = -\int d^4x \sqrt{-g_h}\, \lambda_h(\Phi^2 - v_h^2)^2, \tag{4}$$

and

$$S_v = -\int d^4x \sqrt{-g_v}\, \lambda_v(\Phi^2 - v_v^2)^2, \tag{5}$$

where g_h and g_v are the determinants of the induced metric on the hidden and visible branes, respectively. Note that Φ and $v_{v,h}$ have mass dimension $3/2$, while $\lambda_{v,h}$ have mass dimension -2. Kinetic terms for the scalar field can be added to the brane actions without changing our results. The terms on the branes cause Φ to develop a ϕ-dependent vacuum expectation value $\Phi(\phi)$ which is determined classically by solving the differential equation

$$\begin{aligned} 0 = &-\frac{1}{r_c^2}\,\partial_\phi(e^{-4\sigma}\partial_\phi\Phi) + m^2 e^{-4\sigma}\Phi \\ &+ 4e^{-4\sigma}\lambda_v\Phi(\Phi^2 - v_v^2)\,\frac{\delta(\phi - \pi)}{r_c} \\ &+ 4e^{-4\sigma}\lambda_h\Phi(\Phi^2 - v_h^2)\,\frac{\delta(\phi)}{r_c}, \end{aligned} \tag{6}$$

where $\sigma(\phi) = kr_c|\phi|$. Away from the boundaries at $\phi = 0, \pi$, this equation has the general solution

$$\Phi(\phi) = e^{2\sigma}[Ae^{\nu\sigma} + Be^{-\nu\sigma}], \tag{7}$$

with $\nu = \sqrt{4 + m^2/k^2}$. Putting this solution back into the scalar field action and integrating over ϕ yields an effective four-dimensional potential for r_c which has the form

$$\begin{aligned} V_\Phi(r_c) = &\,k(\nu + 2)A^2(e^{2\nu kr_c\pi} - 1) \\ &+ k(\nu - 2)B^2(1 - e^{-2\nu kr_c\pi}) \\ &+ \lambda_v e^{-4kr_c\pi}[\Phi(\pi)^2 - v_v^2]^2 + \lambda_h[\Phi(0)^2 - v_h^2]^2. \end{aligned} \tag{8}$$

The unknown coefficients A and B are determined by imposing appropriate boundary conditions on the 3-branes. We obtain these boundary conditions by inserting Eq. (7) into the equations of motion and matching the delta functions:

$$k[(2 + \nu)A + (2 - \nu)B] - 2\lambda_h\Phi(0)[\Phi(0)^2 - v_h^2] = 0, \tag{9}$$

$$\begin{aligned} ke^{2kr_c\pi}[(2 + \nu)e^{\nu kr_c\pi}A + (2 - \nu)e^{-\nu kr_c\pi}B] + \\ 2\lambda_v\Phi(\pi)[\Phi(\pi)^2 - v_v^2] = 0. \end{aligned} \tag{10}$$

Rather than solve these equations in general, we consider the simplified case in which the parameters λ_h and λ_v are large. It is evident from Eq. (8) that in this limit, it is energetically favorable to have $\Phi(0) = v_h$ and $\Phi(\pi) = v_v$ [11]. Thus, from Eq. (7) we get for large kr_c

$$A = v_v e^{-(2+\nu)kr_c\pi} - v_h e^{-2\nu kr_c\pi}, \tag{11}$$

$$B = v_h(1 + e^{-2\nu kr_c\pi}) - v_v e^{-(2+\nu)kr_c\pi}, \tag{12}$$

where subleading powers of $\exp(-kr_c\pi)$ have been neglected. Now suppose that $m/k \ll 1$ so that $\nu = 2 + \epsilon$, with $\epsilon \simeq m^2/4k^2$ a small quantity. In the large kr_c limit, the potential becomes

$$\begin{aligned} V_\Phi(r_c) = &\,k\epsilon v_h^2 + 4ke^{-4kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})^2\left(1 + \frac{\epsilon}{4}\right) \\ &- k\epsilon v_h e^{-(4+\epsilon)kr_c\pi}(2v_v - v_h e^{-\epsilon kr_c\pi}), \end{aligned} \tag{13}$$

where terms of order ϵ^2 are neglected (but ϵkr_c is not treated as small). Ignoring terms proportional to ϵ, this potential has a minimum at

$$kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]. \tag{14}$$

With $\ln(v_h/v_v)$ of order unity, we need only m^2/k^2 of order $1/10$ to get $kr_c \sim 10$. Clearly, no extreme fine-tuning of parameters is required to get the right magnitude for kr_c. For example, taking $v_h/v_v = 1.5$ and $m/k = 0.2$ yields $kr_c \simeq 12$.

The stress tensor for the scalar field can be written as $T_s^{AB} = T_k^{AB} + T_m^{AB}$, where for large kr_c

$$\begin{aligned} T_k^{\phi\phi} \simeq &-\frac{k^2}{2r_c^2}[(4 + \epsilon)(v_v - v_h e^{-\epsilon kr_c\pi}) \\ &\times e^{-(4+\epsilon)(kr_c\pi-\sigma)} - \epsilon v_h e^{-\epsilon\sigma}]^2, \end{aligned} \tag{15}$$

$$\begin{aligned} T_k^{\mu\nu} \simeq &\,\frac{k^2}{2}\,e^{2\sigma}\eta^{\mu\nu}[(4 + \epsilon)(v_v - v_h e^{-\epsilon kr_c\pi}) \\ &\times e^{-(4+\epsilon)(kr_c\pi-\sigma)} - \epsilon v_h e^{-\epsilon\sigma}]^2, \end{aligned} \tag{16}$$

and

$$\begin{aligned} T_m^{\phi\phi} \simeq &-\frac{2k^2\epsilon}{r_c^2}[(v_v - v_h e^{-\epsilon kr_c\pi})e^{-(4+\epsilon)(kr_c\pi-\sigma)} \\ &+ v_h e^{-\epsilon\sigma}]^2, \end{aligned} \tag{17}$$

$$\begin{aligned} T_m^{\mu\nu} \simeq &-2k^2e^{2\sigma}\eta^{\mu\nu}\epsilon[(v_v - v_h e^{-\epsilon kr_c\pi})e^{-(4+\epsilon)(kr_c\pi-\sigma)} \\ &+ v_h e^{-\epsilon\sigma}]^2. \end{aligned} \tag{18}$$

As long as v_h^2/M^3 and v_v^2/M^3 are small, T_s^{AB} can be neglected in comparison to the stress tensor induced by the bulk cosmological constant. It is therefore safe to ignore the influence of the scalar field on the background geometry for the computation of $V(r_c)$. A similar criterion ensures that the stress tensor induced by the bulk cosmological constant is dominant for $kr_c \sim 1$.

One might worry that the validity of Eqs. (13) and (14) requires unnaturally large values of λ_h and λ_v. We will check that this is not the case by computing the leading $1/\lambda$ correction to the potential. To obtain this correction, we linearize Eqs. (9) and (10) about the large λ solution. Neglecting terms of order ϵ, the vacuum expectation values (VEVs) are shifted by

$$\delta\Phi(0) = \frac{k}{\lambda_h v_h^2} e^{-(4+\epsilon)kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi}), \quad (19)$$

$$\delta\Phi(\pi) = -\frac{k}{\lambda_v v_v^2}(v_v - v_h e^{-\epsilon kr_c\pi}), \quad (20)$$

and thus (neglecting subleading exponentials of $kr_c\pi$)

$$\delta A = -\frac{k}{\lambda_v v_v^2} e^{-(4+\epsilon)kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi}), \quad (21)$$

$$\delta B = e^{-(4+\epsilon)kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})\left[\frac{k}{\lambda_v v_v^2} + \frac{k}{\lambda_h v_h^2}\right]. \quad (22)$$

Hence, the correction to the potential is

$$\delta V_\Phi(r_c) = -\frac{4k^2}{\lambda_v v_v^2} e^{-4kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})^2. \quad (23)$$

This has the same form as the leading $\epsilon \to 0$ behavior of Eq. (13) and therefore does not significantly affect the location of the minimum.

Note that the forms of the potentials in Eqs. (13) and (23) are valid only for large kr_c. For small kr_c, the potential becomes

$$V_\Phi(r_c) = \frac{(v_v - v_h)^2}{\pi r_c} \quad (24)$$

when terms of order ϵ and $1/\lambda$ are neglected. The singularity as $r_c \to 0$ is removed by finite λ corrections which become large for small r_c, and yield

$$V_\Phi(0) = \frac{\lambda_h \lambda_v}{\lambda_h + \lambda_v}(v_v^2 - v_h^2)^2. \quad (25)$$

In the scenario of Randall and Sundrum, the action is the sum of the five-dimensional Einstein-Hilbert action plus world-volume actions for the 3-branes:

$$S = \int d^4x\, d\phi\, \sqrt{G}\,[-\Lambda + 2M^3 R] - \int d^4x \sqrt{-g_h}\, V_h - \int d^4x \sqrt{-g_v}\, V_v\,. \quad (26)$$

For Eq. (1) to be a solution of the field equations that follow from Eq. (26), one must arrange $V_h = -V_v = 24M^3k$, where $\Lambda = -24M^3k^2$. This amounts to having a vanishing four-dimensional cosmological constant plus an additional fine-tuning which causes the r_c potential to vanish. However, imagine perturbing the 3-brane tensions by small amounts [12]:

$$V_h \to V_h + \delta V_h\,, \quad (27)$$

$$V_v \to V_v + \delta V_v\,. \quad (28)$$

As long as $|\delta V_h|$ and $|\delta V_v|$ are small compared to $-\Lambda/k$, these shifts in the brane tensions induce the following potential for r_c:

$$V_\Lambda(r_c) = \delta V_h + \delta V_v e^{-4kr_c\pi}. \quad (29)$$

For δV_v small, the sum of potentials $V_\Phi(r_c) + V_\Lambda(r_c)$ has a minimum for large kr_c. The effective four-dimensional cosmological constant can be tuned to zero by adjusting the value of δV_h. For $\delta V_v < k\epsilon v_v^2$ the minimum is global, while for $\delta V_v > k\epsilon v_v^2$ the minimum is a false vacuum since $r_c \to \infty$ is a configuration of lower energy.

We have seen that a bulk scalar with a ϕ-dependent VEV can generate a potential to stabilize r_c without having to fine-tune the parameters of the model (there is still one fine-tuning associated with the four-dimensional cosmological constant, however). This mechanism for stabilizing r_c is a reasonably generic effect caused by the presence of a ϕ-dependent vacuum bulk field configuration. It may be worthwhile to work out other features of the specific toy model presented here, such as the back reaction of the scalar field and shifts of the brane tensions on the spacetime geometry. Also, the use of the large λ limit was purely for convenience and the finite λ case could be considered.

With r_c stabilized, effective field theory reasoning suggests that the cosmology associated with this scenario should be standard for temperatures below the weak scale. However, for temperatures above this scale, it will be different (see Ref. [13]) from the usual Friedmann cosmology.

The scenario presented in Ref. [4] represents an attractive solution to the hierarchy puzzle. However, it also has some features that are less appealing than the standard model with minimal particle content. In this scenario, higher dimension operators are suppressed by the weak scale and unlike the standard model, where the suppression can be by powers of the grand unified theory scale, there is no explanation for the smallness of neutrino masses and the long proton lifetime based simply on dimensional analysis.

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*Email address: walter@theory.caltech.edu
†Email address: wise@theory.caltech.edu

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